Inter-university Attraction Pole meeting Vrije Universiteit Brussel, October 2nd, 2009

# Strings vs Condensed Matter An overview



Can string theory be used to access experimentally testable systems?



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Our best hope resides in the gauge/string duality



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Near horizon region of D-brane geometry includes an Anti-de Sitter factor

$$ds^{2} = \frac{L^{2}}{r^{2}} \left( -dt^{2} + d\vec{x}^{2} + dr^{2} \right)$$



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Typical application of the duality: *fundamental particle interactions* described by *quantum gauge theories* (*confinement*,  $\chi$ SB, quark-gluon plasma, ...)



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However we are still **quite far** from a full-fledged **string dual** of the **theory of strong interactions** (non supersymmetric, non conformal, ...)





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New applications to *condensed matter systems* 





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New applications to *condensed matter systems* 

#### WHY?

Many examples of *strongly coupled* 

scale invariant systems in condensed matter physics



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#### WHY?

Many examples of *strongly coupled scale invariant* systems in condensed matter physics These *quantum critical systems* are not described by the usual paradigms of *order parameters* and *quasiparticles* 



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*Quantum critical point* : continuous phase transition at T = 0(driven by quantum fluctuations rather than thermal fluctuations)



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**Quantum critical point**: continuous phase transition at T = 0(driven by *quantum fluctuations* rather than thermal fluctuations)

Simple example: *quantum antiferromagnet* (e.g. *TICuCl<sub>3</sub>*)  $H = \sum J_{ij} \vec{S}_i \cdot \vec{S}_j$ 



 $\langle ij \rangle$ 

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# **Quantum critical systems** *Quantum critical point* : continuous phase transition at T = 0(driven by *quantum fluctuations* rather than thermal fluctuations) Simple example: *quantum antiferromagnet* (e.g. *TICuCl*<sub>2</sub>) $H = \sum J_{ij} \vec{S}_i \cdot \vec{S}_j$ $\langle ij \rangle$



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Order parameter  $\vec{\varphi} = (-1)^i \vec{S_i}$ Ground state has Néel order  $\langle \vec{\varphi} \rangle \neq 0$ 



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Ground state is "quantum paramagnet" Spins locked in *valence bond singlets* 



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 $\langle ij \rangle$ 

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 $\langle ij \rangle$ 

#### QUANTUM CRITICAL POINT

 $g_c$ 



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**O(3)** order parameter  $\vec{\varphi}$ described by QFT action





 $g_c$ 



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**O(3)** order parameter  $\vec{\varphi}$  described by QFT action

er 
$$\vec{\varphi}$$
  
ction  $S = \int d\tau d^2 x \left[ (\partial_\tau \vec{\varphi}^2) + c^2 (\nabla_x \vec{\varphi})^2 + (g - g_c) \vec{\varphi}^2 + u (\vec{\varphi}^2)^2 \right]$ 

BECOMES QUANTUM CRITICAL AND STRONGLY COUPLED AT  $g \rightarrow g_c$ 





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BECOMES QUANTUM CRITICAL AND STRONGLY COUPLED AT  $g \to g_c$ ENERGY SCALE  $\Delta \sim (g - g_c)^{\nu z}$  COHERENCE LENGTH  $\xi \sim (g - g_c)^{-\nu}$ 



What happens if we add *temperature* to the system?





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What happens if we add *temperature* to the system?





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What happens if we add *temperature* to the system?





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What happens if we add *temperature* to the system?



Quantum "perfect fluid": transport coefficients depend only on universal constants

$$\frac{\text{Relaxation time}}{\tau \ge \frac{\hbar}{k_B T} \times [\text{universal } O(1)]}$$



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What happens if we add *temperature* to the system?



Quantum "perfect fluid": transport coefficients depend only on universal constants

Relaxation time	Electrical conductivity	
$\tau \ge \frac{\hbar}{k_B T} \times [\text{universal } O(1)]$	$\sigma = \frac{e^2}{h} \times [\text{universal } O(1)]$	
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What happens if we add *temperature* to the system?



Quantum "perfect fluid": transport coefficients depend only on universal constants **Relaxation time** Electrical conductivity Viscosity / entropy density  $\frac{\eta}{s} = \frac{\hbar}{k_B} \times [\text{universal } O(1)]$  $au \ge \frac{\pi}{k_B T} \times [\text{universal } O(1)]$  $\sigma = \frac{\sigma}{L} \times [\text{universal } O(1)]$ SCHRÖDINGER QUANTUM HOLOGRAPHIC SUPER AdS / CMT **INTRODUCTION CONCLUSIONS** IAP Brussels 02/10/09 CRITICALITY **CONDUCTORS** SYSTEMS METHODS

### Holography 1: temperature and black holes

*Quantum critical region* described by *conformal strongly coupled* 3d QFT



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*Quantum critical region* described by *conformal strongly coupled* 3d QFT

Ideal set-up for holographic duality



# Holography 1: temperature and black holes

*Quantum critical region* described by *conformal strongly coupled* 3d QFT

Ideal set-up for *holographic duality* 

**3d Conformal Field Theory** 

Gravity in (Asymptotically) 4d Anti-de Sitter

































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Condensed matter systems are *electron systems* : add a U(1) symmetry



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(Global) symmetry in QFT





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(Global) symmetry in QFT

$$\begin{array}{c} \textit{EINSTEIN-MAXWELL} \\ \textit{ACTION} \end{array} S = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G_N} \left( R - 2\Lambda \right) - \frac{1}{4g^2} F^2 \right] \end{array}$$



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Condensed matter systems are *electron systems* : add a U(1) symmetry

(Global) symmetry in QFT (Gauged) symmetry in gravity  
EINSTEIN-MAXWELL 
$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G_N} \left( R - 2\Lambda \right) - \frac{1}{4g^2} F^2 \right]$$
  
ELECTROMAGNETIC FIELD STRENGTH  $F = dA$ 



Condensed matter systems are *electron systems* : add a U(1) symmetry

(Global) symmetry in QFT

(Gauged) symmetry in gravity

ELECTROMAGNETIC FIFI D STRENGTH F = dA

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$$ELECTROMAGNETIC FIRE COMMAGNETIC FIRE COMMAGNE$$

**EINSTEIN-MAXWELL** ΛΟΤΙΟΝΙ

Charged AdS black hole solution

$$\begin{split} ds^2 &= \frac{L^2}{r^2} \left( -f(r)dt^2 + d\vec{x}^2 + \frac{dr^2}{f(r)} \right) \\ A_t &= \mu \left( 1 - \frac{r}{r_H} \right) \end{split}$$

Condensed matter systems are *electron systems* : add a U(1) symmetry

(Global) symmetry in QFT

FT (Gauged) symmetry in gravity

EINSTEIN-MAXWELL

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$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G_N} \left( R - 2\Lambda \right) - \frac{1}{4g^2} F^2 \right]$$
  
ELECTROMAGNETIC FIELD STRENGTH  $F = dA$   
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Condensed matter systems are *electron systems* : add a U(1) symmetry

(Global) symmetry in QFT

EINSTEIN-MAXWELL ACTION



 $S = \int d^{4}x \sqrt{-g} \left[ \frac{1}{16\pi G_{N}} \left( R - 2\Lambda \right) - \frac{1}{4g^{2}} F^{2} \right]$  ELECTROMAGNETIC FIELD STRENGTH F = dA  $i = \frac{L^{2}}{r^{2}} \left( -f(r)dt^{2} + d\vec{x}^{2} + \frac{dr^{2}}{f(r)} \right) f(r) = 1 - M\frac{r^{3}}{r_{H}^{3}} + Q\frac{r^{4}}{r_{H}^{4}}$   $A_{t} = \mu \left( 1 - \frac{r}{r_{H}} \right)$  CHEMICAL POTENTIAL (COULD ALSO ADD A

MAGNETIC FIELD)



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Condensed matter systems are *electron systems* : add a U(1) symmetry

(Global) symmetry in QFT

EINSTEIN-MAXWELL ACTION



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(Gauged) symmetry in gravity  $S = \int d^4x \sqrt{-g} \left| \frac{1}{16\pi G_N} \left( R - 2\Lambda \right) - \frac{1}{4g^2} F^2 \right|$ ELECTROMAGNETIC FIFI D STRENGTH F = dACharged AdS black hole solution  $ds^{2} = \frac{L^{2}}{r^{2}} \left( -f(r)dt^{2} + d\vec{x}^{2} + \frac{dr^{2}}{f(r)} \right) f(r) = 1 - M\frac{r^{3}}{r_{H}^{3}} + Q\frac{r^{4}}{r_{H}^{4}}$  $A_t = \mu \left( 1 - \frac{r}{r_H} \right)$ Now  $T/\mu$  is a *physical* parameter and can be taken CHEMICAL POTENTIAL to *zero* continuously (COULD ALSO ADD A

MAGNETIC FIELD)

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Condensed matter systems are *electron systems* : add a U(1) symmetry

(Global) symmetry in QFT

(Gauged) symmetry in gravity

$$\frac{d^4x\sqrt{-g}\left[\frac{1}{16\pi G_N}\left(R-2\Lambda\right)-\frac{1}{4g^2}F^2\right]}{\text{ELECTROMAGNETIC FIELD STRENGTH }F=dA$$



**EINSTEIN-MAXWELL** 

**ACTION** 

Charged AdS black hole solution

$=\frac{L^2}{r^2}\left(-f(r)dt^2+d\vec{x}\right)$	$f(r) = 1 - M \frac{r^3}{r_H^3} + 0$	$Qrac{r}{r_{j}^{2}}$
$= \mu \left( 1 - \frac{r}{r_H} \right)$	Now $T/\mu$ is a <i>physical</i>	_

CHEMICAL POTENTIAL

Now  $T/\mu$  is a *physical parameter* and can be taken to *zero* continuously

What we have done for *temperature* and *chemical potential* can be generalized for any *perturbation* of the CFT by a *relevant operator* 

 $ds^2$ 

 $A_{t}$ 



Condensed matter systems are *electron systems* : add a U(1) symmetry

(Global) symmetry in QFT

(Gauged) symmetry in gravity

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Charged AdS black hole solution



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CHEMICAL POTENTIAL

Field theory operator *O* 

**Dynamical bulk field**  $\phi$ 



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Condensed matter systems are *electron systems* : add a U(1) symmetry

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(Global) symmetry in QFT

(Gauged) symmetry in gravity

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Charged AdS black hole solution



**EINSTEIN-MAXWELL** 

ACTION

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**Dynamical bulk field**  $\phi$ 

$$Z_{\text{gravity}}[\phi \to \delta \phi_0] = \left\langle \exp\left(i \int d^3 x \sqrt{-g_0} \delta \phi_0 \mathcal{O}\right) \right\rangle_{\text{CF}}$$



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Condensed matter systems are *electron systems* : add a U(1) symmetry

(Global) symmetry in QFT

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EINSTEIN-MAXWELL 
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Condensed matter systems are *electron systems* : add a U(1) symmetry

(Global) symmetry in QFT

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Charged AdS black hole solution



EINSTEIN-MAXWELL

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CHEMICAL POTENTIAL

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Condensed matter systems are *electron systems* : add a U(1) symmetry

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Charged AdS black hole solution



EINSTEIN-MAXWELL

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## Holography 3: transport properties

Real time dynamics and transport very sensitive to interactions
 Holographic methods are often our only handle at strong coupling



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# Holography 3: transport properties Real time dynamics and transport very sensitive to interactions Image: Holographic methods are often our only handle at strong coupling Perturb system away from equilibrium $\Delta S = \int d^3x \mathcal{O}\delta\phi_0^{0} \int \mathcal{O} \delta\phi_0^{0} \mathcal{O} \partial \mathcal{O}^{0} \mathcal{$

 $\begin{array}{c} \checkmark \quad \textbf{Electrical conductivity} \\ \delta \langle \vec{J} \rangle \stackrel{k, \omega \to 0}{\longrightarrow} i \omega \chi \vec{A} = \sigma \vec{E} \end{array}$ 





$$\delta \langle \vec{J} \rangle \stackrel{_{k,\omega\to 0}}{\longrightarrow} i\omega\chi \vec{A} = \sigma \vec{E}$$

OHM'S LAW

$$\sigma(\omega) = \frac{iG^R_{\vec{J}\vec{J}}(\omega)}{\omega}$$













## Holographic superconductors

Superconductivity is spectacular application of U(1) spontaneous symmetry breaking





## Holographic superconductors

**Superconductivity** is spectacular application of *U(1) spontaneous symmetry breaking* 

- Vanishing resistance at low temperatures
- Expulsion of magnetic field (Meissner effect)





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## Holographic superconductors

**Superconductivity** is spectacular application of *U(1) spontaneous symmetry breaking* 

- Vanishing resistance at low temperatures
- Expulsion of magnetic field (Meissner effect)

Conventional *superconductors* are well explained by *BCS theory* 











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$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G_N} \left( R - 2\Lambda \right) - \frac{1}{4g^2} F^2 \right]$$





$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G_N} \left( R - 2\Lambda \right) - \frac{1}{4g^2} F^2 - \left| \left( \partial - iqA \right) \phi \right|^2 - m^2 |\phi|^2 \right]$$

$$ADD \ CHARGED \ SCALAR \ TO \ SERVE \ AS \ ORDER \ PARAMETER \ ADD \ CHARGED \ SCALAR \ TO \ SERVE \ AS \ ORDER \ PARAMETER \ ADD \ CHARGED \ SCALAR \ TO \ SERVE \ AS \ ORDER \ PARAMETER \ ADD \ CHARGED \ SCALAR \ TO \ SERVE \ AS \ ORDER \ PARAMETER \ ADD \ CHARGED \ SCALAR \ TO \ SERVE \ AS \ ORDER \ PARAMETER \ ADD \ CHARGED \ SCALAR \ TO \ SERVE \ AS \ ORDER \ PARAMETER \ ADD \ CHARGED \ SCALAR \ TO \ SERVE \ AS \ ORDER \ PARAMETER \ ADD \ SCALAR \ ADD \ ADD$$







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## Non-relativistic systems



Recall that at a *quantum critical point* 

**ENERGY**  $\Delta \sim (g - g_c)^{\nu z}$  **LENGTH**  $\xi \sim (g - g_c)^{-\nu}$ Implies scale invariance  $t \to \lambda^z t$   $x \to \lambda x$ 



## Non-relativistic systems



Recall that at a *quantum critical point ENERGY*  $\Delta \sim (g - g_c)^{\nu z}$  *LENGTH*  $\xi \sim (g - g_c)^{-\nu}$ Implies *scale invariance*  $t \rightarrow \lambda^z t$   $x \rightarrow \lambda x$  *z DYNAMICAL EXPONENT* In *relativistic* case treated so far: z = 1



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SCHRÖDINGEF

CONCLUSIONS


Recall that at a *quantum critical point ENERGY*  $\Delta \sim (g - g_c)^{\nu z}$  *LENGTH*  $\xi \sim (g - g_c)^{-\nu}$ Implies *scale invariance*  $t \rightarrow \lambda^z t$   $x \rightarrow \lambda x$  *z DYNAMICAL EXPONENT* In *relativistic* case treated so far: z = 1Many *non-relativistic condensed matter systems* have  $z \neq 1$ 





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Example: Fermions at unitarity (experimentally realized in trapped cold atoms) realize Schrödinger algebra (z = 2)









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 $\times$ 



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1 / k<sub>F</sub>a

-0.5

-1.0

-1.3

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$$ds^{2} = \frac{L^{2}}{r^{2}} \left( -\frac{L^{2}}{r^{2}} dt^{2} - 2dt dv + \frac{d\vec{x}^{2} + dr^{2}}{space} \right) SCHRÖDINGER$$

$$space-Time$$

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Can construct asymptotically Schrödinger (charged) black h and compute thermodynamics, transport, ...

with results appropriate for a non-relativistic theory







2.0



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$$introduction Market M$$

### **Conclusions**





**QUANTUM** CRITICALITY

HOLOGRAPHIC METHODS

SUPER **CONDUCTORS** 

CONCLUSIONS