HEP@VUB BRUSSELS

Higgsploding the Hierarchy Problem

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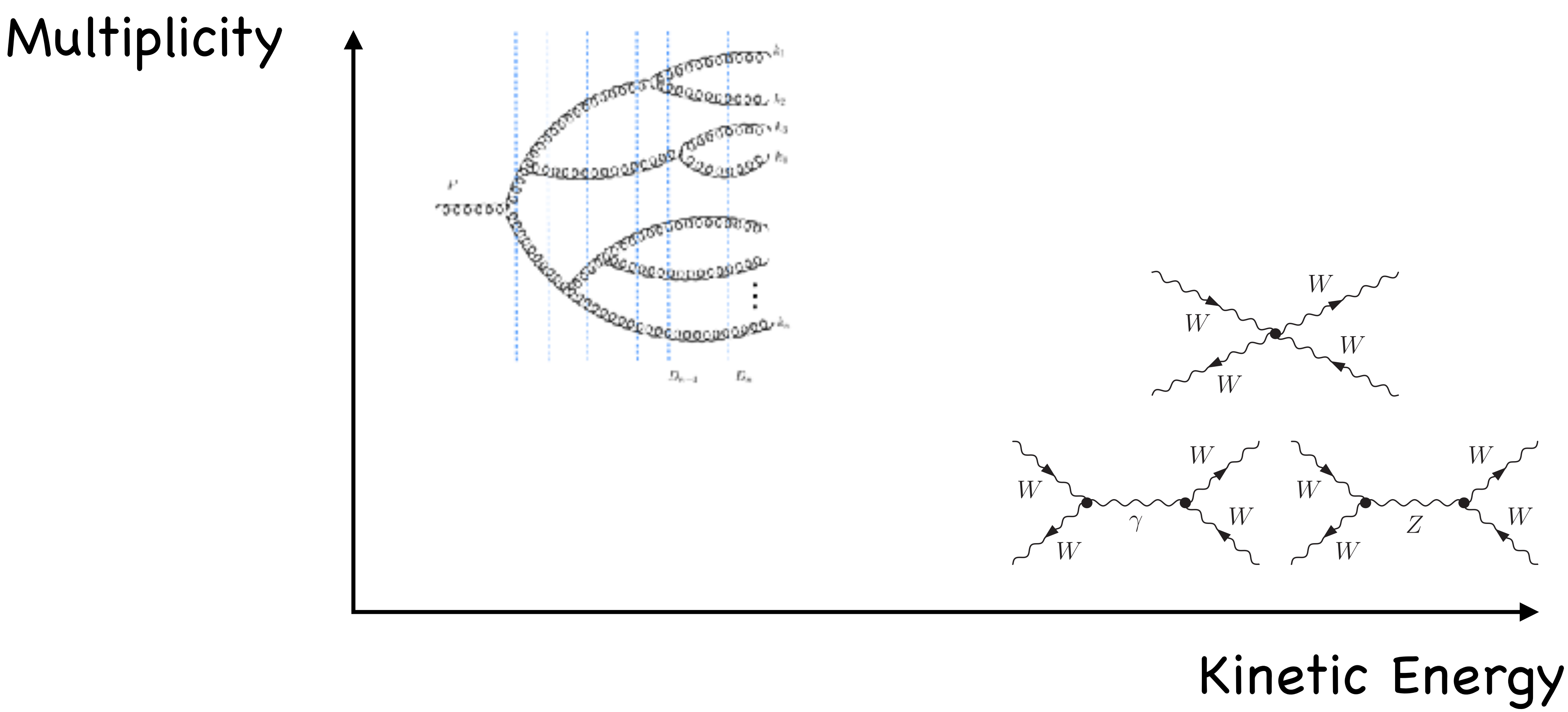
A Standard Model Tale

Before the discovery of the Higgs boson - (Yang-Mills theories)

Situation at tree-level



Perturbative unitarity violated at high energies → model inconsistent (at high energies)

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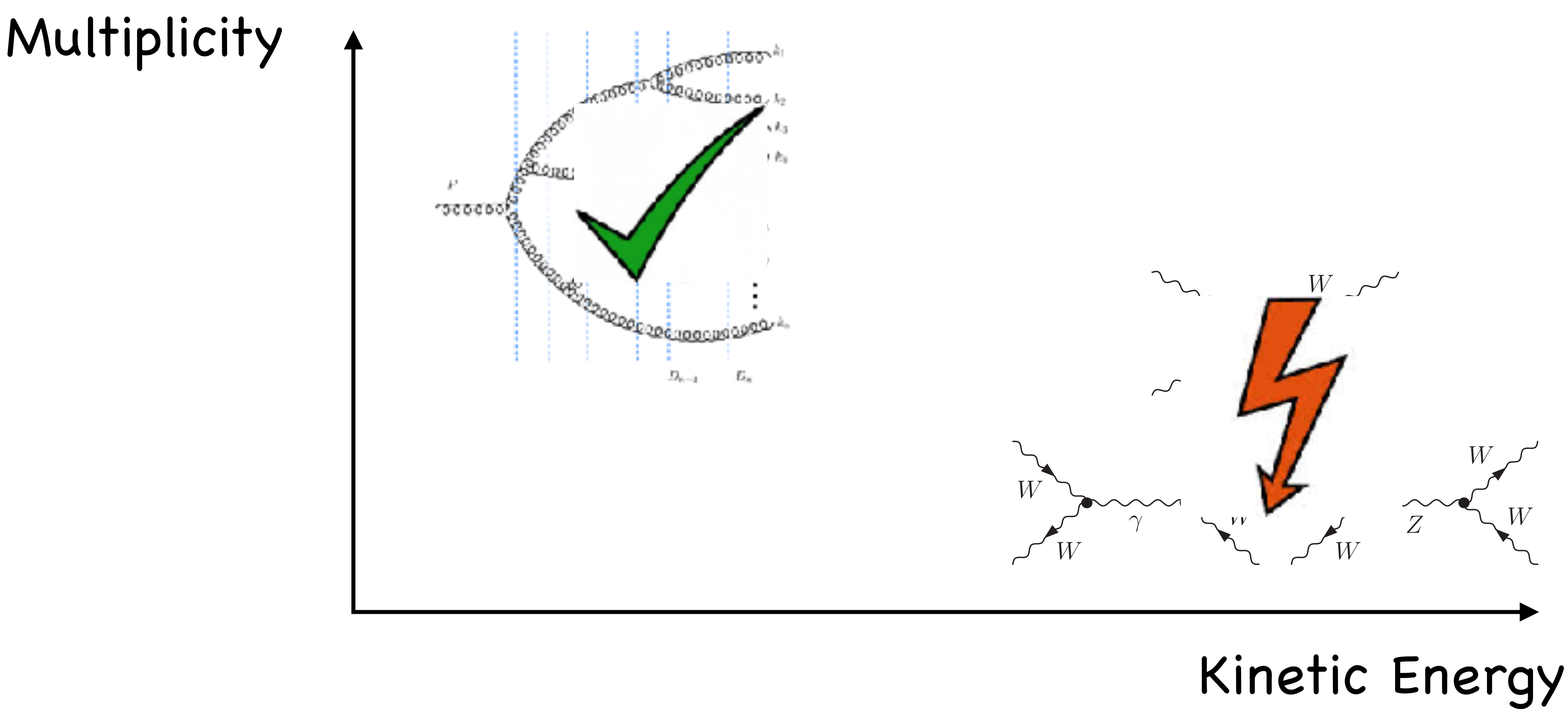

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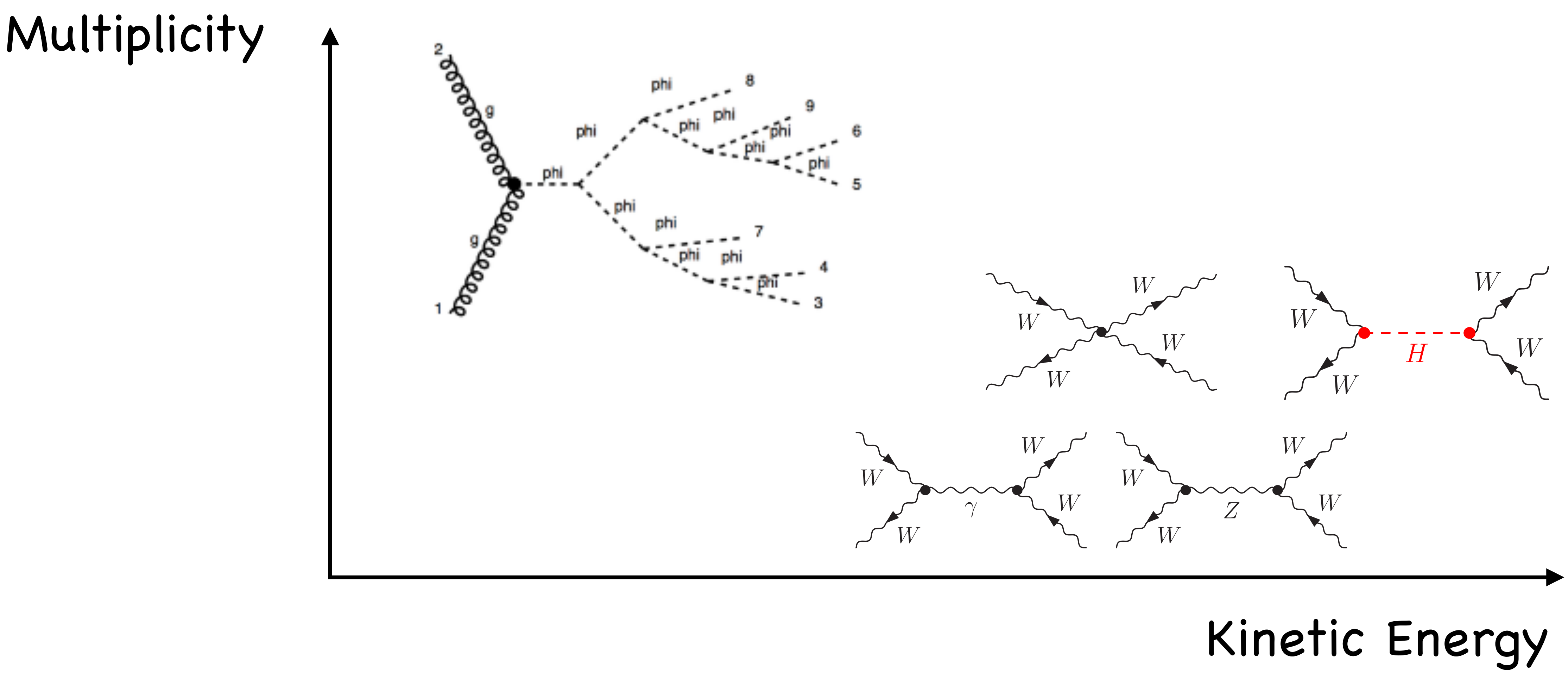
After the discovery of the Higgs boson - complete Standard Model

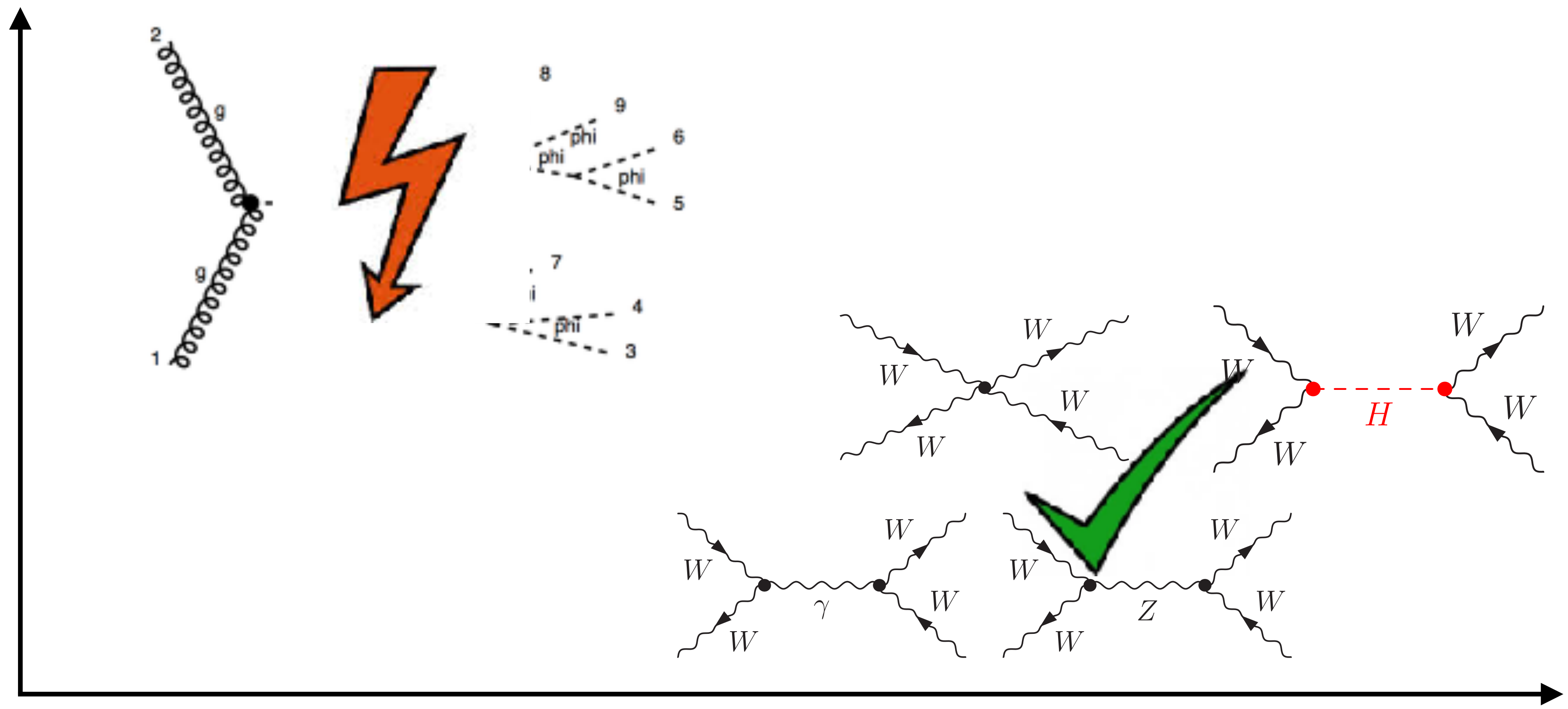
Situation at tree-level



After the discovery of the Higgs boson - complete Standard Model

Situation at tree-level

Multiplicity



Kinetic Energy

Perturbative unitarity violated
at very high multiplicities → model inconsistent
(at high multiplicities)

+ Hierarchy problem (Loop level)

Calculation of 1* -> n amplitudes

Assume Lagrangian

$$\mathcal{L}_\rho(\phi) \,=\, \frac{1}{2}\,(\partial\phi)^2 - \frac{1}{2}M^2\phi^2 - \frac{1}{4}\lambda\phi^4 +\, \rho\,\phi$$

The amplitude is calculated using the LSZ reduction technique [Brown '92]

$$\langle n|\phi(x)|0\rangle \,=\lim_{\rho\to 0}\left[\prod_{j=1}^{n}\lim_{p_j^2\to M^2}\int d^4x_j e^{ip_j\cdot x_j}(M^2-p_j^2)\frac{\delta}{\delta\rho(x_j)}\right]\langle 0_{\rm out}|\phi(x)|0_{\rm in}\rangle_\rho$$

where the tree-level approximation is obtained via $\langle 0_{\rm out}|\phi(x)|0_{\rm in}\rangle_\rho \longrightarrow \phi_{\rm cl}(x)$

and $\phi_{\rm cl}(x)$ is a solution to the classical field equation

IDEA: Fill whole phase-space with particles, i.e. produce all particles at mass threshold

with $\vec{p}_j = 0$ $\quad p_j^\mu = (\omega,\vec{0})$ $\quad$ and $\quad \rho(x) = \rho(t) = \rho_0(\omega)\,e^{i\omega t}$

Here QFT -> time-dep QM:

$$(M^2-p_j^2)\frac{\delta}{\delta\rho(x_j)} \longrightarrow (M^2-\omega^2)\frac{\delta}{\delta\rho(t_j)} = \frac{\delta}{\delta z(t_j)}$$

$$z(t) \,:=\, \frac{\rho_0(\omega)\,e^{i\omega t}}{M^2-\omega^2-i\epsilon} \,:=\, z_0\;e^{i\omega t}\,, \quad z_0 = \text{finite const}$$

Hence, the generating function of tree amplitudes on multi-particle thresholds is a classical solution. It solves an ordinary differential equation with no source term

$$d_t^2\phi + M^2\phi + \lambda\phi^3 \;=\; 0$$

with $$\phi_{\rm cl}(t) \;=\; z(t) \;+\; \sum_{n=2}^{\infty} d_n \, z(t)^n \,, \qquad z := \; z_0 \, e^{iMt}$$

The coefficients d_n determine the actual amplitudes by differentiation w.r.t. z

$$\mathcal{A}_{1\to n} \;=\; \left(\frac{\partial}{\partial z}\right)^n \phi_{\rm cl}\Big|_{z=0} \;=\; n!\, d_n \qquad \textbf{Factorial growth!!}$$

$$\phi_{\rm cl}(t) \;=\; \frac{z(t)}{1 - \frac{\lambda}{8M^2}\, z(t)^2} \qquad \mathcal{A}_{1\to n} \;=\; n! \left(\frac{\lambda}{8M^2}\right)^{\frac{n-1}{2}}$$

Several generalisations of this approach:

- **Higgs like, ie. phi^4 with vev:** [Brown 9209203]

$$\mathcal{L}(h) = \frac{1}{2}(\partial h)^2 - \frac{\lambda}{4}\left(h^2 - v^2\right)^2 \quad \longrightarrow \quad \mathcal{A}_{1\to n} = \left(\frac{\partial}{\partial z}\right)^n h_{\rm cl}\Big|_{z=0} = n!\,(2v)^{1-n}$$

- **Gauge-Higgs theory:** [Khoze 1404.4876]

Higgs process $$\mathcal{A}(h \to n \times h + m \times Z_L) = (2v)^{1-n-m}\, n!\, m!\, d(n,m)$$

Z process $$\mathcal{A}(Z_L \to n \times h + (m+1) \times Z_L) = \frac{1}{(2v)^{n+m}}\, n!\,(m+1)!\, a(n,m)$$

- **Go beyond mass threshold (needs space-dep sol.):** [Khoze 1411.2925]

DGL: $$-(\partial^\mu \partial_\mu + M_h^2)\,\varphi = 3\lambda v\,\varphi^2 + \lambda\,\varphi^3$$

$$\varepsilon = \frac{1}{n\,M_h} E_n^{\rm kin} = \frac{1}{n}\frac{1}{2M_h^2}\sum_{i=1}^n \vec{p}_i^{\,2}$$

↓

$$\mathcal{A}_n(p_1 \ldots p_n) = n!\,(2v)^{1-n}\left(1 - \frac{7}{6}n\,\varepsilon - \frac{1}{6}\frac{n}{n-1}\varepsilon + \mathcal{O}(\varepsilon^2)\right)$$

From amplitudes to cross sections

$$\sigma_{n,m} = \int d\Phi_{n,m} \frac{1}{n!\, m!} \left|\mathcal{A}_{h^* \to n\times h + m\times Z_L}\right|^2 \quad \text{x} \quad \text{flux factor}$$

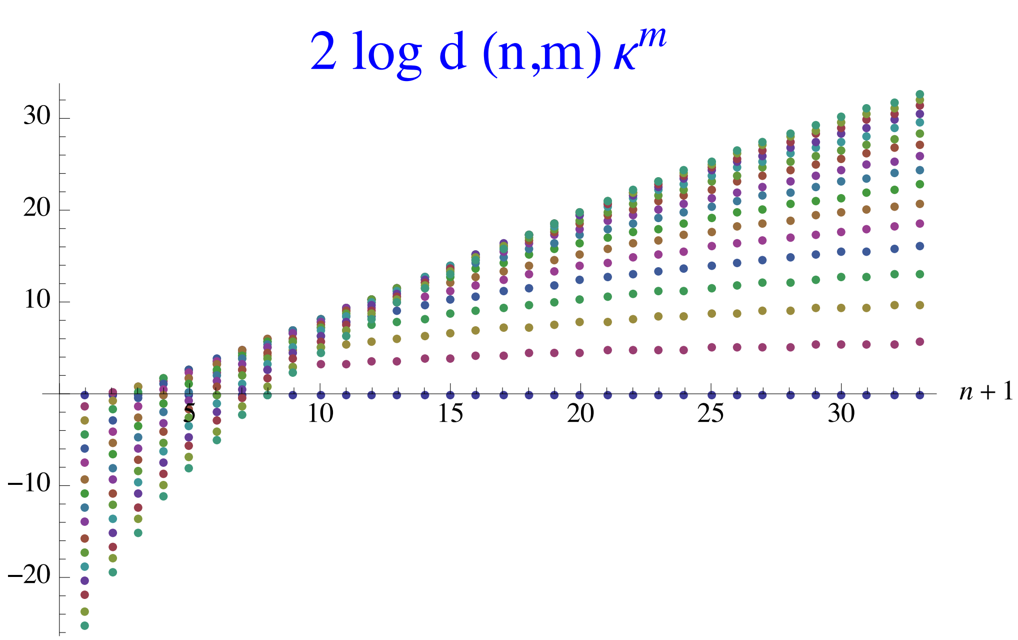
$$\int d\Phi_n = (2\pi)^4 \delta^{(4)}(P_{\text{in}} - \sum_{j=1}^{n} p_j) \prod_{j=1}^{n} \int \frac{d^3 p_j}{(2\pi)^3\, 2p_j^0}$$

Bose statistics factors for n identical Higgs and m identical long. Vec.

Integration with $n\varepsilon_h$ fixed $\Phi_n \simeq \frac{1}{\sqrt{n}} \left(\frac{M_h^2}{2}\right)^n \exp\left[\frac{3n}{2}\left(\log\frac{\varepsilon_h}{3\pi} + 1\right) + \frac{n\varepsilon_h}{4} + \mathcal{O}(n\varepsilon_h^2)\right]$

$$\sigma_{n,m} \sim \exp\Big[2\log(\kappa^m d(n,m)) + n\log\frac{\lambda n}{4} + m\log\frac{\lambda m}{4}$$

$$+ \frac{n}{2}\left(3\log\frac{\varepsilon_h}{3\pi} + 1\right) + \frac{m}{2}\left(3\log\frac{\varepsilon_V}{3\pi} + 1\right) - \frac{25}{12}\, n\,\varepsilon_h - 3.15\, m\,\varepsilon_V + \mathcal{O}(n\varepsilon_h^2 + m\varepsilon_V^2)\Big]$$



How about loops?

Usual criticism: need to include loops to render cross section finite.

Keep in mind, we calculate exclusive rate of massive internal and outgoing particles -> **no mass-divergencies and objects IR-safe**

However, leading loop contributions can be resummed (only valid when $n\lambda < 1$):

Follow Brown's program after Wick-rotating complex operator and using

[Smith '92]
[Voloshin '92]

$$\phi_{0+1}(t) = \frac{z(t)}{1-(\bar{\lambda}/8\bar{m}^2)z(t)^2}\left(1-\frac{3\lambda}{4}F\frac{(\lambda/8m^2)^2 z(t)^4}{(1-(\lambda/8m^2)z(t)^2)^2}\right)$$

one obtains for scalar loops

$$A_n = n!\,(2v)^{1-n}\left[1+n(n-1)\,\frac{\sqrt{3}\,\lambda}{8\pi}+O(\lambda^2)\right]$$

and including fermion loops it is argued cancellations can occur

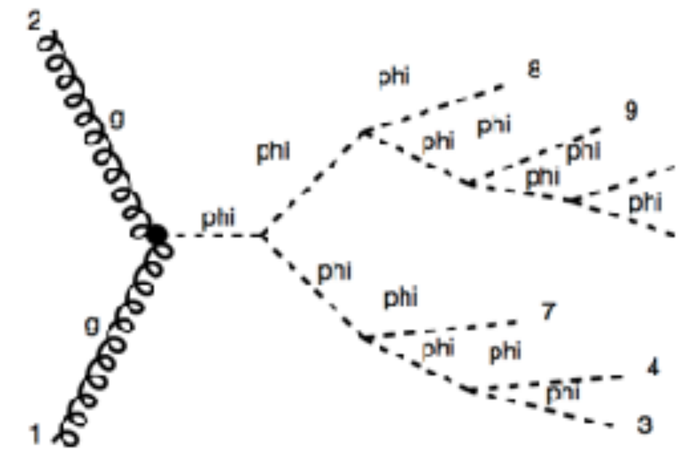
$$A_n \to A_n \times \left[1+(-1)^{2r}\,C(r)\,n^{4r-4}\,\lambda\right]$$

[Voloshin '17]

(exponentiate for $n\lambda > 1$)? in SM subleading to scalar loops

Assuming $\mathcal{L} = \frac{1}{2}\partial^\mu h\, \partial_\mu h - \frac{\lambda}{4}\left(h^2 - v^2\right)^2$

where $\langle h \rangle = v$ and in unit. gauge $\varphi(x) = h(x) - v$



In non-rel. limit the LO cross section for n-Higgs production scales like:

$$\sigma_n \propto \exp\left[\frac{1}{\lambda} F_{\rm h.g.}(\lambda n, \varepsilon)\right] \quad \text{with} \quad \frac{1}{\lambda} F_{\rm h.g.}(\lambda n, \varepsilon) = \frac{\lambda n}{\lambda}\left(f_0(\lambda n) + f(\varepsilon)\right)$$

for a scalar theory with SSB: $f_0(\lambda n) = \log\frac{\lambda n}{4} - 1$ at tree level

[Libanov, Rubakov, Son, Troitsky '94] $f(\varepsilon) \to \frac{3}{2}\left(\log\frac{\varepsilon}{3\pi} + 1\right) - \frac{25}{12}\varepsilon$ for $\varepsilon \ll 1$

Resummed 1-loop contribution:

$$\mathcal{A}_{1\to n} = \mathcal{A}^{\rm tree}_{1\to n} \times \exp\left[B\,\lambda n^2 + \mathcal{O}(\lambda n)\right] \quad \text{with} \quad \mathrm{B} = +\lambda n \frac{\sqrt{3}}{4\pi}$$

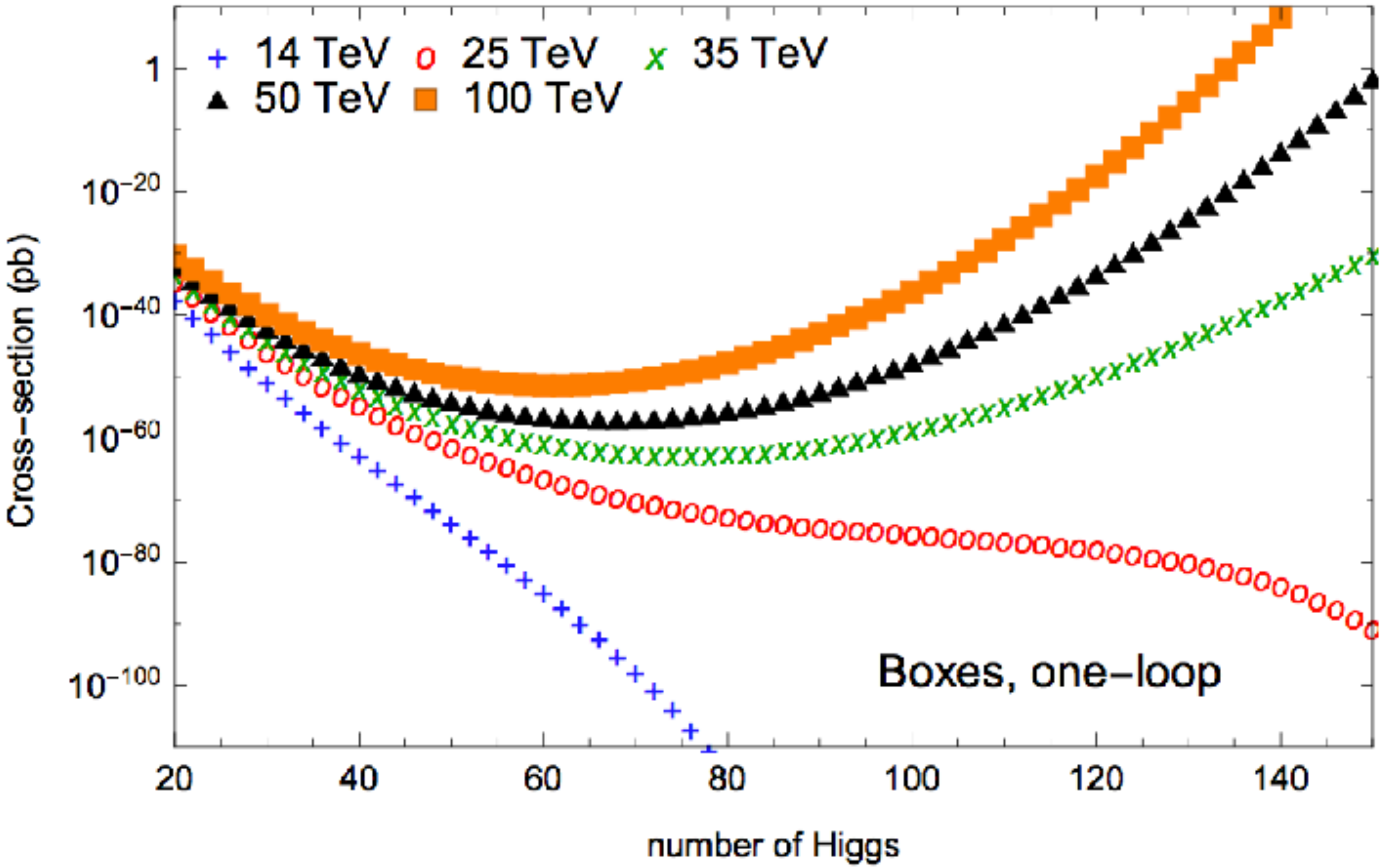
$$f_0(\lambda n) = \log\frac{\lambda n}{4} - 1 + \lambda n \frac{\sqrt{3}}{4\pi}$$

significant enhancement
(but higher corrections unknown)

$$f(\epsilon) \to \frac{3}{2}\left(\log\frac{3}{3\pi} + 1\right) - \frac{25}{12}\epsilon \quad \text{for } \epsilon \ll 1$$

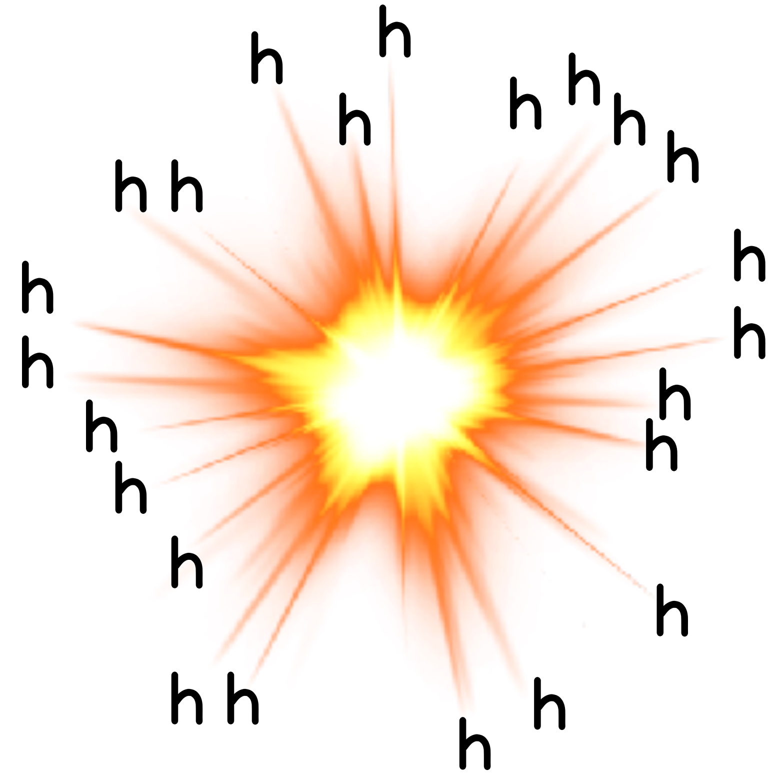
[Smith '92, Voloshin '92]
[Voloshin '17]

Was argued that it could be used to assess what collider energy needed to test a breakdown of perturbativity



[Degrande, Khoze, Mattelaer '16]

Explosive growth of 1->n process

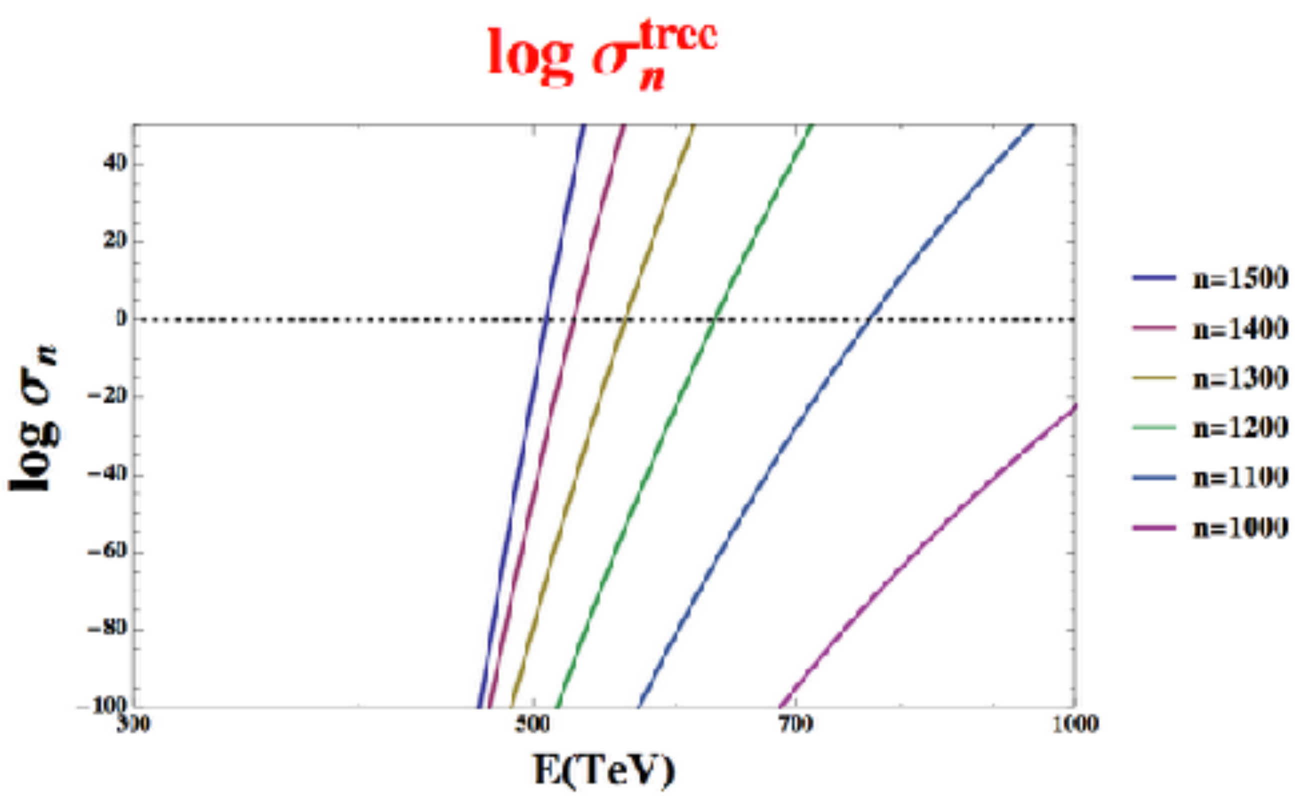


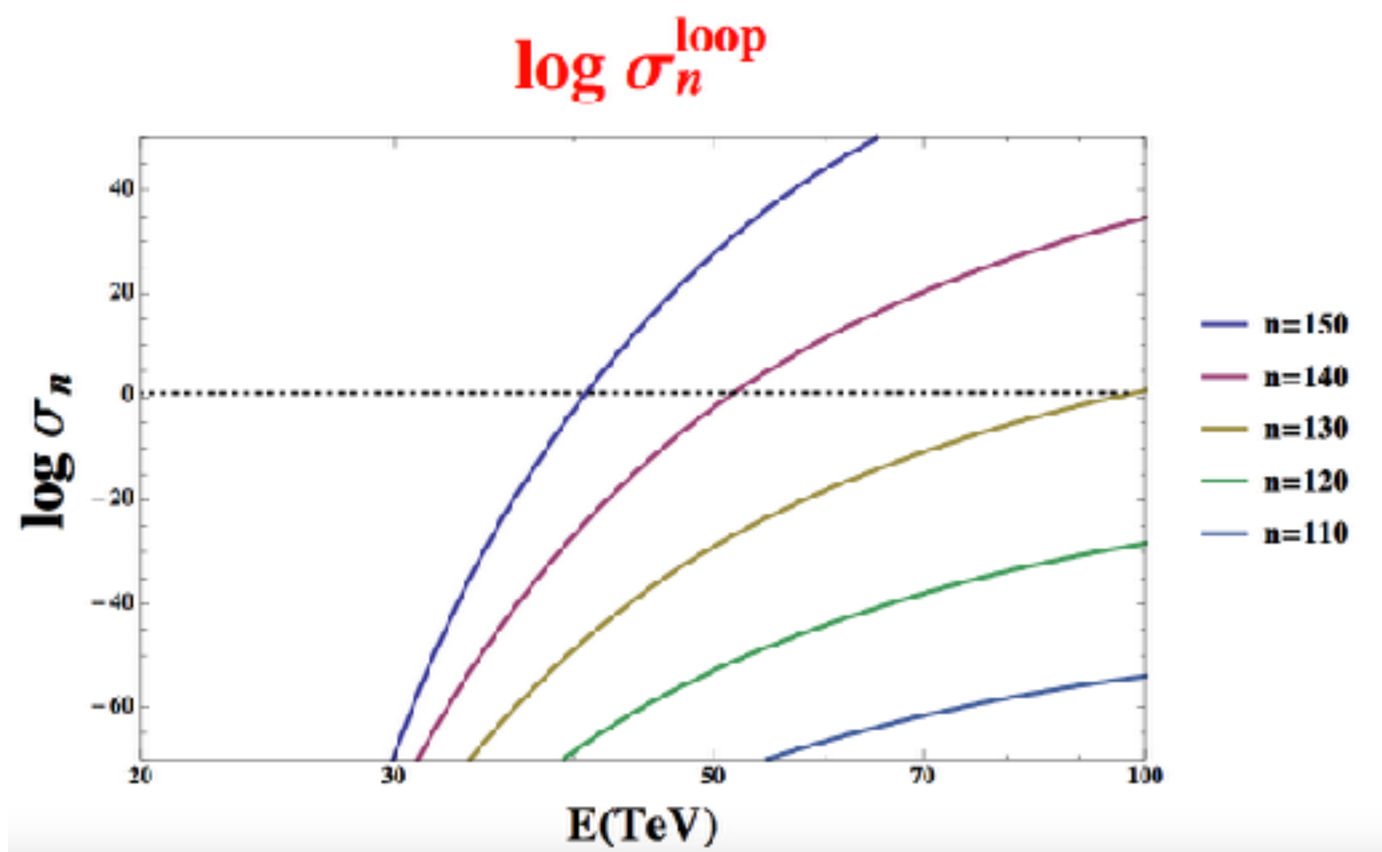
$$\mathcal{R}_n(s) := \frac{1}{2M_h^2}\int d\Pi_n |\mathcal{M}(1\to n)|^2$$

$$\mathcal{R}(\lambda; n, \varepsilon) = \exp\left[n\left(\log\frac{\lambda n}{4} - 1\right) + \frac{3n}{2}\left(\log\frac{\varepsilon}{3\pi} + 1\right) - \frac{25}{12}n\varepsilon\right]$$

Extreme energy dependence for 1* -> n cross section

including 1-loop result reduces 'ignition' scale





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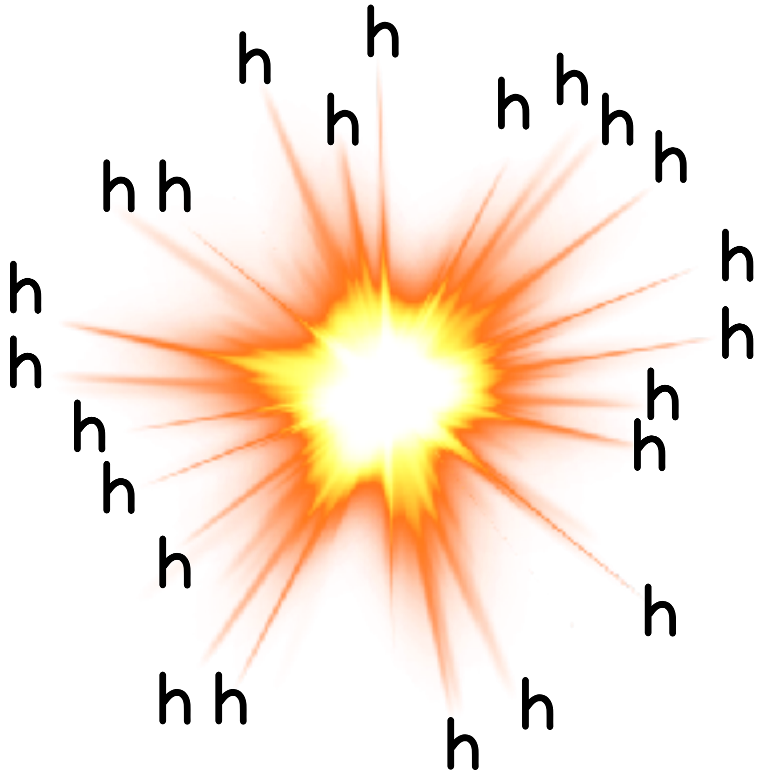

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Extreme energy dependence for 1^* -> n cross section

including 1-loop result reduces 'ignition' scale

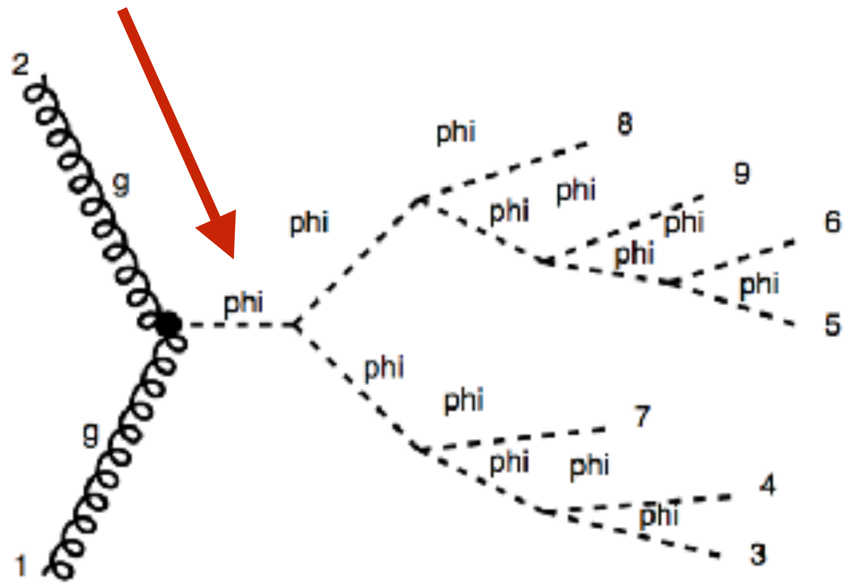
energy low

energy beyond threshold

Higgspersion

[Khoze, MS '17]

However, previous calculations neglected 'width'/self-energy contribution to scalar propagator



Analytic expression for process

$$\mathcal{M}_{gg\to h^*} \times \frac{i}{p^2 - M_h^2 + i\, M_h\, \Gamma(p^2)} \times \mathcal{M}_{h^*\to n\times h}$$

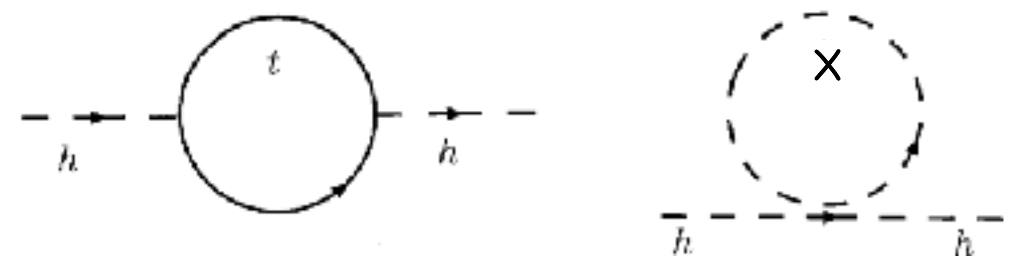
results in limit $s \gg M_h^2,\, m_t^2$ in

$$\sigma^{\Delta}_{gg\to n\times h} \sim y_t^2 m_t^2 \log^4\left(\frac{m_t}{\sqrt{s}}\right) \times \frac{1}{s^2 + M_h^4 \mathcal{R}^2} \times \mathcal{R}_n$$

For large R cross section small $\sigma_{gg\to n\times h} \sim \begin{cases} \mathcal{R} & : \text{ for } \mathcal{R} \lesssim 1 \\ 1/\mathcal{R} \to 0 & : \text{ for } \mathcal{R} \gg 1 \text{ at } s \to \infty \end{cases}$

-> no violation of perturbative unitarity for large multiplicities

Higgsploding the Hierarchy Problem [Khoze, MS '17]



$$\Delta M_h^2 \sim \lambda_P \int \frac{d^4p}{16\pi^4} \frac{1}{M_X^2 - p^2 + i\,\mathrm{Im}\,\Sigma_X(p^2)}$$

$$= \lambda_P \int \frac{d^4p}{16\pi^4} \left(\frac{M_X^2 - p^2}{(M_X^2 - p^2)^2 + (\mathrm{Im}\,\Sigma_X(p^2))^2} - \frac{i\,\mathrm{Im}\,\Sigma_X(p^2)}{(M_X^2 - p^2)^2 + (\mathrm{Im}\,\Sigma_X(p^2))^2} \right)$$

Due to Higgsplosion the multi-particle contribution to the width of X explode at $p^2 = s_\star$ where $\sqrt{s_\star} \simeq \mathcal{O}(25)\mathrm{TeV}$

→ It provides a sharp UV cut-off in the integral, possibly at $s_\star \ll M_X^2$

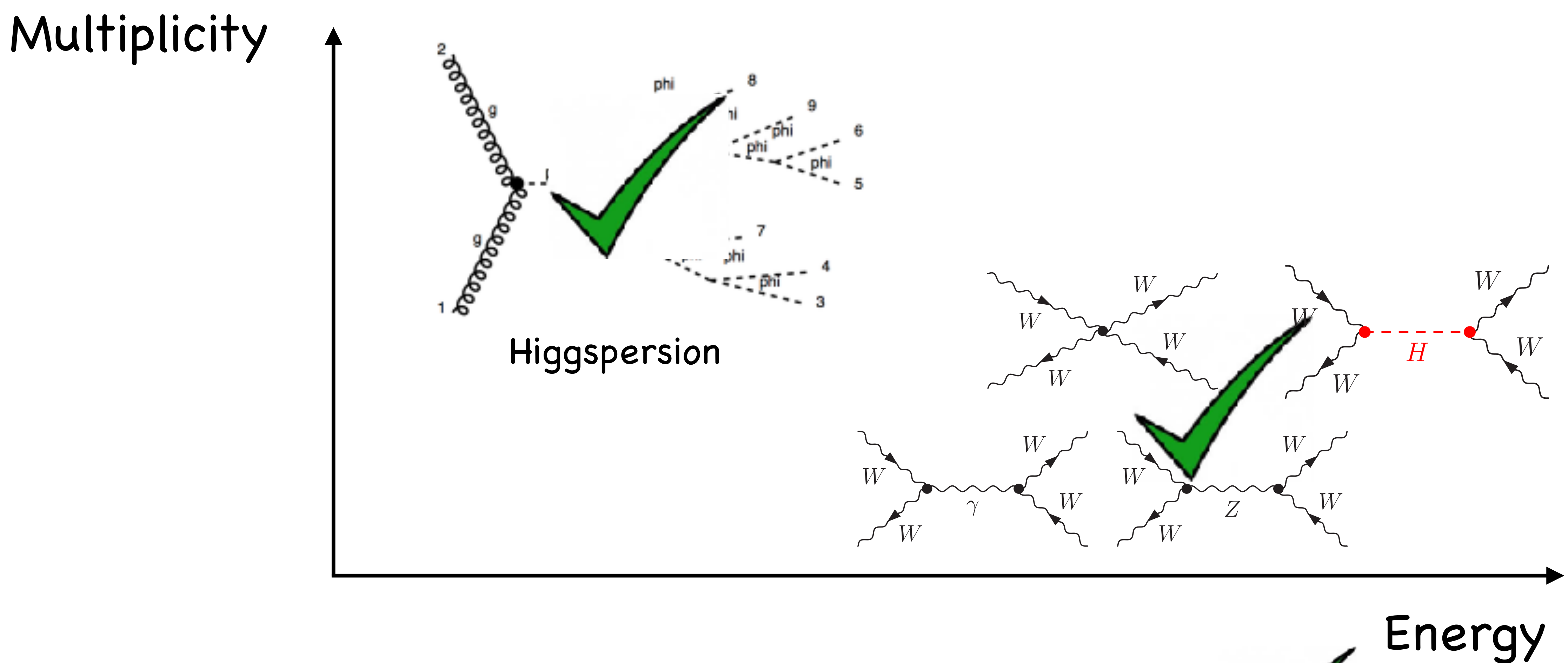
Hence, the contribution to the Higgs mass amounts to

$$\Delta M_h^2 \propto \lambda_P \frac{s_\star}{M_X^2} s_\star \ll \lambda_P M_X^2$$

and thus mends the Hierarchy problem by $\left(\frac{\sqrt{s_\star}}{M_X}\right)^4 \simeq \left(\frac{25\,\mathrm{TeV}}{M_X}\right)^4$

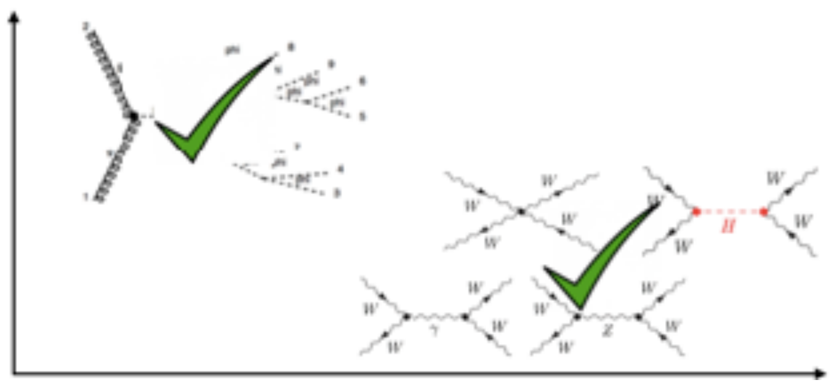
If Higgsplosion is not a mathematical artefact but realised in nature:

Situation at tree-level



+ Hierarchy problem (Loop level)

→ SM heals itself, retains self-consistency to very high energies and multiplicities



Summary

Higgs boson can be cause and cure for its Hierarchy Problem

Obvious question:
If Higgsplosion realised in nature, what does it imply for physics beyond O(100) TeV?

You can join one of two camps

Believer	Denier
-> build O(100) TeV collider	-> work on QFT

But you cannot be indifferent