# Towards an Asymptotically Safe SM

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# Outline

- Motivation; the hierarchy versus triviality problem
- RG flows and the asymptotic safety idea
- Asymptotically safe 4D QFTs
- Adding relevant operators
- Radiative symmetry breaking
- An ASSM ?

# Motivation: two problems to do with scalars

### **The hierarchy problem:**

Why is the Weak Scale so much lower than the Planck Scale - and how is it protected?

More precisely perturbation theory with a higgs scalar is suspect: very "massive states" dominate any perturbative calculation to do with higgs physics.

Actually don't even need a heavy resonance: this can be true for some other rapid change (in e.g. beta functions) at a high scale.

### **The hierarchy problem:**

#### Candidate symmetries:

- Higgs is a Goldstone mode of *some broken global symmetry* (like the pions in chiral symmetry breaking) with breaking scale of a few TeV
- *Supersymmetry* relates boson to fermions. Divergences cancel level by level. Phenomenology requires soft (a.k.a. dimensionful) breaking.
- Scaling symmetry Higgs is the Goldstone mode of a broken scale invariance (a.k.a. dilaton) (a trivial perturbative example of this is the Standard Model with vanishing higgs mass, but it can occur in nonperturbative models based on AdS/CFT). (*The subject here but not Coleman-Weinberg!!*)
- Misaligned Supersymmetry even non-supersymmetric non-tachyonic strings are finite. (Alternative route to naturalness) (SAA+Dienes+Mavroudi)

## The triviality problem:

Scalars lead to Landau poles:

=> the theory is UV incomplete

But trying to UV complete it result in the hierarchy problem again! (see previous comments)

### Hints from QCD

QCD is (unlike SUSY) a UV complete theory. Why?

- 1. *There is no hierarchy problem:* quark masses are protected by chiral symmetry
- 2. There is no triviality problem: QCD is asymptotically free



Note the philosophy of QCD: we do not mind running masses because they do not upset the Gaussian UV fixed point. We simply measure them and let them run. Or to put it another way: they are "relevant" operators that are effectively zero in the UV. They do not need to run to zero in the UV! (We also don't care too much about couplings blowing up in the IR.)

# RG flows and the asymptotic safety idea

### The Basic idea

Weinberg used this as a basis for his proposal of UV complete theories



### **Categorise the content of a theory as follows**

**Irrelevant operators:** would disrupt the fixed point - therefore asymptotically safe theories have to emanate precisely from UV fixed point where they are assumed zero (exactly renormalizable trajectory)

**Marginal operators:** can be involved in determining the UV fixed point where they become *exactly* marginal. Or can be marginally relevant (asymptotically free) or irrelevant.

**Relevant operators:** become "irrelevant" in the UV but may determine the IR fixed point.

Note relevant or marginally relevant operators still have "infinities" at the FP - just as quark masses, they still run at the FP just like any other relevant operator: but being relevant they do not affect the FP. (And by definition they become less important the higher you go in energy.)

**Simple example of flow - normal QCD:** 

$$\partial_t lpha = -B lpha^2$$
  $t = \log \mu/\mu_0$ 

This theory has *unstable* fixed point at  $\alpha = 0$ . Asymptotically free if B > 0



Weinberg's original set-up

$$\alpha$$
$$\partial_t \alpha = A \,\alpha - B \,\alpha^2$$

Gastmans et al '78 W€inbdug µø Λ Peskin '80 Gawedski, Kupiainen '85 Kawai et al '90 de Calan\*et al '91 Morris '04

If A>0, B>0, this theory has *unstable* UV fixed point at  $\alpha = A/B$  and *stable* one at IR  $\alpha = 0$ 



#### **Caswell-Banks-Zaks fixed point:**

Take QCD with  $SU(N_C)$  and  $N_F$  fermions but very large numbers of colours+flavours

$$\partial_t \alpha = -B\alpha^2 + C\alpha^3$$
  $B \propto \epsilon = \frac{N_F}{N_C} - \frac{1}{2}$ 

Turns out C>0, B>0: theory has *stable* IR fixed point at  $\alpha = B/C$  and *unstable* one in UV  $\alpha = 0$ 



### What about Asymptotic safety in 4D 2FT?

Again would have ...

$$\partial_t \alpha = -B\alpha^2 + C\alpha^3$$

But requires C<0, B<0, this theory has *stable* IR fixed point at  $\alpha = 0$  and *unstable* UV one at  $\alpha = B/C$ 



# Asymptotic safety in 4D QFT

#### That was a one coupling cartoon: real situation requires several couplings to realise Litim & Sannino '14

In order to get this behaviour need to add scalars and Yukawa couplings

$$\mathcal{L} = -\frac{1}{2} \operatorname{Tr} F^{\mu\nu} F_{\mu\nu} + \operatorname{Tr} \left( \overline{Q} \, i \not D \, Q \right) + y \operatorname{Tr} \left( \overline{Q} \, H \, Q \right) + \operatorname{Tr} \left( \partial_{\mu} H^{\dagger} \, \partial^{\mu} H \right) - u \operatorname{Tr} \left[ (H^{\dagger} H)^{2} \right] - v \left( \operatorname{Tr} \left[ H^{\dagger} H \right] \right)^{2},$$

*H* is an  $N_F \times N_F$  scalar Initially have  $U(N_F)_L \times U(N_F)_R$  flavour symmetry

$$\left( \alpha_g = \frac{g^2 N_C}{(4\pi)^2}, \quad \alpha_y = \frac{y^2 N_C}{(4\pi)^2} \right)$$

Effect of Yukawa ....

$$\beta_g = \alpha_g^2 \left[ \frac{4}{3} \epsilon + \left( 25 + \frac{26}{3} \epsilon \right) \alpha_g \left[ -2 \left( \frac{11}{2} + \epsilon \right)^2 \alpha_y \right] \right]$$

$$\beta_y = \alpha_y \left[ (13 + 2\epsilon) \, \alpha_y - 6 \, \alpha_g \right]$$



Four couplings - flow could in principle be four dimensional

$$\alpha_g = \frac{g^2 N_C}{(4\pi)^2}, \quad \alpha_y = \frac{y^2 N_C}{(4\pi)^2}, \quad \alpha_h = \frac{u N_F}{(4\pi)^2}, \quad \alpha_v = \frac{v N_F^2}{(4\pi)^2}$$

but ...



Four couplings - flow could in principle be four dimensional

$$\alpha_g = \frac{g^2 N_C}{(4\pi)^2}, \quad \alpha_y = \frac{y^2 N_C}{(4\pi)^2}, \quad \alpha_h = \frac{u N_F}{(4\pi)^2}, \quad \alpha_v = \frac{v N_F^2}{(4\pi)^2}$$





Along the separatrix/critical-curve/exact-trajectory can parameterise the flow in terms of  $\alpha_g(t)$ 



At the fixed point it is arbitrarily weakly coupled,  $\alpha_g^* = 0.4561 \epsilon$ , where  $\epsilon = \frac{N_F}{N_C} - \frac{11}{2}$ 



# Coleman Weinberg? (no)

### **Recap of the idea**

- *The SM is "classically" scale invariant* tree level Lagrangian has no mass
- Coleman Weinberg mechanism leads to spontaneous breaking at a scale because the scale invariance is anomalous. (Huge amount of interest since 2012)
- Compute effective potential and renormalize it

$$\begin{split} V_{eff} &= \frac{\lambda}{4!} |\phi|^4 + \frac{3g^4}{64\pi^2} |\phi|^4 \left( \log \frac{|\phi|}{\mu} - \frac{25}{6} \right) \qquad \frac{\partial^2 V}{\partial \phi^2} |_{\phi=0} = 0 \quad \frac{\partial^4 V}{\partial \phi^4} |_{\phi=\mu} = \lambda \\ \text{We imposed by hand no generation of mass terms!} \\ \text{Minimization leads to dimensional transmutation} \\ &\langle \phi \rangle = \mu e^{\frac{11}{6} - \frac{4\pi^2 \lambda}{9g^4}} \end{split}$$

- *Heuristically unlikely to work from a UV fixed point:* CW is all about *IR* scale invariance where  $\phi = 0$  which is why it is a strange starting point for solving the problems of large UV thresholds.
- Proof (already shown numerically by Litim, Mojaza, Sannino but can do it analytically): for example choose the real trace direction ...

$$H = \frac{\phi}{\sqrt{2N_F}} \mathbb{1}_{N_F \times N_F} \implies V_{class}^{(4)} = \frac{4\pi^2}{N_F^2} (\alpha_h + \alpha_v) \phi^4$$

- Effectively  $\lambda = 32\pi^2 \frac{3}{N_F^2} (\alpha_h + \alpha_v)$
- Also define

$$\kappa = 32\pi^2 \frac{1}{N_F^2} \left( 3\alpha_h + \alpha_v \right)$$

$$\begin{split} V &= \frac{\lambda}{4!} \phi^4 + \frac{m_{\phi}^2}{2} \phi^2 + \frac{1}{64\pi^2} \left( m_{\phi}^2 + \frac{\lambda}{2} \phi^2 \right)^2 \left( \log \frac{m_{\phi}^2 + \frac{\lambda}{2} \phi^2}{\mu^2} - \frac{3}{2} \right) \\ &- \frac{(4\pi)^2}{4N_F N_C} \alpha_y^2 \phi^4 \left( \log \frac{(4\pi)^2 \alpha_y \phi^2}{\sqrt{N_F N_C} \mu^2} - \frac{3}{2} \right) \\ &+ \frac{(N_F^2 - 1)}{64\pi^2} \left( \frac{\kappa}{2} \phi^2 \right)^2 \left( \log \frac{\frac{\kappa}{2} \phi^2}{\mu^2} - \frac{3}{2} \right) + \frac{N_F^2}{64\pi^2} \left( \frac{\lambda}{6} \phi^2 \right)^2 \left( \log \frac{\frac{\lambda}{6} \phi^2}{\mu^2} - \frac{3}{2} \right) \end{split}$$

• Effectively 
$$\lambda = 32\pi^2 \frac{3}{N_F^2} (\alpha_h + \alpha_v)$$

• Also define 
$$\kappa = 32\pi^2 \frac{1}{N_F^2} (3\alpha_h + \alpha_v)$$

Corrections all of order  $\alpha\lambda$ , so no perturbative minimum without a mass-squared for  $\phi$ 

$$\begin{split} V &= \frac{\lambda}{4!} \phi^4 + \frac{m_{\phi}^2}{2} \phi^2 + \frac{1}{64\pi^2} \left( m_{\phi}^2 + \frac{\lambda}{2} \phi^2 \right)^2 \left( \log \frac{m_{\phi}^2 + \frac{\lambda}{2} \phi^2}{\mu^2} - \frac{3}{2} \right) \\ &- \frac{(4\pi)^2}{4N_F N_C} \alpha_y^2 \phi^4 \left( \log \frac{(4\pi)^2 \alpha_y \phi^2}{\sqrt{N_F N_C \mu^2}} - \frac{3}{2} \right) \\ &+ \frac{(N_F^2 - 1)}{64\pi^2} \left( \frac{\kappa}{2} \phi^2 \right)^2 \left( \log \frac{\frac{\kappa}{2} \phi^2}{\mu^2} - \frac{3}{2} \right) + \frac{N_F^2}{64\pi^2} \left( \frac{\lambda}{6} \phi^2 \right)^2 \left( \log \frac{\frac{\lambda}{6} \phi^2}{\mu^2} - \frac{3}{2} \right) \end{split}$$



# Adding relevant operators mass-squareds

#### Solve Callan Symanzik eqn for them as usual =>

warm-up; first restrict ourselves to the diagonal direction where mass-squared term looks like the following operator:

$$V \supset \frac{m_{\phi}^2}{4N_F} \left( \text{Tr}(H + H^{\dagger}) \right)^2$$



#### Solve Callan Symanzik eqn for them as usual =>

For mass-squareds, by dimensions have contributions from cross-terms only ...

$$V \supset \frac{m_{\phi}^2}{2} \phi^2 \left(1 + \frac{\lambda t}{16\pi^2}\right)$$

Using the solutions along the separatrix:

$$\begin{split} \beta_{m_{\phi}^{2}} &= m_{\phi}^{2} \left( \frac{\lambda}{16\pi^{2}} + 2\gamma \right) ,\\ \frac{1}{m_{\phi}^{2}} \beta_{m_{\phi}^{2}} &= 2\alpha_{y} + \frac{6}{N_{F}^{2}} (\alpha_{v} + \alpha_{h}) \\ &= f\alpha_{g} , \qquad \left( f = \frac{12}{13} \left[ 1 + \frac{3}{4N_{F}^{2}} \left( \sqrt{20 + 6\sqrt{23}} - 1 - \sqrt{23} \right) \right] \right) \end{split}$$

i.e. mass-squared scales with the gauge coupling like all the marginal couplings ...

#### in the end ...

We find *multiplicative* renormalisation ...

$$m_{\phi}^2(t) = m_*^2 \left(\frac{\alpha_g^*}{\alpha_g} - 1\right)^{-\frac{3f}{4\epsilon}} \qquad \alpha_g^* = 0.4561 \epsilon$$

In principle ...  $m_*^2 = m_{\phi}^2(0) \left( \alpha_g^* / \alpha_g(0) - 1 \right)^{\frac{3f}{4\epsilon}}$  but you should just think of it as an RG invariant that defines this particular trajectory. (Every relevant operator will have an associated invariant.) It has the same status as the chiral quark masses.

# Radiative symmetry breaking

### Criticism of the simplest example...

- **Purely** multiplicative: Hence the mass-squared has to be negative along the whole trajectory
- We cheated: in the sense that we ignored all the orthogonal directions!! These also get contributions at one-loop even though their masses were zero at tree-level

In order to address both these, organise the discussion in terms of the  $U(N_F) \times U(N_F)$ flavour symmetry that we break with the mass-squareds:

$$H = \frac{(h_0 + ip_0)}{\sqrt{2N_F}} \mathbb{1}_{N_F \times N_F} + (h_a + ip_a)T_a$$

### Non-trivial simple example...

Seek to add a set of mass-squared operators whose flavour structure is closed under RG: simple example



Following the same procedure and after some work find the following answer in terms of two RG invariants (one for each independent bit of the flavour structure) (where  $v = (1-1/NF^2)$ ):



Starting values get *relatively* closer in UV (note the masses are all shrinking *in absolute* terms in the IR) - full flavour symmetry restored precisely at fixed point





$$\begin{array}{ccc} \alpha_{g,min} & \xrightarrow{\epsilon \to 0} & \frac{1}{2} \alpha_g^* \\ m_{0,min}^2 \sim -\tilde{m}_*^2 \end{array}$$

# The ASSM via radiative breaking...

#### • To embed the SM - focus on breaking SU(Nc) to SU(3) colour with Ns new scalars ...

c.f. Pelaggi, Sannino Strumia Vigiani

	$SU(N_C)$	$SU(N_F)_L \supset SU(2)_L$	$SU(N_F)_R \supset SU(2)_R$	$SU(N_S) \supset SU(2)_R$
$Q_{L,a}^i$			1	1
$Q^a_{R,i}$	$\tilde{\Box}$	1		1
$H_i^j$	1		$\sim$	1
$\tilde{Q}_{j=1N_S}$	$\square$	1	1	

The new scalars give a similar UVFP

Extension of Pati-Salam (XPS) - breaks to SU(3) if we choose  $N_S = N_C - 3$ .

$$\langle \tilde{Q} \rangle = \left( \begin{matrix} 0 & 0 & 0 & 1 \\ \vdots \vdots \vdots & \ddots & \\ 0 & 0 & 0 & 1 \end{matrix} \right)$$

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The new scalars give a similar UVFP ...

Extension of Pati-Salam (XPS) - breaks to SU(3) if we choose  $N_S = N_C - 3$ .

• Explicit embedding looks like P-S

$$Q_{L} = \begin{pmatrix} q_{1} \ell_{1} \cdots \\ q_{2} \ell_{2} \cdots \\ q_{3} \ell_{3} \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}; \quad Q_{R} = \begin{pmatrix} \begin{pmatrix} u_{R} \\ d_{R} \end{pmatrix} \begin{pmatrix} \nu_{R}^{e} \\ e_{R} \end{pmatrix} \cdots \\ \begin{pmatrix} c_{R} \\ s_{R} \end{pmatrix} \begin{pmatrix} \nu_{R}^{\mu} \\ \mu_{R} \end{pmatrix} \cdots \\ \begin{pmatrix} t_{R} \\ b_{R} \end{pmatrix} \begin{pmatrix} \nu_{R}^{\tau} \\ \tau_{R} \end{pmatrix} \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$
$$H^{\dagger} = \begin{pmatrix} \begin{pmatrix} h_{d}^{0} h_{d}^{-} \\ h_{u}^{+} h_{u}^{0} \end{pmatrix}_{11} \begin{pmatrix} h_{d}^{0} h_{d}^{-} \\ h_{u}^{+} h_{u}^{0} \end{pmatrix}_{12} \begin{pmatrix} h_{d}^{0} h_{d}^{-} \\ h_{u}^{+} h_{u}^{0} \end{pmatrix}_{13} \\ \begin{pmatrix} h_{d}^{0} h_{d}^{-} \\ h_{u}^{+} h_{u}^{0} \end{pmatrix}_{21} \begin{pmatrix} h_{d}^{0} h_{d}^{-} \\ h_{u}^{+} h_{u}^{0} \end{pmatrix}_{22} \begin{pmatrix} h_{d}^{0} h_{d}^{-} \\ h_{u}^{+} h_{u}^{0} \end{pmatrix}_{23} \\ \begin{pmatrix} h_{d}^{0} h_{d}^{-} \\ h_{u}^{+} h_{u}^{0} \end{pmatrix}_{31} \begin{pmatrix} h_{d}^{0} h_{d}^{-} \\ h_{u}^{+} h_{u}^{0} \end{pmatrix}_{32} \begin{pmatrix} h_{d}^{0} h_{d}^{-} \\ h_{u}^{+} h_{u}^{0} \end{pmatrix}_{33} \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

• Assignment implies 9 pairs of Higgses one for each choice of generation

• Extend Lagrangian ...

$$\mathcal{L} \supset \mathcal{L}_{YM} + \mathcal{L}_{KE} + \frac{y}{\sqrt{2}} \operatorname{Tr} \left[ H \left( Q_L \cdot Q_R \right) \right] - u_1 \operatorname{Tr} \left[ H^{\dagger} H \right]^2 - u_2 \operatorname{Tr} \left[ H^{\dagger} H H^{\dagger} H \right] - v_1 \operatorname{Tr} \left[ H^{\dagger} H \right] \operatorname{Tr} [\tilde{Q}^{\dagger} \cdot \tilde{Q}] - w_1 \operatorname{Tr} [\tilde{Q}^{\dagger} \cdot \tilde{Q}]^2 - w_2 \operatorname{Tr} [\tilde{Q}^{\dagger} \cdot \tilde{Q} \tilde{Q}^{\dagger} \cdot \tilde{Q}],$$

and find the theory flows as below (we have almost no freedom) developing VEVs to break XPS and electroweak symmetry at same time...



#### • What about the SU(2)xSU(2) gauge groups?

As we saw, these have a large number of flavours (Nf (small f) of order order Nc)?

Not necessarily Landau pole. SU(2) with large number of flavours has a "well-known" UVFP. Gracey, Holdom, Shrock, Pica Sannino,

Resum first terms gives a pole near where beta function vanishes => Need Nf > 16

$$\begin{aligned} \frac{3}{2}\frac{\beta(\alpha)}{A} &= 1 &+ \frac{1}{\tilde{N}}(.3516\tilde{A} - .8057\tilde{A}^2 - 1.567\tilde{A}^3 + 5.342\tilde{A}^4 + \\ &+ \frac{1}{\tilde{N}^2}(-.0206\tilde{A}^2 - 1.602\tilde{A}^3 - 3.244\tilde{A}^4 + ...) \\ &+ \frac{1}{\tilde{N}^3}(-.0555\tilde{A}^3 + ...) + \frac{1}{\tilde{N}^4}(.1198\tilde{A}^4 + ...) \end{aligned}$$

$$\tilde{A} = \frac{2}{15} N_f \frac{\alpha}{\pi}$$
$$\tilde{N} = N_f / 16$$

- The difficult part ...
- Can show in the Veneziano limit the corrections to these terms go like epsilon. Can neglect everything but gauge couplings when determining the SU(2) fixed points.
- By simple power-counting, the SU(2) gauge couplings are subdominant (by 1/Nc) in the original UVFP. Can neglect the SU(2) gauging for this UVFP.

# Summary

- Considered perturbative asymptotically safe QFTs (gauge-Yukawa theories that require scalars)
- UV fixed points do not prefer *any* mass-squared they are relevant operators so simply take any value described by an RG-invariant (multiplicative renormalisation)
- Deviation from zero = breaking of scale invariance, c.f. non-zero quark masses = breaking of chiral symmetry
- Positive mass-squareds can be driven negative in the IR, akin to radiative symmetry breaking in MSSM
- Minimum generated radiatively
- The effect depends on the explicit breaking of flavour structure in the RG invariants.
- Using mechanism to embed the SM looks promising