String theory on AdS_3 with Ramond-Ramond fluxes

Raphael Benichou
VUB

Outline

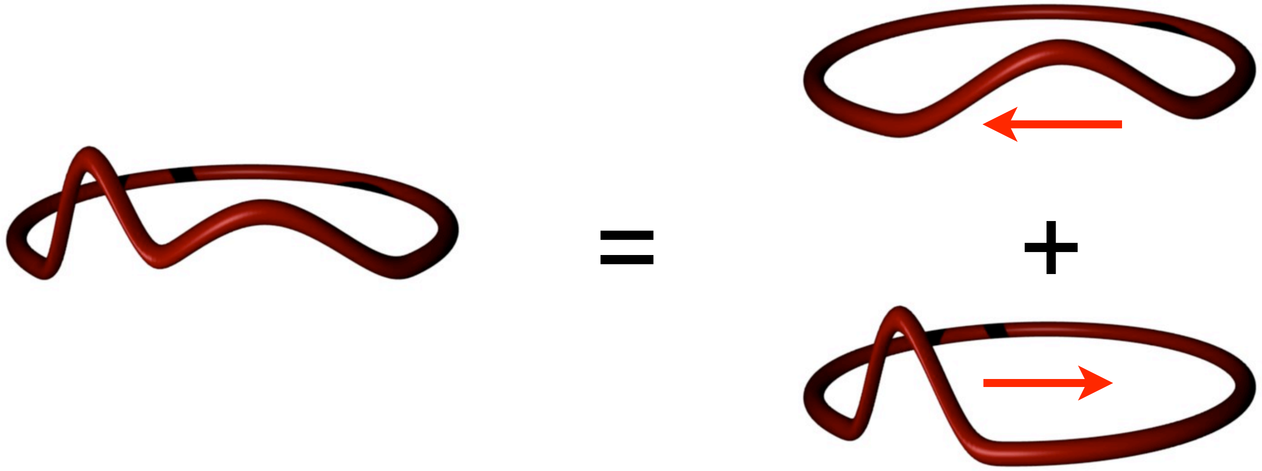
1. **Introduction & motivations**
   → String quantization in spacetime with RR fluxes
   → $AdS_3 \times S^3 \times T^4$
2. **Quantum symmetries of string theory in $AdS_3 \times S^3 \times T^4$**
   → Worldsheet
   → Spacetime

Part one

→ String quantization in spacetime with RR fluxes

→ $AdS_3 \times S^3 \times T^4$

Any excitation of a closed string is the sum of a left-moving and a right-moving excitations.



If both excitations are bosonic, we get a bosonic mode of the superstring.

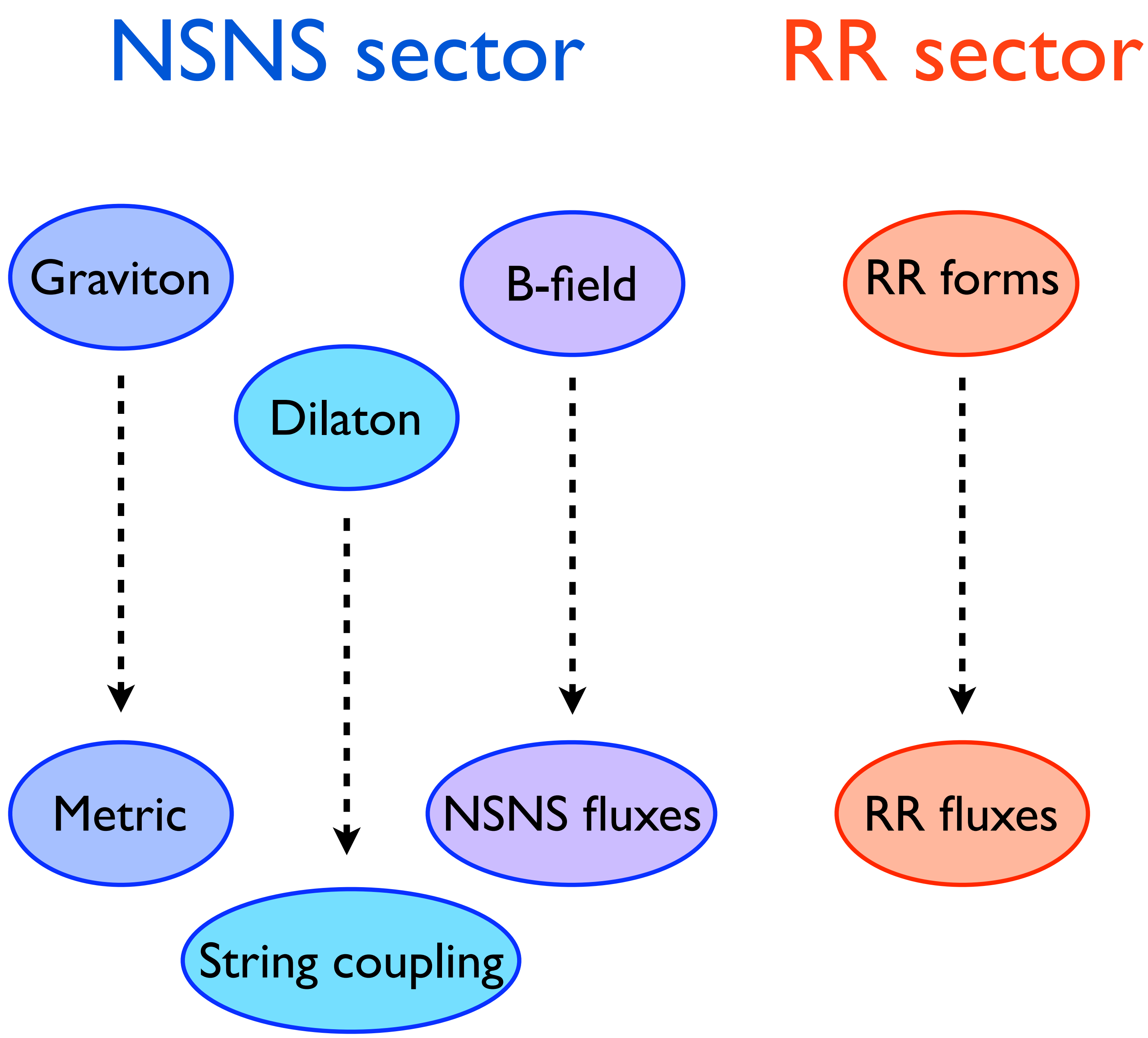
NSNS sector

If both excitations are fermionic, we also get a bosonic mode of the superstring.

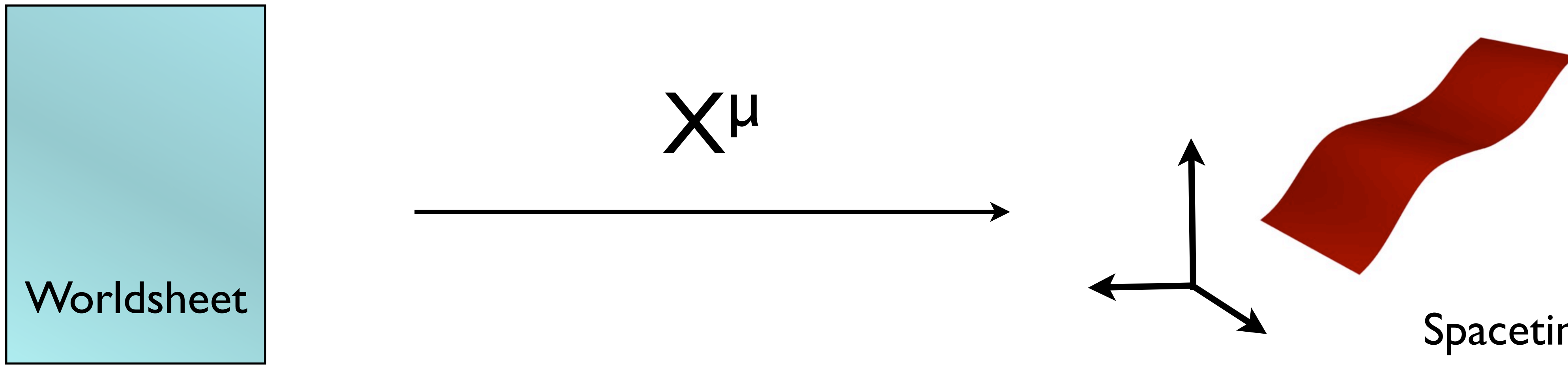
RR sector

The massless bosonic modes of the string are typically :

They can take non-zero expectation value (satisfying generalized Einstein equations).



String quantization : the usual formalism



We consider the quantum theory of maps from the worlsheet to the target space.

The expectation value of the background fields appears as couplings for the fields X^μ :

$$S = \frac{1}{2\pi\alpha'}\int d^2 z \frac{G_{\mu\nu}(X) + B_{\mu\nu}(X)}{2} \partial X^\mu \bar{\partial} X^\nu + \dots \qquad P^{\alpha\beta}(X)$$

NSNS fields

Spacetime coordinates

Problem : how to couple the fields from the RR sector ?

String Theory Landscape

Supergravity
(Curvature < String scale)

?

RNS Formalism
(No RR fluxes)

String quantization in RR backgrounds

An idea is to embed the physical spacetime into a superspace.

Worldsheet fields : X^μ θ^α, p_α

X^μ: Bosonic coordinate

θ^α: Fermionic coordinate

p_α: Conjugate momentum

Green & Schwarz

Berkovits et al.

Then we can write a worldsheet coupling to the RR fields :

$$S = \frac{1}{2\pi\alpha'}\int d^2z \left[\frac{G_{MN}(Z) + B_{MN}(Z)}{2}\partial Z^M \bar{\partial} Z^N + P^{\alpha\beta}(Z) p_\alpha p_\beta + \ldots\right]$$

$G_{MN}(Z) + B_{MN}(Z)$: NSNS fields

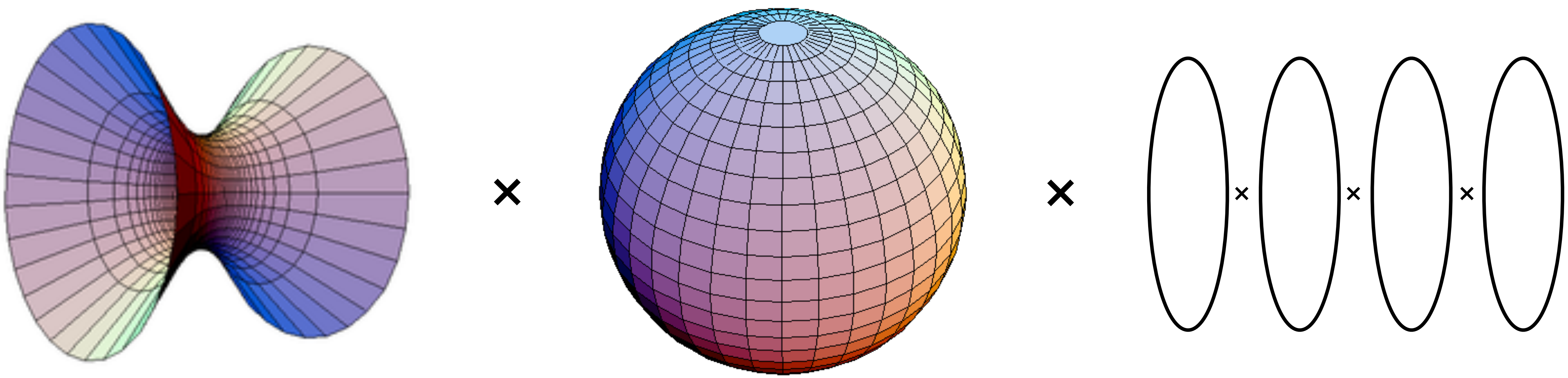
Z^M: Coordinates on superspace

$P^{\alpha\beta}(Z)$: RR fluxes

But no concrete example has been worked out yet.

We would like to quantize the string in a spacetime with RR fluxes.

A good candidate is :



$$AdS_3 \times S^3 \times T^4$$

The spacetime $AdS_3 \times S^3 \times T^4$ has a lot of symmetries :

AdS_3 → SU(1,1)

S^3 → SU(2)

16 supercharges

The symmetry algebra of string theory in this spacetime is even bigger.

↓

N=4 SuperConformal Algebra

For a theory of gravity in AdS_3 : SU(1,1)

↓

Virasoro

Brown & Henneaux, 1986

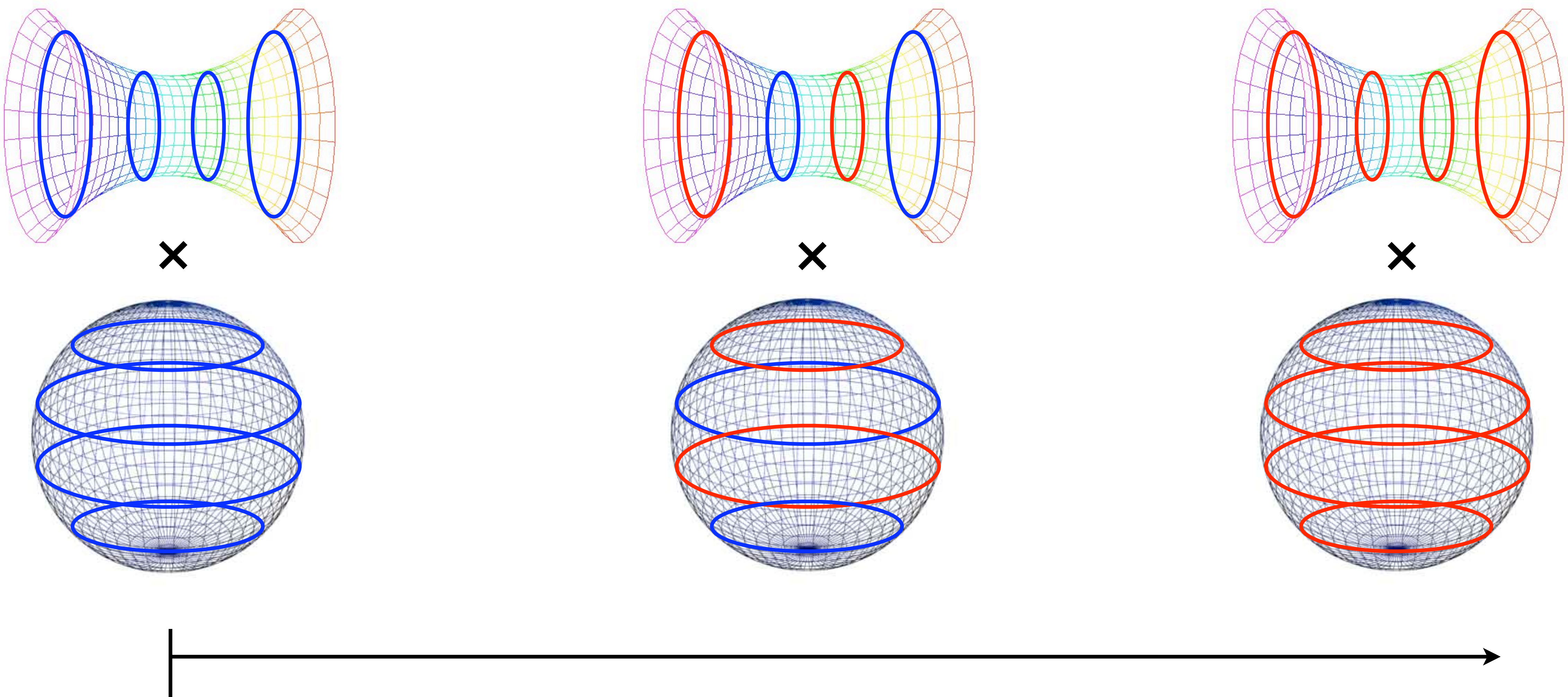
➛ Another motivation is the investigation of the AdS/CFT correspondence beyond the supergravity approximation.

$AdS_3 \times S^3$ can be supported by NSNS and RR fluxes :

NSNS fluxes only

Both NSNS and RR fluxes

RR fluxes only



We know how to quantize the string here !

Maldacena & Ooguri, 2000

Part 2

String theory on $AdS_3 \times S^3 \times T^4$:

→ Symmetry algebra of the worldsheet theory

→ Generators of spacetime symmetry algebra

First we have to embed $AdS_3 \times S^3$ in a superspace.

We choose a superspace with 8 supercharges so that $AdS_3 \times S^3$ can be embedded in a supergroup manifold.

$$AdS_3 \times S^3 \subset PSU(1,1|2)$$

$$g \in PSU(1,1|2) \longrightarrow g = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

Bosonic: A, D

Fermionic: B, C

$$B = C = 0 \Longrightarrow A \in SU(1,1), \; D \in SU(2)$$

The relevant formalism to quantize the string in this superspace is the six-dimensional hybrid formalism.

Berkovits & Vafa, 1994

Berkovits, Vafa & Witten, 1999

We consider the 2D non-linear sigma model on the supergroup manifold of PSU(1,1|2).

It is a Conformal Field Theory with two parameters :

- ➛ NSNS fluxes
- ➛ RR fluxes

Bershadsky, Zhukov & Vaintrob, 1999

Berkovits, Vafa & Witten, 1999

The global symmetry group is $PSU(1,1|2)_L \times PSU(1,1|2)_R$.

$$g \longrightarrow hg \qquad g \longrightarrow gh$$

We will study the currents associated to this symmetry.

$$\begin{pmatrix} j_{L,z} \\ j_{L,\bar{z}} \end{pmatrix} \qquad \begin{pmatrix} j_{R,z} \\ j_{R,\bar{z}} \end{pmatrix}$$

In the case where there are no RR fluxes : the sigma model is a Wess-Zumino-Witten model.

Gotz, Quella & Schomerus, 2006

The currents simplify :

$$\begin{pmatrix} j_{L,z} \\ j_{L,\bar{z}} = 0 \end{pmatrix} \quad \begin{pmatrix} j_{R,z} = 0 \\ j_{R,\bar{z}} \end{pmatrix}$$

They satisfy :

$$j^a_{L,z}(z) j^b_{L,z}(w) \approx \frac{k\kappa^{ab}}{(z-w)^2} + i f^{ab}{}_c \frac{j^c_{L,z}(w)}{z-w} + \ldots$$

$$j_{L,z} = j^a_{L,z} t_a$$

Generators of the Lie algebra

The current algebra allows for the complete solution of the model :

➢ The spectrum is generated by the currents.

➢ The stress-tensor can be written in terms of the currents. As a consequence correlation functions can be computed thanks to a first-order differential equation.

Knizhnik & Zamolodchikov, 1984

$$j^a_{L,z}(z) j^b_{L,z}(w) \approx \frac{k\kappa^{ab}}{(z-w)^2} + i f^{ab}{}_c \frac{j^c_{L,z}(w)}{z-w} + \ldots$$

RR fluxes = 0

When we turn on RR fluxes, the current algebra becomes :

RR fluxes $\neq$ 0

$$j^a_z(z) j^b_w(w) \approx \frac{(1+kf^2)^2 f^2 \kappa^{ab}}{(z-w)^2} + f^2(1+kf^2)(\frac{3}{2} - \frac{kf^2}{2}) i f^{ab}{}_c \frac{j^c(w)}{z-w} + f^2(1+kf^2)^2 \frac{1}{2} i f^{ab}{}_c \frac{\bar{z}-\bar{w}}{(z-w)^2} j^c_{\bar{w}}(w) + \ldots$$

$$j^a_{\bar{z}}(z) j^b_{\bar{w}}(w) \approx \frac{f^2(1-kf^2)^2 \kappa^{ab}}{(\bar{z}-\bar{w})^2} + f^2(1-kf^2)(\frac{3}{2} + \frac{kf^2}{2}) i f^{ab}{}_c \frac{j^c_{\bar{w}}(w)}{\bar{z}-\bar{w}} + f^2 \frac{1}{2}(1-kf^2)^2 i f^{ab}{}_c \frac{z-w}{(\bar{z}-\bar{w})^2} j^c(w) + \ldots.$$

$$j^a_z(z) j^b_{\bar{w}}(w) \approx -2\pi f^2(1+kf^2)(1-kf^2)\kappa^{ab}\delta^{(2)}(z-w) + f^2\frac{1}{2}(1+kf^2)^2 i f^{ab}{}_c \frac{j^c_{\bar{w}}(w)}{z-w} + f^2\frac{1}{2}(1-kf^2)^2 i f^{ab}{}_c \frac{1}{\bar{z}-\bar{w}} j^c(w) + \ldots$$

Ashok, Benichou & Troost
arXiv:0903.4277

We computed this current algebra using two different methods :

Ashok, Benichou & Troost
arXiv:0903.4277

Perturbative computation of currents 2 and 3 points functions to all orders in perturbation theory.

$$\langle j^a_{L,z/\bar{z}}(z,\bar{z}) j^b_{L,z/\bar{z}}(w,\bar{w})\rangle$$

$$\langle j^a_{L,z/\bar{z}}(z,\bar{z}) j^b_{L,z/\bar{z}}(w,\bar{w}) j^c_{L,z/\bar{z}}(x,\bar{x})\rangle$$

Computation of the OPEs in conformal perturbation theory near the WZW point.

$$S = S_{WZW} + \epsilon S_{RR}$$

$$\begin{aligned} j^a_L(z,\bar{z}) j^b_L(w,\bar{w}) &= \{j^a_L(z,\bar{z}) j^b_L(w,\bar{w})\}_{WZW} \\ &+ \epsilon \{j^a_L(z,\bar{z}) j^b_L(w,\bar{w})\}_{(1)} + \ldots \end{aligned}$$

Thanks to the current algebra we can give a constructive proof of conformal invariance :

$$T = \gamma \; Str(j_z j_z) \qquad T(z)T(w) = \frac{dim\ G}{2(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w}$$

The current algebra we identified is realized in all sigma models on supergroup manifolds with vanishing dual Coxeter number.

PSL(n|n)   OsP(2n+2|2n)   etc.

<u>These theories are relevant for :</u>

String theory

Integer quantum Hall effect

Zirnbauer, 1999

Quenched disorder systems

Polymers

Percolation

etc.

Parisi & Sourlas, 1980

Let us go back to string theory on $AdS_3 \times S^3$.

The AdS/CFT correspondance implies that it is dual to a 2D CFT realizing the N=4 superconformal algebra.

In the gravity theory, this symmetry algebra is generated by diffeomorphisms that do not vanish at spatial infinity.

Brown & Henneaux, 1986

We can construct these generators :

$$\begin{aligned} \delta X^\mu &= \xi^\mu \\ \delta G_{\mu\nu} &= \partial_{(\mu}\xi_{\nu)} \end{aligned}$$

$$S = \frac{1}{4\pi\alpha'} \int d^2z \; G_{\mu\nu}(X)\partial X^\mu \bar{\partial} X^\nu + \ldots$$

$$V = \frac{1}{4\pi\alpha'} \int d^2z \; \partial_{(\mu}\xi_{\nu)}\partial X^\mu \bar{\partial} X^\nu$$

Explicitly the bosonic generators of the N=4 superconformal algebra reads :

R-currents :

$$J^a(x) = \frac{1}{\pi}\int d^2z\,\left(j^a_{L,z}\bar{\partial}\Lambda(x,\bar{x};z,\bar{z}) + j^a_{L,\bar{z}}\partial\Lambda(x,\bar{x};z,\bar{z})\right)$$

Ashok, Benichou & Troost arXiv:0907.1242

Stress-tensor :

$$T(x) = \frac{1}{2\pi}\int d^2z\,\left(\partial_x j_{L,z}\partial_x\bar{\partial}\Lambda(x,\bar{x};z,\bar{z}) + 2\partial_x^2 j_{L,z}\bar{\partial}\Lambda(x,\bar{x};z,\bar{z}) + \partial_x j_{L,\bar{z}}\partial_x\partial\Lambda(x,\bar{x};z,\bar{z}) + 2\partial_x^2 j_{L,\bar{z}}\partial\Lambda(x,\bar{x};z,\bar{z})\right)$$

with $j_{L,z} = j^+_{L,z} - 2xj^3_{L,z} + x^2 j^-_{L,z}$

The variables $(x,\bar{x})$ code the Sl(2) spin. They are interpreted as coordinates on the boundary.

These generators generalize the ones constructed in the case of NSNS fluxes only.

Giveon, Kutasov & Seiberg, 1998

Kutasov & Seiberg, 1999

De Boer, Ooguri, Robins & Tannenhauser, 1998

Thanks to the worldsheet current algebra, we can show that the generators $J^a_;\ T$ satisfy the expected OPEs :

$$J^a(x)J^b(y) \approx \frac{\kappa^{ab}}{(x-y)^2}I + \frac{if^{ab}{}_c J^c(y)}{x-y} + \ldots$$

$$T(x)J^a(y) \approx \frac{J^a(y)}{(x-y)^2} + \frac{\partial J^a(y)}{x-y} + \ldots$$

$$T(x)T(y) \approx \frac{3I}{(x-y)^4} + \frac{2T(y)}{(x-y)^2} + \frac{\partial T(y)}{x-y} + \ldots$$

Ashok, Benichou & Troost arXiv:0907.1242

Work in progress

➢ Investigation of the sigma model thanks to the current algebra (sprectrum, correlation functions...)

➢ Current algebra for coset models.

Thank you.