# *A Clockwork Tale*

DANIELE TERESI

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Service de Physique Théorique, Université Libre de Bruxelles, Belgium

THE FATE OF NATURALNESS, VUB, 12/06/17

# *A Medieval Tale*

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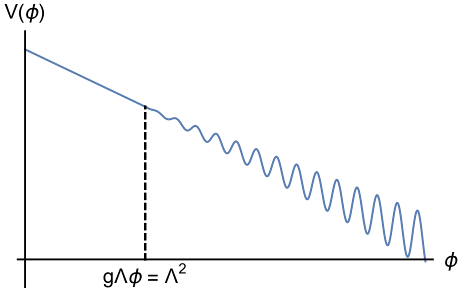
# *Prologue:*
# *Making numbers small dynamically*

# Naturalness from a relaxion [Graham, Kaplan, Rajendran, '15]

- slow-rolling pseudo NG boson: **relaxion** $\phi$, slope from $g\Lambda^3\phi$
- relaxion-dependent $H$ mass term: $(\Lambda^2 - g\Lambda\phi)\, H^\dagger H$
- backreaction when $H$ vev: $f_\pi^2\, m_\pi^2(v) \cos(\phi/f)$
- $\implies v \ll \Lambda$ **dynamically selected**:



# Clockworking the relaxion

- it requires **tiny** $g$, e.g. $g = O(v/\Lambda)^4 \approx 10^{-50}$
- technically natural, NG shift symmetry for $g \to 0$, but still ...
- it requires **trans-planckian** $\Delta\phi$
- Solution: [Choi, Im , '15; Kaplan, Rattazzi, '15]
- $g$ from much larger period $F \gg f$:

$$-\mathcal{L} \supset \left[\Lambda^2 - \Lambda^2 \cos\left(\frac{\phi}{F} + \alpha\right)\right] H^\dagger H \; - \; \Lambda^4 \cos\left(\frac{\phi}{F} + \alpha\right) \; - \; m_{BR}^4(v) \cos\frac{\phi}{f}$$

$$\Longrightarrow g = \Lambda/F$$

- $F = 3^N f$ from **clockwork** chain:

$$-\mathcal{L} \supset \epsilon \left(\Phi_0^\dagger \Phi_1^3 + \Phi_1^\dagger \Phi_2^3 + \ldots + \Phi_{N-1}^\dagger \Phi_N^3\right) \; + \; \frac{\phi_1}{\hat{f}} G\widetilde{G} \; + \; \frac{\phi_N}{\hat{f}} G\widetilde{G}$$

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# Get rid of the relaxion, keep the clockwork

- the last step itself is sufficient to **generate hierarchies**! [Giudice, McCullough, '16]
- **clockwork mechanism** $\rightarrow$ an elegant and economical way to generate **tiny numbers**/large hierarchies $X$ with only $\mathcal{O}(1)$ **couplings** and $N \sim \log X$ **fields**
- a **framework** for model building [Giudice, McCullough, '16], useful for:
  - low-scale invisible axions [Giudice, McCullough, '16; Farina, Pappadopulo, Rompineve, Tesi, '16]
  - **hierarchy problem** [Giudice, McCullough, '16]
  - flavour puzzle?
  - inflation [Kehagias, Riotto, '16]
  - **dark matter** [Hambye, DT, Tytgat, '16] (used in **this talk** to explain main features)
- dark matter cosmologically stable if decays by dim-5 ($\Lambda \gg M_{PL}$), dim-6 ($\Lambda \sim M_{GUT}$), tiny couplings $\Longrightarrow$ **all difficult to test**
- **clockwork** mechanism $\rightarrow$ dark matter cosmologically **stable** although it **decays into SM** via $\mathcal{O}(1)$ **interactions** with **TeV-scale** particles!
- large interactions $\Longrightarrow$ dark matter is a **thermal relic**, i.e. a **WIMP**

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# *Chapter 1:*
# *How to do multiplications in QFT*

# The clockwork mechanism

Based on the simple observation that:

$1/2 \quad \times \quad 1/2 \quad \times \quad 1/2 \quad \times \quad 1/2 \quad \times \quad \ldots \quad \times \quad 1/2$ can **easily** be **tiny**

Use a **chain** of N fields

$$\phi_0 \ \overset{1/q}{\text{———}} \ \phi_1 \ \overset{1/q}{\text{———}} \ \phi_2 \ \overset{1/q}{\text{———}} \ \phi_3 \ \overset{1/q}{\text{———}} \ \ldots \ \overset{1/q}{\text{———}} \ \phi_N \ \text{———} \ \mathbf{SM}$$

if clever **symmetry** $\longrightarrow \quad \phi_{light} \approx \phi_0 \quad \Longrightarrow \quad \phi_{light} - \mathbf{SM} \sim \mathbf{1/q^N} \quad (q > 1)$

For **fermions** use chiral symmetries

$$R_0 \ \overset{m}{\text{——}} \ \underbrace{L_1 \quad R_1}_{qm} \ \overset{m}{\text{——}} \ \underbrace{L_2 \quad R_2}_{qm} \ \overset{m}{\text{——}} \ \underbrace{L_3 \quad R_3}_{qm} \ \overset{m}{\text{——}} \ \ldots \ \overset{m}{\text{——}} \ \underbrace{L_N \quad R_N}_{qm} \ \text{——} \ L_{SM}$$

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# Clockwork scalar

- For **scalars**, use a chain of $N+1$ symmetries: $U(1)_0 \times U(1)_1 \times \ldots \times U(1)_N$
- broken by $N$ spurions $m_k^2 \equiv m^2$ with $\quad Q_k(m_k^2) = 1, \quad Q_{k+1}(m_k^2) = -q \quad (q > 1)$
- $\mathcal{L} = -\frac{f^2}{2} \sum_{k=0}^{N} |\partial U_k|^2 \; + \; \frac{m^2 f^2}{2} \sum_{k=0}^{N-1} \left( U_k^\dagger U_{k+1}^q + h.c. \right)$
- for the Goldstones $\phi_k$, $U_k \propto e^{i\phi_k/f}$: $\quad -\mathcal{L} \supset \frac{m^2}{2} \sum_{k=0}^{N-1} \left( \phi_k - q\phi_{k+1} \right)^2$
- **unbroken** $U(1)$ with $\mathcal{Q} = \sum_k \frac{Q_k}{q^k} \quad \Longrightarrow \quad$ **massless** $\varphi_0 = \mathcal{N} \sum_k \frac{\phi_k}{q^k}$
- For instance, if $\mathcal{L} \supset \frac{\phi_N}{16\pi^2 f} G\widetilde{G} \quad \Longrightarrow \quad \frac{\varphi_0}{16\pi^2 F} G\widetilde{G}$ with $\quad F = f\,\frac{q^N}{\mathcal{N}} \gg f$

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# Clockwork dark matter [Hambye, DT, Tytgat, '16]

- chiral symmetry group:

$$U(1)_{R_0} \times U(1)_{L_1} \times U(1)_{R_1} \times \ldots \times U(1)_{L_N} \times U(1)_{R_N} \quad \text{with} \quad U(1)_{R_N} \equiv U(1)_{L_{SM}}$$

- **scalars**:

$S_i \sim (-1, 1)$ under $U(1)_{R_i} \times U(1)_{L_{i+1}}$ $\qquad C_i \sim (1, -1)$ under $U(1)_{L_i} \times U(1)_{R_i}$

- chain of fields:

$$\mathbf{R_0} \;\underline{\;\; S_1 \;\;}\; L_1 \;\underline{\;\; C_1 \;\;}\; R_1 \;\underline{\;\; S_2 \;\;}\; L_2 \;\underline{\;\; C_2 \;\;}\; \ldots \;\underline{\;\; C_N \;\;}\; R_N \;\underline{\qquad}\; \mathbf{L_{SM}}$$

- **clockwork** mechanism when scalars acquire a **vev**:

$$m = y_S \langle S_i \rangle \qquad qm = y_C \langle C_i \rangle$$

- **Majorana mass** $m_N$ for $R_0$, eigenstate $N \approx R_0$ is the **dark-matter** candidate

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# Clockwork fermion

- the Lagrangian is

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{kinetic}} - \sum_{i=1}^{N} \left( y_S S_i \bar{L}_i R_{i-1} - y_C C_i \bar{L}_i R_i + h.c. \right) \\ - \left( y \, \bar{L}_{SM} \widetilde{H} R_N + h.c. \right) - \frac{1}{2} \left( m_N \overline{R_0^c} \, R_0 + h.c. \right)$$

- after the scalars acquire vevs $m = y_S \langle S_i \rangle \,, \quad qm = y_C \langle C_i \rangle$:

$$\mathcal{L} \supset -m \sum_{i=1}^{N} \left( \bar{L}_i R_{i-1} - q \, \bar{L}_i R_i \right) - \frac{m_N}{2} \overline{R_0^c} \, R_0 + h.c.$$

- for $m_N = 0$, the "right-handed" mass matrix satisfies $M^\dagger M \equiv M^2_{scalar}$

- clockwork mechanism for $m_N \lesssim qm \quad$ (for $q \gg 1$)

# *Chapter 2:*
# *How clockwork matter became dark*

# The spectrum

Take $q \gg 1$ for simplicity

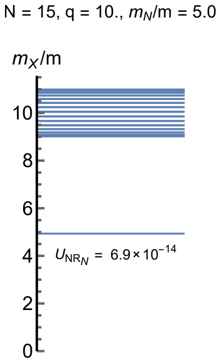
- the **dark-matter** Majorana fermion $N$ with mass $\approx m_N$:

$$N \approx R_0 + \frac{1}{q^1}R_1 + \frac{1}{q^2}R_2 + \ldots + \frac{1}{q^N}R_N$$

- a **band** of N **pseudo-Dirac** $\psi_i$ with mass $\approx qm$:

$$\psi_i \approx \frac{1}{\sqrt{N}}\sum_k \mathcal{O}(1)L_k + \mathcal{O}(1)R_k$$

- $N$ scalars $S_i$ and $C_i$ expected in the same mass range (not necessarily dynamic, but not discussed here)

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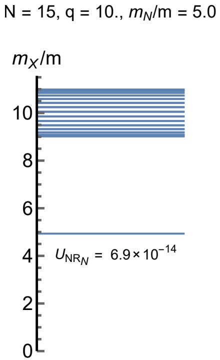


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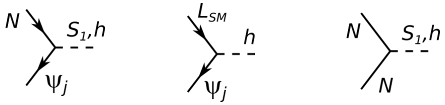

# Cosmological (meta)stability of dark matter

$$\mathbf{R_0} \ \underline{\ y_S\langle S_1\rangle\ } \ L_1 \ \underline{\ y_C\langle C_1\rangle\ } \ R_1 \ \underline{\ y_S\langle S_2\rangle\ } \ L_2 \ \underline{\ y_C\langle C_2\rangle\ } \ \ldots \ \underline{\ y_C\langle C_N\rangle\ } \ R_N \ \underline{\ y\,h\ } \ \mathbf{L_{SM}}$$

$N$ can **decay**, e.g. $N \to \nu h, \nu Z, lW$, but

The coupling of **dark matter** to **SM fermions** is **clockwork suppressed**:

$$\mathcal{L} \supset -\frac{y}{q^N}\,\bar{L}_{SM}\widetilde{H}N_R$$

**Dark matter cosmologically stable**

The decay lifetime of $N$ longer than the age of the Universe with $\mathcal{O}(1)$ **couplings** and $\lesssim$ **TeV-scale** states

- indirect detection $\Longrightarrow q^{2N} > 1.5 \times 10^{50}\ \left(\frac{m_N}{\mathrm{GeV}}\right) y^2$
  for example: $m_N \sim 100\,\mathrm{GeV}$, $y \sim 1$, $q \sim 10$, $N \sim 26$
- effect of **clockwork gears** $\psi_j$ in loop diagrams also **clockwork-suppressed**

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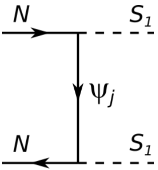
# Scenario A: $m_S < m_N$

Dominant process:



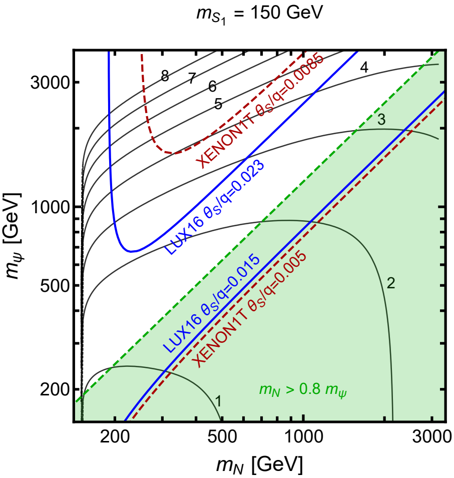

from $N \sim R_0$, $\Psi_j \supset L_1$
and $y_S\, S_1 \overline{L}_1 R_0$

**not clockwork-suppressed!**

$\Longrightarrow$ **N is a WIMP**

perturbative $y_S < \sqrt{4\pi} \simeq 3.5$

$\Longrightarrow$ **N and $\psi_j$ light enough**




$y_S$ needed for correct $\Omega_{DM}$

# Scenario B: $m_N < m_S$ and $2m_N < m_S + m_h$

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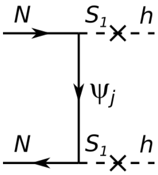
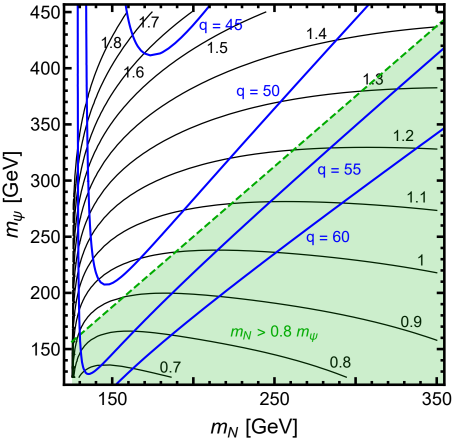



$\propto \quad (y_S\theta_S)^4$

$\theta_S \lessapprox 0.4$ **from colliders**

$y_S$ non-perturbative for universal $\theta_S$:
$\theta_S \lessapprox 0.4/\sqrt{N}$

it works also near the $h$ and $S$ resonances, for universal $\theta_S$ too




$y_S\theta_S$ needed for correct $\Omega_{DM}$

# Other limits and prospects

- Indirect detection: annihilation is p-wave, but decays $N \to h\nu$ monochromatic
- $\psi_j$ in the hundreds of GeV range, coupled via $y\,\overline{L}_{SM}HR_N$ and $\psi_j \supset R_N$
  $\Longrightarrow$ pseudo-Dirac **RH neutrinos** in the **observable range**, **y sizeable**
  - EWPT: $|B_{l\psi}|^2 \equiv y^2v^2/(2m_\psi^2) \lesssim 10^{-3}$
  - LFV: $BR(\mu \to e\gamma) \;\approx\; 8\times 10^{-4}\,|B_{e\Psi}|^2\,|B_{\mu\Psi}|^2 < 4.2\times 10^{-13}$
  - direct L-conserving searches: up to $m_\psi \approx 200$ GeV with 300 $\mathrm{fb}^{-1}$ [Das, Dev, Okada, '14]
  - if $m_N \lll m_\psi$ L-violating searches: up to $m_\psi \approx 300$ GeV with 300 $\mathrm{fb}^{-1}$ [Deppisch, Dev, Pilaftsis, '15]
- In scenario B $S_1$ **needs** to have **large mixing with $h$**, in A it can
  $\Longrightarrow$ limits and searches for scalar singlets [Falkowski, Gross, Lebedev, '15; Robens, Stefaniak, '15]
  - for $m_S < 500\,\mathrm{GeV}$: $\theta_S < 0.3-0.4$ from direct searches
  - for $m_S > 500\,\mathrm{GeV}$: $\theta_S \lesssim 0.3-0.4$ from EWPT

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  $\Longrightarrow$ pseudo-Dirac **RH neutrinos** in the **observable range**, **y sizeable**
  - EWPT: $|B_{l\psi}|^2 \equiv y^2v^2/(2m_\psi^2) \lesssim 10^{-3}$
  - LFV: $BR(\mu \to e\gamma) \;\approx\; 8 \times 10^{-4}\,|B_{e\Psi}|^2\,|B_{\mu\Psi}|^2 < 4.2 \times 10^{-13}$
  - direct L-conserving searches: up to $m_\psi \approx 200$ GeV with 300 $\mathrm{fb}^{-1}$ [Das, Dev, Okada, '14]
  - if $m_N \not\ll m_\psi$ L-violating searches: up to $m_\psi \approx 300$ GeV with 300 $\mathrm{fb}^{-1}$ [Deppisch, Dev, Pilaftsis, '15]
- In scenario B $S_1$ **needs** to have **large mixing with** $h$, in A it can
  $\Longrightarrow$ limits and searches for scalar singlets [Falkowski, Gross, Lebedev, '15; Robens, Stefaniak, '15]
  - for $m_S < 500\,\mathrm{GeV}$: $\theta_S < 0.3 - 0.4$ from direct searches
  - for $m_S > 500\,\mathrm{GeV}$: $\theta_S \lesssim 0.3 - 0.4$ from EWPT

# Other limits and prospects

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# Majorana neutrino masses [Hambye, DT, Tytgat, '16]

- SM leptons interact with TeV-scale $\psi_i$ with large Yukawas $\Longrightarrow$ **huge $m_\nu$**???
- Clockwork at work: if there were no $R_0 \Longrightarrow$ no chiral partner for $\nu$s
  but effect of $R_0$ has to go through the **whole clockwork chain**:

$$m_\nu \simeq \frac{m_D^2}{q^{2N} m_N}$$

- suppression here is smaller than for DM: $q = 10, m_N = 1\,\text{TeV} \Longrightarrow N \approx 7$
- $\geq 2$ nonzero $m_\nu \Longrightarrow$ at least **2 clockwork chains**
- a **suggestive possibility**: 1 chain for dark matter, 2 chains for neutrino masses
- + resonant leptogenesis? Starting from 2 degenerate, a small mass splitting can be generated by interactions with the clockwork gears...
- many model-building variants (not discussed here)

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# *Chapter 3:*
# *One more dimension*

# Clockwork from a flat extra dimension [Hambye, DT, Tytgat, '16]

- the clockwork Lagrangian can come from a **discretized 5th dimension**
- **flat-spacetime** construction for fermion:
- 1 Dirac fermion with mass M in the 5D bulk $\to L_i, R_i$

$$\mathcal{L}_5 \supset \bar{\psi}(i\overleftrightarrow{\not\partial}_D - M)\psi = i\overline{\psi}\gamma^\mu \partial_\mu \psi + \left[\frac{1}{2}\left(\overline{L}\,\partial_Z R - (\partial_Z \overline{L})R\right) - M\overline{L}R + h.c.\right]$$

- + **Wilson term** $-\frac{a}{2}\,\partial_Z\overline{\psi}\,\partial_Z\psi = -\frac{a}{2}\,\partial_Z\overline{L}\,\partial_Z R$ removes 1 hopping direction
- to get light mode, **1 chiral fermion** on one brane $\to R_0$ (or Dirichlet b.c. $L(0) = 0$)
- **SM chiral leptons** on the other brane $\to L_{SM}$
- discretized Lagrangian $\mathcal{L} \supset \sum_{i=0}^{N-1} \frac{1}{a}\,\overline{L}_{i+1}R_i - \sum_{i=1}^{N}\left(\frac{1}{a} + M\right)\overline{L}_i R_i$
- clockwork with $m = \frac{1}{a}$, $\quad qm = \frac{1}{a} + M$, $\quad q^N = \left(1 + \frac{\pi R M}{N}\right)^N \to e^{\pi R M}$

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## Clockwork from the metric [Giudice, McCullough, '16]

- **curved-spacetime** construction for scalar:
- curved **metric** $ds^2 = X(|Z|)\,dx^2 \;+\; Y(|Z|)\,dZ^2$
- **massless** scalar in the 5D bulk:

$$\mathcal{S} = -2\int_0^R dZ \int d^4x\,\sqrt{-g}\,\frac{1}{2}\,g^{MN}\partial_M\phi\,\partial_N\phi = -\int_0^R dZ\int d^4x\,X^2Y^{1/2}\left[\frac{(\partial_\mu\phi)^2}{X} + \frac{(\partial_Z\phi)^2}{Y}\right]$$

- discretized Lagrangian:

$$\mathcal{L} \supset \sum_{j=0}^{N-1} m_j^2(\phi_j - q_j\phi_{j+1})^2 \qquad \text{with } m_j^2 = \frac{X_j}{a^2Y_j}, \quad q_j = \frac{X_j^{1/2}Y_j^{1/4}}{X_{j+1}^{1/2}Y_{j+1}^{1/4}}$$

- clockwork if $X_j \propto Y_j$
  finite for $N \to \infty$ if $X_j \propto Y_j \propto e^{-\frac{4}{3}k\,a\,j}$
- $m = \dfrac{1}{a}, \quad q = e^{k\,a}, \quad q^N = e^{\pi kR}$

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# The clockwork metric

- in the **continuum**: $ds^2 = e^{\frac{4}{3}k|Z|}\,(dx^2 + dZ^2)$
- Kaluza-Klein modes for massless scalar

$$\psi_0(Z) \simeq \sqrt{k\pi R}\, e^{-k\pi R} \qquad \Longrightarrow \qquad \frac{dP}{dZ} \propto e^{2kZ}$$

$$\psi_n(Z) = e^{-kZ} \times \text{oscillatory} \qquad \Longrightarrow \qquad \frac{dP}{dZ} = \text{oscillatory}$$

- what about **Large Extra Dimension** or **Randall-Sundrum**?

| | $m_j$ | $q_j$ |
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| LED | $\frac{1}{a}$ | 1 |
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# *Chapter 4:*
# *Clockwork naturalness*

# Clockwork graviton

- discrete clockwork: $N+1$ **copies** of **4D gravity** $g_j^{\mu\nu}$
- linear approximation: $g_j^{\mu\nu} = \eta_j^{\mu\nu} + 2h_j^{\mu\nu}/M_j$
- clockwork Pauli-Fierz mass terms

$$\mathcal{L} = -\frac{m^2}{2}\sum_{j=0}^{N-1}\left([h_j^{\mu\nu} - q\, h_{j+1}^{\mu\nu}]^2 - [\eta_{\mu\nu}(h_j^{\mu\nu} - q\, h_{j+1}^{\mu\nu})]^2\right)$$

- invariant under $h_j^{\mu\nu} \to h_j^{\mu\nu} + \frac{1}{q^j}(\partial^\mu A^\nu + \partial^\nu A^\mu)$
- $\Longrightarrow$ **massless graviton** $\mathfrak{h}_0^{\mu\nu}$ **localized** at $j=0$:

$$\frac{1}{M_N} h_N^{\mu\nu} T_{\mu\nu} \longrightarrow \frac{1}{M_P}\mathfrak{h}_0^{\mu\nu} T_{\mu\nu} \quad \text{with} \quad M_P = \frac{q^N M_N}{\mathcal{N}}$$

- but... multi-gravity theories are dodgy $\to$ **continuum limit**

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# The metric from the linear dilaton

- we want **massless 5D gravity** with a clockwork **metric** $ds^2 = e^{\frac{4}{3}k|Z|}(dx^2 + dZ^2)$
- clockwork gravity $\to$ metric should not be treated as a background
- can we **obtain** the metric?
- linear **dilaton** model (Jordan frame):

$$\mathcal{S} = \int d^4x\, dZ \sqrt{-g}\, \frac{M_5^3}{2}\, e^S (\mathcal{R} + g^{MN}\partial_M S\, \partial_N S + 4k^2) + \text{brane } \Lambda s$$

- $k$ breaks global Weyl $g_{MN} \to e^{-2\alpha} g_{MN}, \quad S \to S + 3\alpha$
- go to Einstein frame, solve EoMs: $S = 2k|Z|, \quad ds^2 = e^{\frac{4}{3}k|Z|}(dx^2 + dZ^2)$
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# A solution to the hierarchy problem

- effective **4D Planck mass**: $M_P^2 = 2M_5^2 \int_0^{\pi R} dZ\, e^{2kZ} = \frac{M_5^3}{k}(e^{2\pi kR} - 1)$
- 4D graviton fluctuations: $ds^2 \;=\; e^{\frac{4}{3}k|Z|}\left[\left(\eta_{\mu\nu} + \frac{2}{M_5^{3/2}} h_{\mu\nu}\right) dx^\mu dx^\nu \;+\; dZ^2\right]$
- action: $\mathcal{S} = -\frac{1}{2}\int d^4x\, dZ\, e^{2\,k|Z|}\left[(\partial_\lambda h_{\mu\nu})(\partial^\lambda h^{\mu\nu}) \;+\; (\partial_Z h_{\mu\nu})(\partial_Z h^{\mu\nu})\right]$
- $\mathcal{L}_{SM}$ at $Z=0$: $\quad \frac{h^{\mu\nu}(x, Z=0)\, T^{SM}_{\mu\nu}(x)}{M_5^{3/2}} \longrightarrow \sum_n \frac{\mathfrak{h}_n^{\mu\nu}(x)\, T^{SM}_{\mu\nu}(x)}{\Lambda_n}$ with

$$\Lambda_0 = M_P\,, \qquad \Lambda_n^2 = M_5^3\,\pi R\,(1 + k^2R^2/n^2)$$

- the cutoff is $M_5 \implies \quad m_h = O(M_5) \ll M_P \;\rightarrow$ **solution** to **hierarchy problem**
- for $k = 1$ TeV, $M_5 = 10$ TeV $\;\rightarrow\; k\,R \simeq 10$

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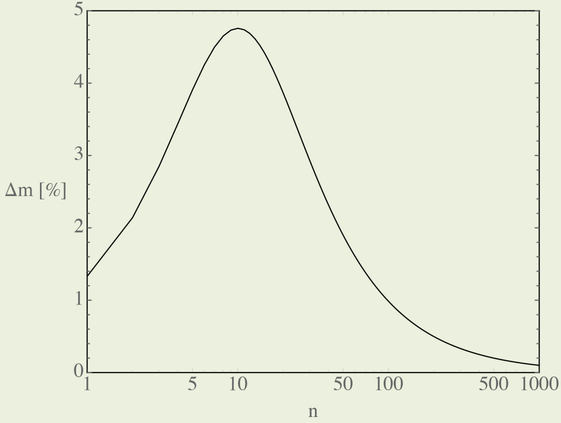
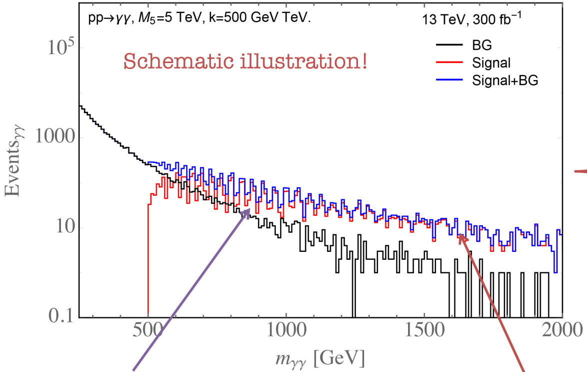
## Clockwork mass splitting:

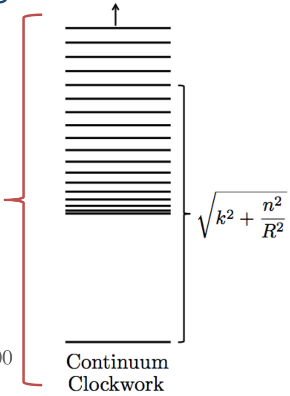



[ slide by M. McCullough ]

# Phenomenology

## At colliders would look something like:



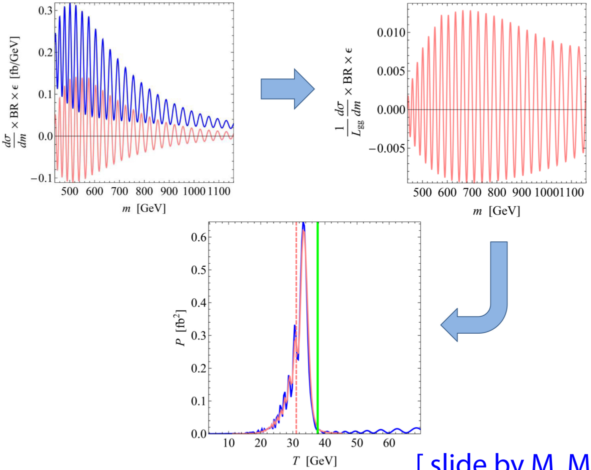

Most interestingly, due to splittings, signal appears to "oscillate". Thus get extra sensitivity by doing spectral analysis... The "power spectrum" of LHC data!

Can search for continuum spectrum at high energies.

[ slide by M. McCullough ]

# Phenomenology

The fourier transform would then exhibit a peak near the inverse radius:




[ slide by M. McCullough ]

# *Epilogue:*
# *Scrambling the clockwork*

# Disassembling the clockwork? [Craig, Garcia Garcia, Sutherland, '17]

- **Disclaimer**: I'm simplifying the argument
- **Definition** of clockwork: *a theory with no exponential hierarchies in fundamental parameters that gives rise to exponentially suppressed* **site-dependent** *couplings to symmetry-protected zero mode*
- **Claim 1**: **no clockwork from geometry**
- for a scalar in curved clockwork metric: $\psi_0 = \text{const.} \equiv \mathcal{C}_0$
- coupling on a brane at $Z = Z_0$: $\dfrac{\phi}{16\pi^2 f_{5D}} G\widetilde{G} \Longrightarrow \dfrac{\varphi_0}{16\pi^2 F} G\widetilde{G}$

  with $F = f_{5D}^{3/2}/\mathcal{C}_0 = M_{PL}\left(\dfrac{f_{5D}}{M_5}\right)^{3/2}$ **independent on** $Z_0 \Longrightarrow$ **no clockwork**
- **Claim 1b**: **clockwork only for flat-spacetime** construction with bulk mass and boundary terms (they do it for scalar, but essentially a re-discovery of construction in [Hambye, Teresi, Tytgat, '16] )

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- coupling on a brane at $Z = Z_0$: $\frac{\phi}{16\pi^2 f_{5D}} G\widetilde{G} \Longrightarrow \frac{\varphi_0}{16\pi^2 F} G\widetilde{G}$

  with $F = f_{5D}^{3/2}/\mathcal{C}_0 = M_{PL}\left(\frac{f_{5D}}{M_5}\right)^{3/2}$ **independent on $Z_0$** $\Longrightarrow$ **no clockwork**
- **Claim 1b**: **clockwork only for flat-spacetime** construction with bulk mass and boundary terms (they do it for scalar, but essentially a re-discovery of construction in [Hambye, Teresi, Tytgat, '16] )

# Disassembling the clockwork? (continued)

- **Claim 2**: **no non-Abelian clockwork** (including gravity)
- non-Abelian Yang-Mills clockwork chain
- kinetic terms:

$$-\mathcal{L}_{kin} = \sum_{j=0}^{N} \frac{1}{4g_j^2} F_j F_j = \sum_{j=0}^{N} \frac{1}{4g_j^2} \left( F_j^{abelian} F_j^{abelian} + 4f\, A_j A_j\, \partial A_j + f f\, A_j A_j A_j A_j \right)$$

- if, in terms of zero mode $\mathcal{A}_0$, $A_j = c_j \mathcal{A}_0 + \ldots$

$$-\mathcal{L}_{kin} \supset \sum_{j=0}^{N} \frac{c_j^2}{4g_j^2} \mathcal{F}_0^{abelian} \mathcal{F}_0^{abelian} + \sum_{j=0}^{N} \frac{c_j^3}{4g_j^2} 4f\, \mathcal{A}_0\, \mathcal{A}_0\, \partial \mathcal{A}_0 + \sum_{j=0}^{N} \frac{c_j^4}{4g_j^2} f f\, \mathcal{A}_0\, \mathcal{A}_0\, \mathcal{A}_0\, \mathcal{A}_0$$

- gauge invariance for $\mathcal{A}_0$: $\sum_{j=0}^{N} \frac{c_j^2}{g_j^2} = \sum_{j=0}^{N} \frac{c_j^3}{g_j^2} = \sum_{j=0}^{N} \frac{c_j^4}{g_j^2} \equiv \frac{1}{g_{eff}^2}$
- $\implies \quad c_j \in \{0, 1\} \quad \rightarrow$ **no clockwork**
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## Reassembling the clockwork? [Giudice, McCullough, '17]

- **Disclaimer**: I'm simplifying the argument
- **answer** to **Claim 2** (here for scalar, given also for gravity):
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- in this basis $g_k = g_0 \, q^{-k} \neq$ const. $\implies \; g_{\mathit{eff}} \approx g_N = g_0 \, q^{-N}$
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- **answer** to **Claim 1b**:
- the **curved** and **flat**-spacetime scalar constructions are the **same**! (related by field redefinition $\phi = e^{-kZ}\pi$)
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- definition of [Craig, Garcia Garcia, Sutherland, '17] is (UV) **model dependent**, e.g.:

  $$\mathcal{S} \supset \int d^5x\,\delta(Z - Z_0)\,e^{nS/2}\frac{\phi}{16\pi^2 f}G\widetilde{G}$$

  $Z_0$-profile of coupling depends on $n$, spurion charge of $f$ under global Weyl
- for **model building** and **hierarchy problem**, relevant definition:
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The End