Flux compactifications

and their application(s)

HEP@VUB Meeting - Brussels, March 28

Markus Dierigl - UU and UvA

based on: several works with W. Buchmuller, F. Ruehle, and J. Schweizer and
1611.03798 (JHEP) with W. Buchmuller, E. Dudas and J. Schweizer
1804.07497 (JHEP) with W. Buchmuller and E. Dudas

Delta Institute for Theoretical Physics

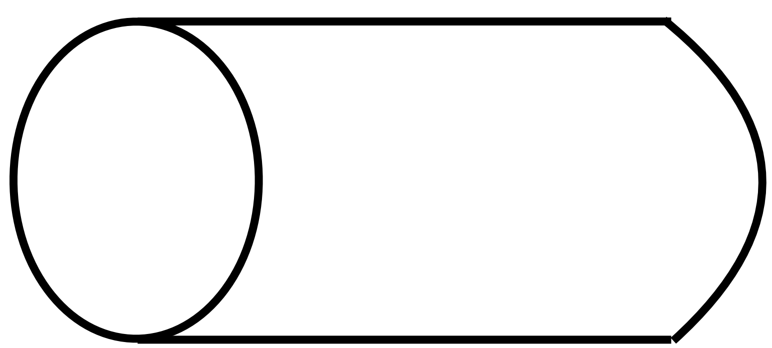
Outlook

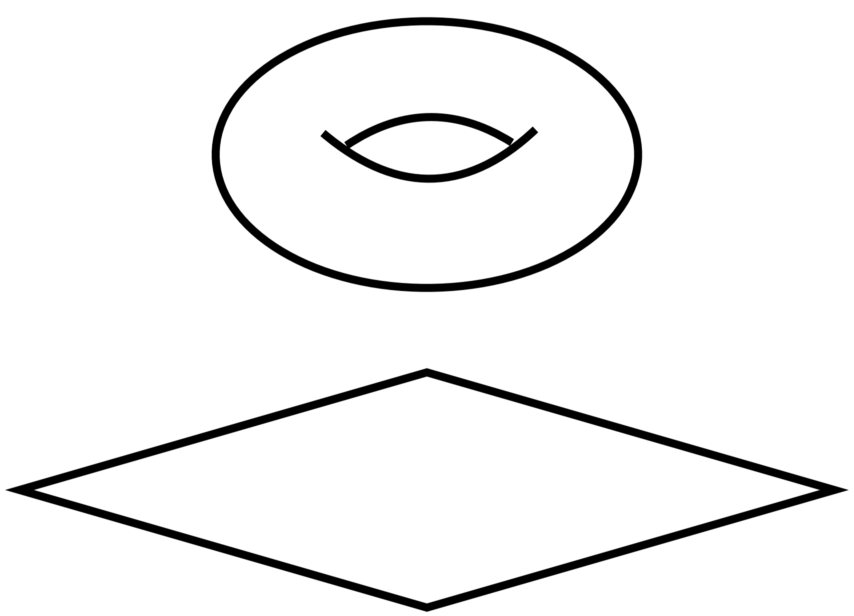
- **Compactifications**
  - Kaluza-Klein (KK) tower of effective fields
  - Lorentz structure of fields
  - Problems for compactifications
- **Magnetic flux**
  - Vector field background
  - Effect on charged states
  - Application for GUTs

- **Massless scalar field**
    - Review: **Hierarchy problem** for scalar fields
    - **5d example** with **finite corrections**
    - **6d with flux** leads to **vanishing corrections**
        - **Explicitly** for **1-loop**
        - **Symmetry arguments for all-loop**
- **Conclusion (Summary and Outlook)**

Compactification

- Start with **higher-dimensional** quantum field theory
- In order to account for our visible dimensions keep **four non-compact dimensions**
- The remaining dimensions should be “invisible” at our energies and have to be **compact** (here: circle or torus)





KK-tower in circle compactification

Assume a **complex scalar field in 5d** compactified **on a circle:**

$$\varphi(x_\mu, y + 2\pi L) = \varphi(x_\mu, y)$$

Can be decomposed into **Fourier modes**

$$\varphi(x_\mu, y) = \frac{1}{\sqrt{2\pi L}} \sum_{n=-\infty}^{\infty} e^{in\frac{y}{L}} \varphi_n(x_\mu)$$

with mode functions only depending on non-compact coordinates

What does this imply in lower-dimensional effective theory?

The 4d action is obtained by **integrating over internal space**:

$$
\begin{aligned}
S &= \int d^4x\, dy \big(\partial_\mu \overline{\varphi} \partial^\mu \varphi - \partial_y \overline{\varphi} \partial_y \varphi\big) \\
&= \int d^4x \int_0^{2\pi L} dy \sum_{n,m} \frac{1}{2\pi L} e^{i(n-m)\frac{y}{L}} \left(\partial_\mu \overline{\varphi}_m \partial^\mu \varphi_n - \frac{nm}{L^2} \overline{\varphi}_m \varphi_n\right) \\
&= \int d^4x \sum_n \left(\partial_\mu \overline{\varphi}_n \partial^\mu \varphi_n - \frac{n^2}{L^2} \overline{\varphi}_n \varphi_n\right)
\end{aligned}
$$

- **Infinite number** of 4d fields
- Most of the fields are **massive** of the **order of the compactification scale**
- The collection of fields is called **Kaluza-Klein tower**

Lorentz structure in compactification

- **All type of fields** have similar **Kaluza-Klein** expansion
- The **Lorentz character** can be **affected by compactification**

Consider **5d theory on circle**:

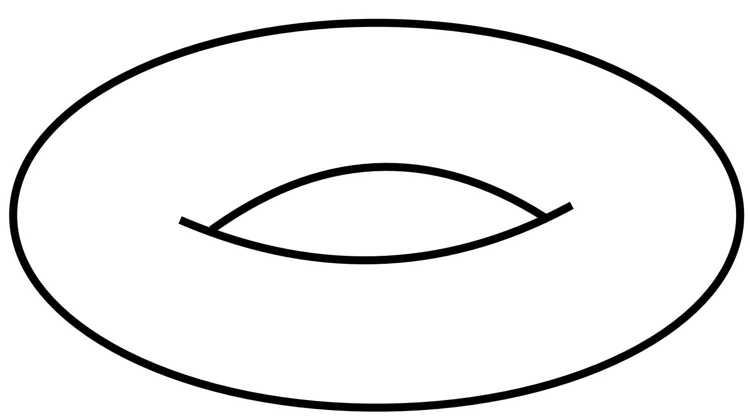
$$\begin{aligned} \text{Scalar:} \quad & \varphi \to \varphi_n \\ \text{Vector:} \quad & A_M \to (A_\mu)_n \,, (A_y)_n \\ \text{Metric:} \quad & g_{MN} \to (g_{\mu\nu})_n \,, (g_{5\mu})_n \,, (g_{55})_n \end{aligned}$$

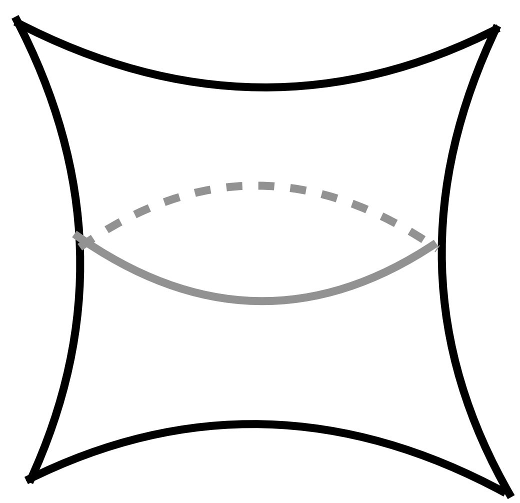
[Kaluza ‘ 21, Klein ’26]

$$\text{Fermion :} \quad \Psi \to (\psi_L)_n \,, (\psi_R)_n$$

Other internal spaces

- **Many properties** of the lower dimensional action depend on the **geometry of the internal space**
- **Torus:** Two circles lead in general to two KK-towers
- **Orbifold:** “Folding” of a smooth space
- **Calabi-Yau compactifications:** Used in string theory (becomes complicated and mathematical fast)





Challenges

- In presence of **gravity** the **geometry** becomes **dynamical** (the internal space has to satisfy Einstein equations, one has to find minima for geometrical parameters with large enough masses) ⟶ **Moduli stabilization**

- **Fermions not chiral** (higher-dimensional give rise to both four-dimensional chiralities)

- Hard to generate **hierarchies** in interactions (fields are spread in internal space, lower-dimensional interactions are determined by overlap)

Magnetic Flux

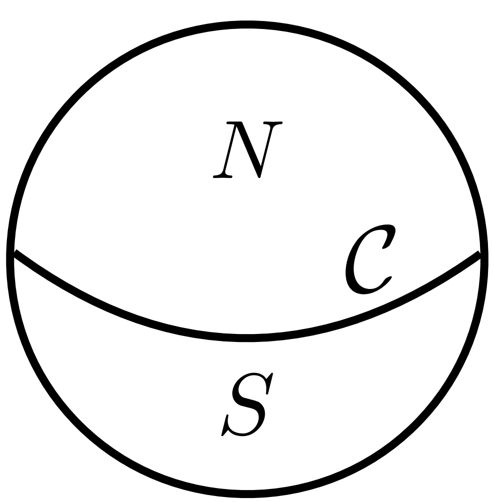
- **Magnetic field in higher dimensions:**

$$F_{MN} = \partial_M A_N - \partial_N A_M$$

$$\text{electric}: \quad F_{0i}\,, \quad \text{magnetic}: \quad F_{ij}$$

- **Internal magnetic field:** at least **two extra dimensions**

- In **compact space** the **magnetic flux is quantized**



$$e^{i\int_N F} = e^{i\oint_C A} = e^{-i\int_S F}$$

$$e^{i\int_{S+N} F} = 1 \quad \Rightarrow \quad \int_{S^2} F = 2\pi N \ \text{ with } \ N \in \mathbb{Z}$$

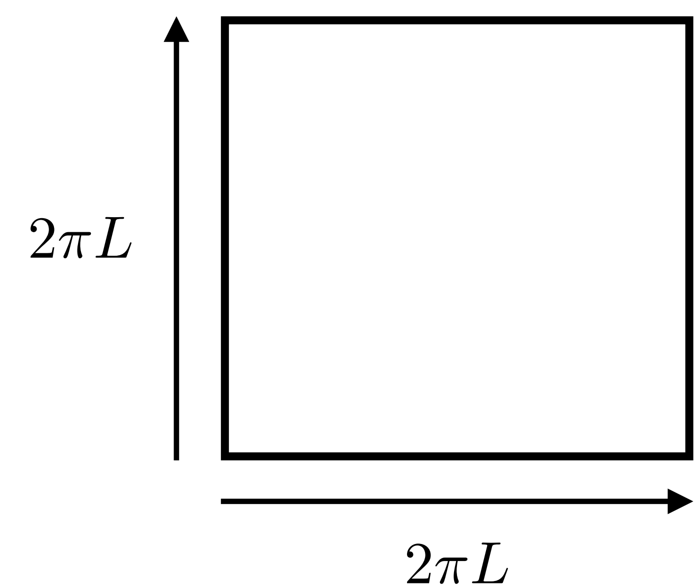
Magnetic flux on torus

We want a **constant magnetic field in torus directions**:

$$\frac{1}{2\pi}\int_{T^2} F = \frac{1}{2\pi}\int_{T^2} dy_5 dy_6\, F_{56} = \frac{1}{2\pi}\text{Area}(T^2)\, f$$

For a **square torus:**

$$f = \frac{N}{2\pi L^2}$$



Possible choice:

$$A_5 = -\tfrac{1}{2} f x_6\,, \qquad A_6 = \tfrac{1}{2} f x_5$$

Needs **patches and transition functions!**

[Buchmuller, MD, Tatsuta ’18]

Wilson lines

There can be **fluctuations around the flux background:**

$$A_m = \langle A_m \rangle + a_m$$

One important fluctuation is the **lowest KK-mode:**

$$a_5(x_\mu)\,, \quad a_6(x_\mu)$$

Does not depend on internal direction → **tree-level massless**

Can be combined into a **complex scalar field** in 4d:

$$\varphi = \frac{1}{\sqrt{2}}(a_6 + i a_5)$$

so-called **Wilson line**

Effect on charged fields

- **Magnetic field has effect on charged fields**
  - Modifies KK-tower to **Landau levels**
  - **Splits masses chirality sensitive**
  - **Localizes field profile in internal space**

In the following we study a **complex scalar field** of **charge q:**

$$\partial_m \mathcal{Q} \to D_M \mathcal{Q} = (\partial_m + iqA_M)\mathcal{Q}$$

Landau levels

Effective 4d masses from **internal part of kinetic term**

$$\mathcal{L}_{6\mathrm{d}} \supset -\overline{D_5}\,\overline{\mathcal{Q}} D_5 \mathcal{Q} - \overline{D_6}\,\overline{\mathcal{Q}} D_6 \mathcal{Q}$$
$$= \overline{\mathcal{Q}}(D_5^2 + D_6^2)\mathcal{Q}$$

Analyse **the algebra of covariant derivatives in flux background:**

$$[D_5, D_6] = iq\partial_5 A_6 - iq\partial_6 A_5 = iqF_{56} = iqf$$

Identical to **harmonic oscillator algebra** with:

$$D_5 \sim \hat{x} = x\,,$$
$$D_6 \sim \hat{p} = -i\hbar\frac{\partial}{\partial x}\,,$$
$$\hbar \sim qf$$

Proceed as for **quantum harmonic oscillator** by introducing **ladder operators:**

$$a = \frac{1}{\sqrt{2qf}}(D_5 + iD_6)\,, \qquad a^\dagger = \frac{1}{\sqrt{2qf}}(D_5 - iD_6)$$

They satisfy:

$$[a, a^\dagger] = -\frac{i}{qf}[D_5, D_6] = 1$$

Rewrite the **internal part of the kinetic term as**:

$$D_5^2 + D_6^2 = qf\left(aa^\dagger + a^\dagger a\right) = 2qf\left(a^\dagger a + \tfrac{1}{2}\right)$$

Leads to **oscillator mass spectrum in 4d.**

A **complex scalar** of **charge q** in **6d** with **flux** leads to:

- **Infinite tower of states** parameterized by **"Landau level" n,** and **multiplicity j**
- **Masses depend only on n** not on j (going from 1 to |N|)
- **Masses** again depend **on compactification scale**

$$m_n^2 = 2qf\left(n + \tfrac{1}{2}\right) = \frac{N}{\pi L^2}\left(n + \tfrac{1}{2}\right)$$

Lowest mass state in 4d:

$$m_0^2 = \frac{N}{2\pi L^2}$$

Fermions with flux

Fermions more complicated due to **non-trivial Lorentz structure:**

$$\mathcal{L}_{6\text{d}} = i\,\overline{\Psi}\Gamma^M D_M \Psi$$

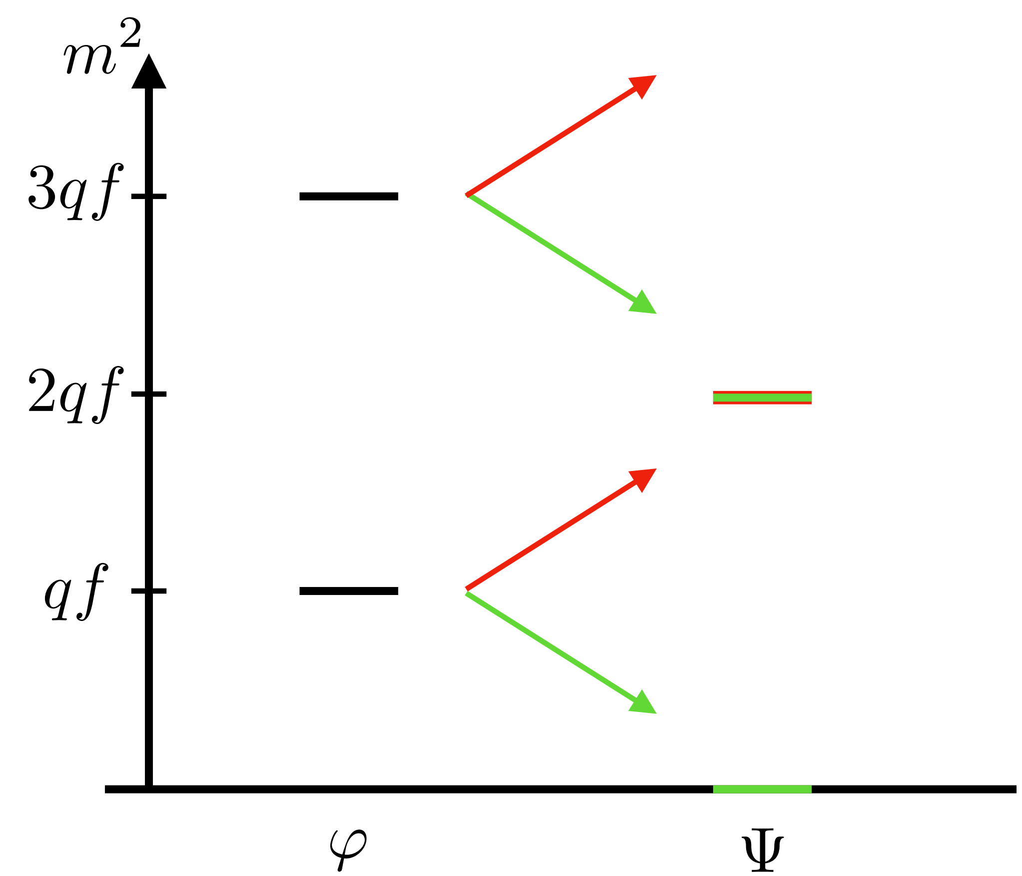
Analyze structure of squared internal Dirac operator

$$M_n^2 = m_n^2 + qf\,\Gamma^5\Gamma^6$$

Depends on internal helicity!

Chiral 4d zero modes (Index theorem)

[Atiyah, Patodi, Singer]



Internal field profiles

see e.g. [Cremades, Ibanez, Marchesano '04]

Push oscillator analogy even further:

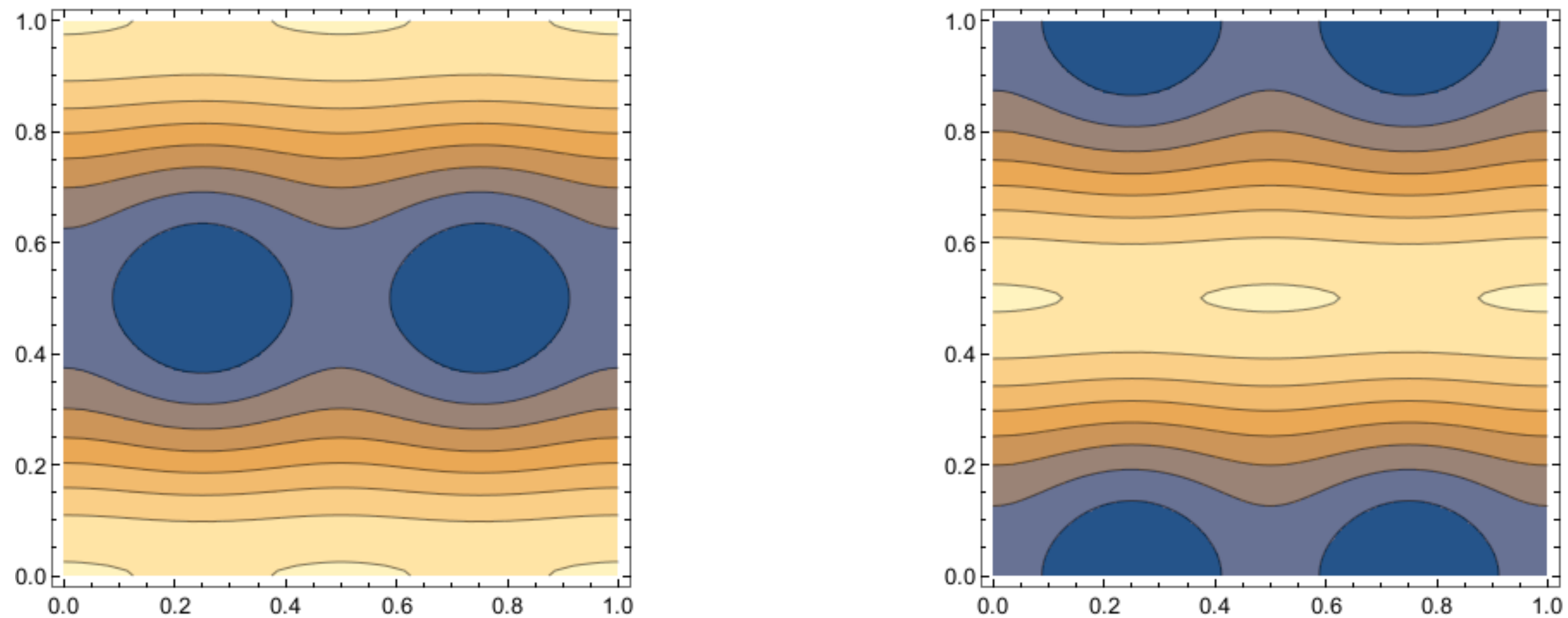
$$a\,\xi_0 = 0$$

"Groundstate annihilated by annihilation operator"

- One finds **N linearly independent field profiles** (degeneracy index j)
- The **same** for **bosons and fermions**
- **Higher modes** from application of **creation operator**

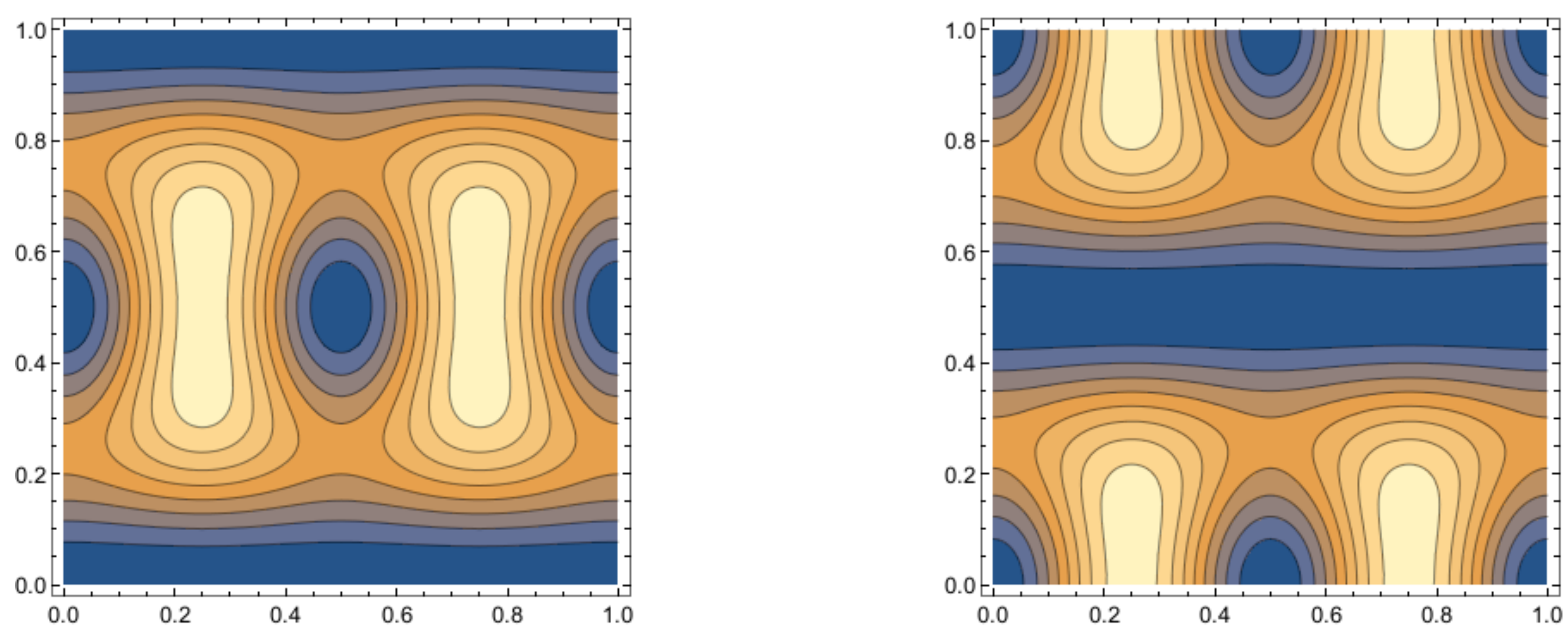
Localized in internal space
(opens up possibilities for hierarchies in overlap integrals)**:**

$|\xi_{0,j}|^2$



Groundstates for N=2

$|\xi_{1,j}|^2$



First excited level for N=2

Application SO(10) GUT

[Asaka, Buchmuller, Covi ’01][Buchmuller, MD, Ruehle, Schweizer ’15]

- **Minimal supersymmetric** model in **6d**
- **Gauge group: SO(10) x U(1)**
- **Compactification on orbifold**
- Wilson line breaking to **SU(3) x SU(2) x U(1) x U(1) x U(1)**
- **Three flux quanta** in additional U(1) (SUSY broken at compactification scale)
- **charged 16-plet** leads to **chiral fermion zero-modes** and **three generations** (separated in bulk)
- **Uncharged 10-plet** only leaves **Higgs doublet as zero-mode**

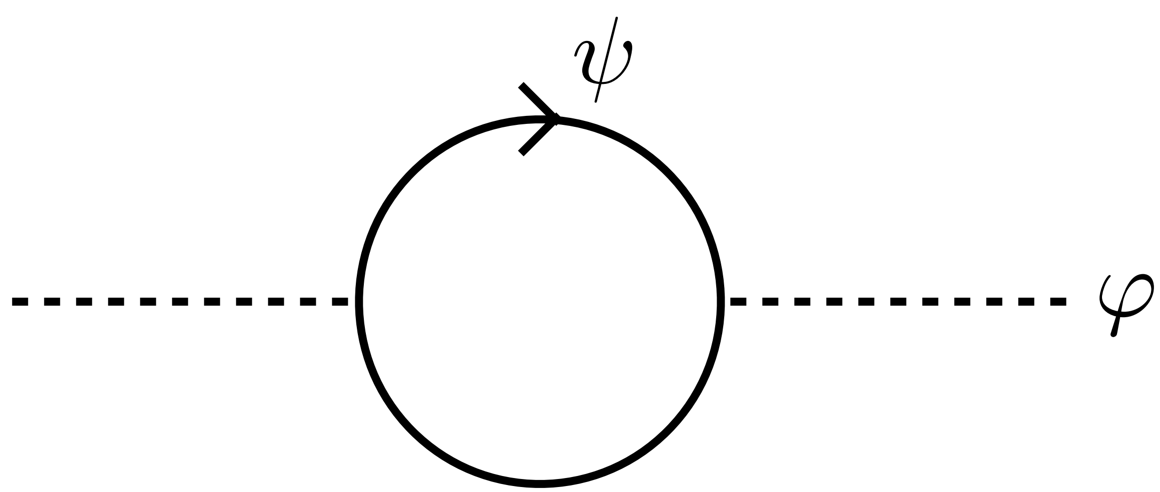
Summary

- **Flux enhances possibilities for model building**
    - **Fermion zero-modes** (scalars lifted)
    - **Bulk localization** (split of different components)
    - **Multiplicity** (number of generations)
- **Flux breaks supersymmetry**
- Can be used to **stabilize extra dimensions** **[Buchmuller, MD, Ruehle, Schweizer '16]**
- **Interesting pattern of quantum corrections**

Mass of scalar fields

- The **bare mass** of a scalar field is subject to **quantum corrections**

- In a theory where the **scalar couples to fermions via Yukawa** interaction $y\bar{\psi}\varphi\psi$ corrections are induced by



- The induced quantum correction is:

$$\delta m^2 \propto -y^2 \int \frac{d^4k}{k^2} \propto -y^2 \int_0^\Lambda dk\, k \propto \Lambda^2$$

- From **experiment** the quantum corrections to the Higgs mass should be

$$\delta m^2 \propto \Lambda^2 \quad \text{with} \quad \Lambda = \mathcal{O}(\text{TeV})$$

- From **consistency** the quantum corrections to the Higgs mass can be

$$\delta m^2 \propto \Lambda^2 \quad \text{with} \quad \Lambda = M_P \sim 10^{19}\text{GeV}$$

- But **measured Higgs mass** is about **125 GeV**

The so-called Hierarchy Problem

(exists for scalar fields with small masses in general)

Theories with extra dimensions

- **Infinite number of fields**
- In principle **all contribute to the mass correction**

Compactification on circle:

$$\mathcal{L}_{5\mathrm{d}} \supset y\phi\overline{\Psi}\Psi \to \sum_n y\phi_0\overline{\psi}_n\psi_n$$

Each contributes quadratically with cut-off.

- **Add supersymmetry** (still some hierarchy necessary)
- Find **different way out**

Wilson line as Higgs

We have seen that the **Lorentz type of field changes under compactifications.**

Yukawa interaction from gauge fields

[Hosotani ’83], [Hatanaka, Inami, Lim ’98], [Hall, Nomura ’01], [Arkani-Hamed, Cohen, Georgi ’01] [Antoniadis, Benakli, Quiros ’01], …

Schematically (again for circle compactification):

$$\mathcal{L}_{5\mathrm{d}} \supset i\,\overline{\Psi}\Gamma^M D_M \Psi \supset -q \sum_n a_5 \overline{\psi}_n \psi_n$$

Wilson line as Higgs scalar with Yukawa interaction

(for complex Higgs go to torus)

Without flux

- Calculate **effective action** in four dimensions **[Antoniadis, Benakli, Quiros ’01], [Buchmuller, MD, Dudas, Schweizer ’16]**

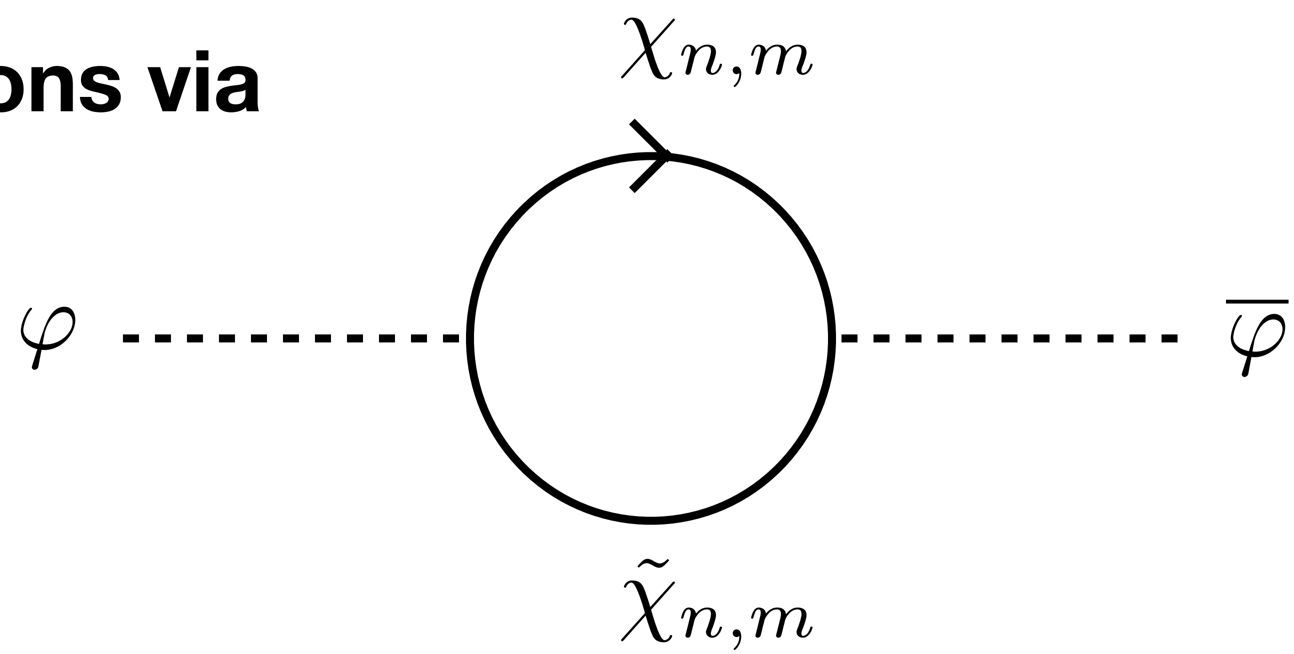
- Calculate **quantum corrections** or **effective potential**

$$\mathcal{L}_{4\mathrm{d}} \subset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \partial_\mu\overline{\varphi}\partial^\mu\varphi$$
$$\sum_{m,n}\Big(-i\chi_{n,m}\sigma^\mu D_\mu\overline{\chi}_{n,m} - i\tilde{\chi}_{n,m}\sigma^\mu D_\mu\overline{\tilde{\chi}}_{n,m}$$
$$+\left(\tfrac{1}{L}(m+in)+\sqrt{2}gq\varphi\right)\tilde{\chi}_{n,m}\chi_{n,m} + \mathrm{h.c.}\Big)$$

- Only **zero-mode** for **6d gauge field**

- **Full tower of charged fermions** (use orthonormality of internal field profiles)

- **Quantum corrections via**



$$\begin{aligned}\delta m^2 &= -4g^2q^2\sum_{n,m}\int\frac{d^4k}{(2\pi)^4}\frac{k^2}{\left(k^2+\frac{1}{L^2}(m^2+n^2)\right)^2}\\ &= -4g^2q^2\sum_{n,m}\int_0^\infty dtte^{-\frac{1}{L}(n^2+m^2)t}\int\frac{d^4k}{(2\pi)^4}k^2e^{k^2t}\\ &= -\frac{g^2q^2}{2\pi^2}\int_0^\infty\frac{dt}{t^2}\Theta_3\left(0;\frac{it}{\pi L^2}\right)^2 = -\frac{g^2q^2}{2\pi^5L^2}\sum_{r,s}\frac{1}{(r^2+s^2)^2}\end{aligned}$$

- **All divergence in r=s=0** (can also be obtained via Poisson resummation), use this as renormalization prescription ⟶ Finite mass

- **Finite result** for the **Higgs mass,** i.e. **no cut-off dependence** (no SUSY)

- **Mass correction of the order of the compactifications scale** (for pheno: large extra dimensions needed)

How is this possible?

- **Higgs** is **“extended object”** (Wilson line)

- **built in cut-off** of the size of object (compactification scale)

With flux

- Do the same calculation **with flux** **[Buchmuller, MD, Dudas, Schweizer ’16] [Buchmuller, MD, Dudas ’18]**

- Combine internal coordinates to complex coordinate, introduce background for gauge field:

$$z = \tfrac{1}{2}(y_5 + iy_6)\,, \qquad \partial_z = \partial_5 - i\partial_6$$

$$\phi = \frac{1}{\sqrt{2}}(A_6 + iA_5)$$

$$\phi = \frac{1}{\sqrt{2}} f\,\bar{z} + \varphi$$

- The **6d action** (flux and one chiral fermion) can be written as:

$$
\begin{aligned}
S_6 = \int d^6x \Big(& - \tfrac{1}{4} F^{\mu\nu} F_{\mu\nu} - \partial^\mu \overline{\varphi} \partial_\mu \varphi - \tfrac{1}{4} (\partial_z \overline{\varphi} + \partial_{\bar{z}} \varphi)^2 - \tfrac{1}{2} f^2 \\
& - \tfrac{1}{2} \partial_{\bar{z}} A^\mu \partial_z A_\mu - \tfrac{i}{\sqrt{2}} \partial_\mu A^\mu (\partial_z \overline{\varphi} - \partial_{\bar{z}} \varphi) \\
& - i \psi \sigma^\mu \overline{D}_\mu \overline{\psi} - i \chi \sigma^\mu D_\mu \overline{\chi} \\
& - \chi \big(\partial_z + q f \bar{z} + \sqrt{2} q \varphi \big) \psi - \overline{\chi} \big(\partial_{\bar{z}} + q f z + \sqrt{2} q \overline{\varphi} \big) \overline{\psi} \Big)
\end{aligned}
$$

- Use **mode decomposition** and **restrict to zero-modes** for **uncharged fields**

- **Uncharged complex scalar field** coupling to the **whole tower** of **charged fermions**

- The **effective action** reads:

$$S_4 = \int d^4x \Big(-\partial^\mu \overline{\varphi}_0 \partial_\mu \varphi_0 + \sum_{n,j} \big(-i\psi_{n,j}\sigma^\mu \overline{D}_\mu \overline{\psi}_{n,j} - i\chi_{n,j}\sigma^\mu D_\mu \overline{\chi}_{n,j}$$

$$- \sqrt{2qf(n+1)}\chi_{n,j}\psi_{n+1,j} - \sqrt{2}q\varphi_0\chi_{n,j}\psi_{n,j} + \text{h.c.}\big)\Big)$$

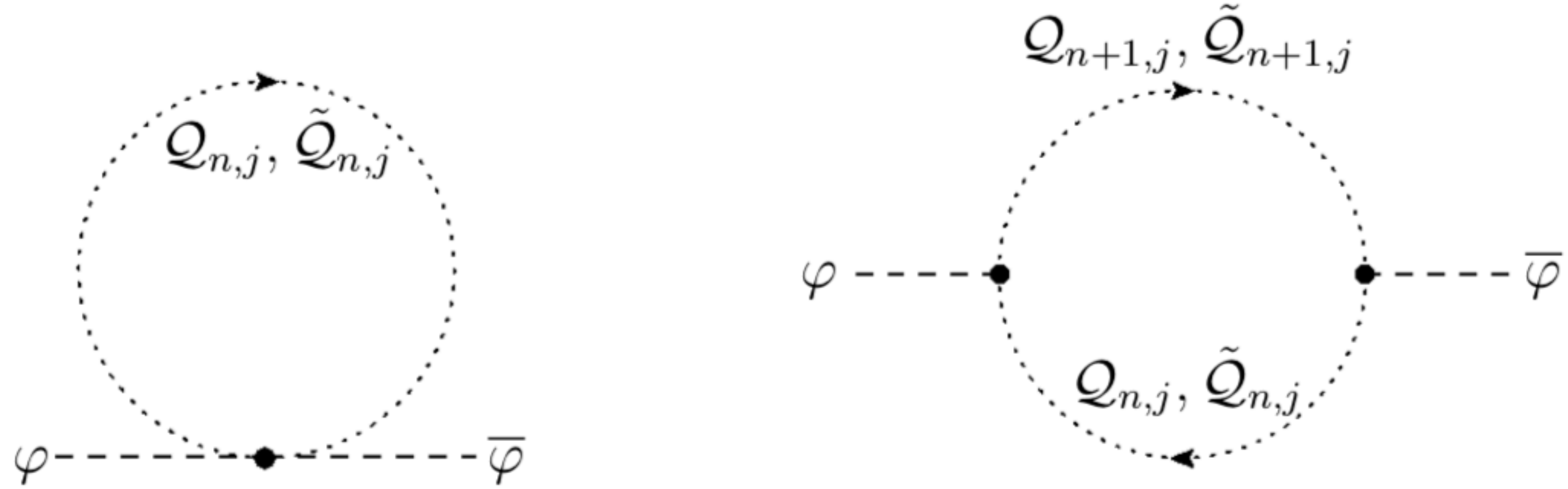
- Similar action can be derived for **charged 6d scalar fields**

- Even a description in terms of **superfields with spontaneously broken SUSY** (D-terms) can be derived **[Buchmuller, MD, Dudas, Schweizer ’16]**

- **Tree-level massless scalar**

- Calculate **quantum corrections** to **mass** and **potential**

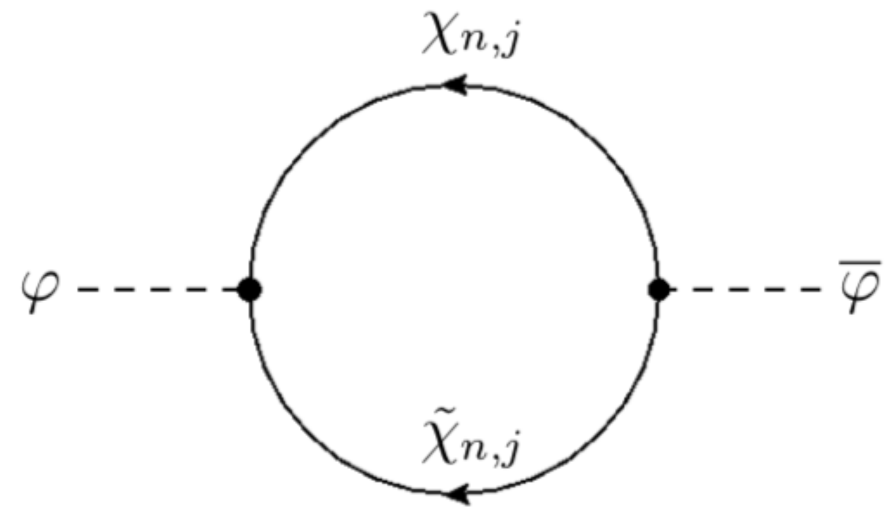
- **Mass-correction** from **charged boson**



$$\begin{aligned}
\delta m_b^2 &= 2q^2g^2|N|\sum_n \int \frac{d^4k}{(2\pi)^4}\Big(\frac{2}{k^2+\alpha(n+\frac{1}{2})} - \frac{2\alpha(n+1)}{(k^2+\alpha(n+\frac{3}{2}))(k^2+\alpha(n+\frac{1}{2}))}\Big) \\
&= -4q^2g^2|N|\sum_n \int \frac{d^4k}{(2\pi)^4}\Big(\frac{n}{k^2+\alpha(n+\frac{1}{2})} - \frac{n+1}{k^2+\alpha(n+\frac{3}{2})}\Big) \\
&= -\frac{q^2g^2}{4\pi^2}|N|\sum_n \int_0^\infty dt\frac{1}{t^2}\big(ne^{-\alpha(n+\frac{1}{2})t} - (n+1)e^{-\alpha(n+\frac{3}{2})t}\big) \\
&= -\frac{q^2g^2}{4\pi^2}|N|\int \frac{dt}{t^2}\Big(\frac{e^{\frac{1}{2}\alpha t}}{(e^{\alpha t}-1)^2} - \frac{e^{\frac{1}{2}\alpha t}}{(e^{\alpha t}-1)^2}\Big) \\
&= \mathbf{0}
\end{aligned}$$

$$\alpha = 2qgf$$

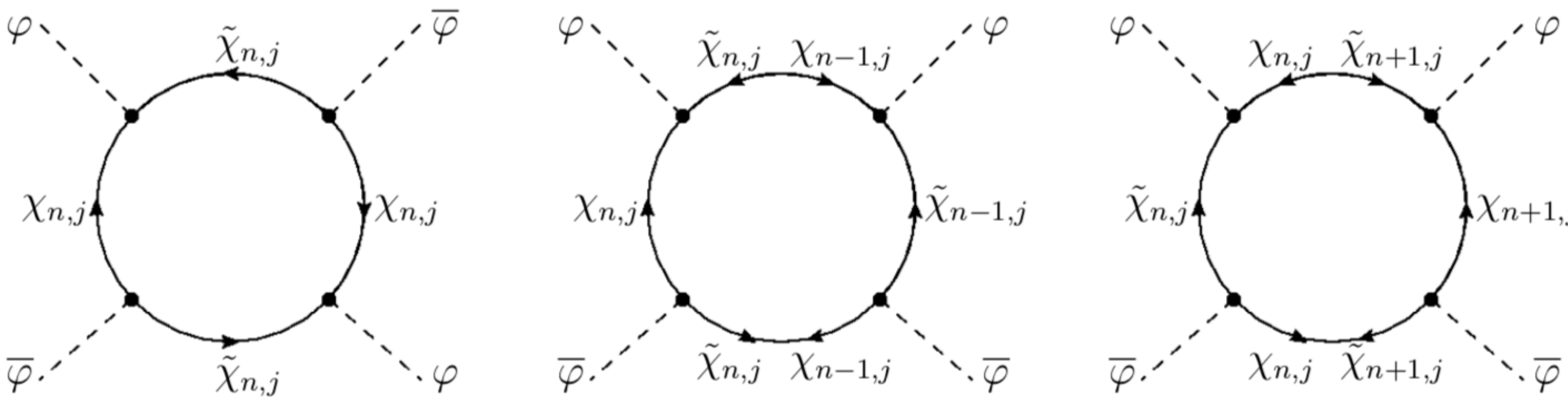
- **Mass-correction** from **charged fermion**



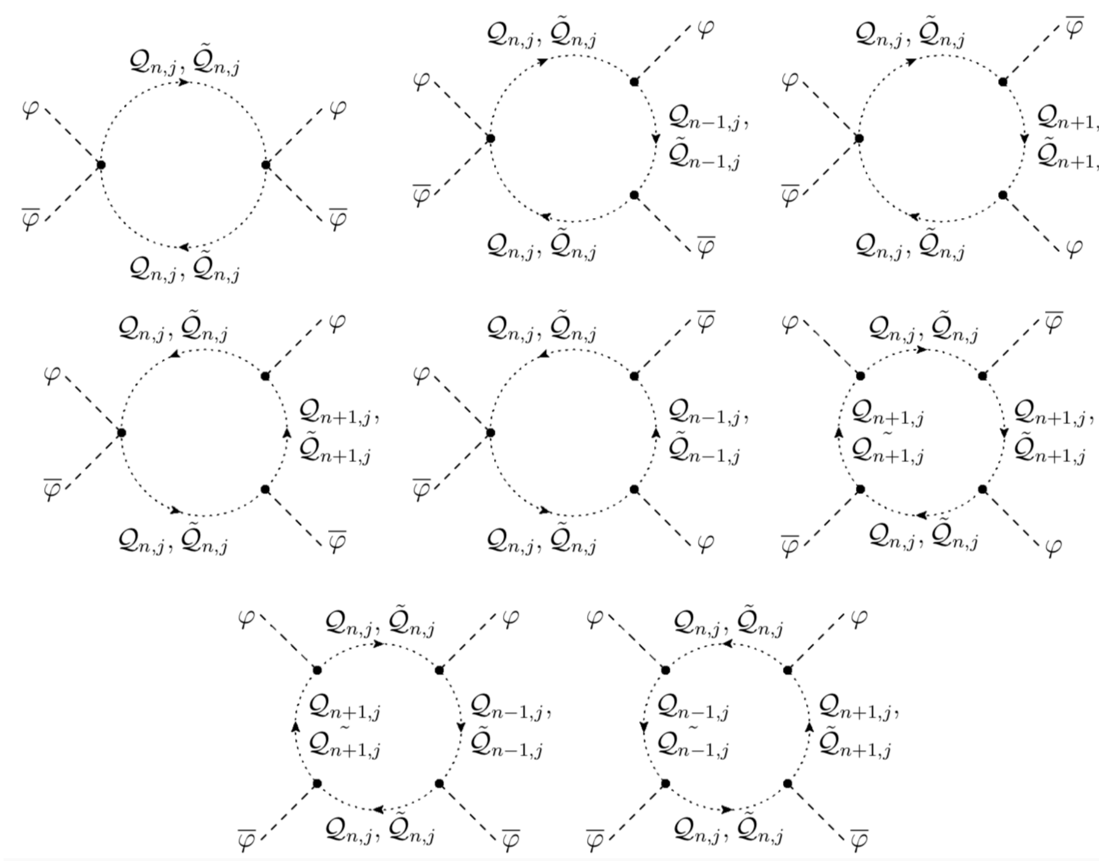
$\tilde{\chi}_{n,j} \sim \psi_{n,j}$

$$
\begin{aligned}
\delta m_f^2 =& -2q^2g^2|N|\sum_n \int \frac{d^4k}{(2\pi)^4}\frac{2k^2}{(k^2+\alpha n)(k^2+\alpha(n+1))} \\
=& 4q^2g^2|N|\sum_n \int \frac{d^4k}{(2\pi)^4}\Big(\frac{n}{k^2+\alpha n}-\frac{n+1}{k^2+\alpha(n+1)}\Big) \\
=& \frac{q^2g^2}{4\pi^2}|N|\sum_n \int_0^\infty dt\frac{1}{t^2}\big(ne^{-\alpha n t}-(n+1)e^{-\alpha(n+1)t}\big) \\
=& \frac{q^2g^2}{4\pi^2}|N|\int \frac{dt}{t^2}\Big(\frac{e^{\alpha t}}{(e^{\alpha t}-1)^2}-\frac{e^{\alpha t}}{(e^{\alpha t}-1)^2}\Big) \\
=& \mathbf{0}
\end{aligned}
$$

- **Correction** to **quartic interaction** by **fermions:**



- **Correction** to **quartic interaction** by **bosons:**



ALL VANISH!

All order from symmetries

[Buchmuller, MD, Dudas '18]

- The **flux background breaks** the **translational symmetry** along the torus directions z

- **6d action invariant** with respect to **modified shift** (including shift of Wilson line)

$$\delta_T X = (\epsilon\partial_z + \bar{\epsilon}\partial_{\bar{z}})X \,, \quad \text{for} \;\; X = A_\mu, \psi, \chi$$

$$\delta_T \varphi = (\epsilon\partial_z + \bar{\epsilon}\partial_{\bar{z}})\varphi + \frac{1}{\sqrt{2}}\bar{\epsilon} f$$

- **6d action invariant** with respect to **transformation** (changes boundary conditions for fermions)

$$\varphi_\Lambda = \varphi - \frac{1}{\sqrt{2}}\partial_z\Lambda \,, \quad \psi_\Lambda = e^{q\Lambda}\psi \,, \quad \chi_\Lambda = e^{-q\Lambda}\chi \,, \quad \Lambda = f(\alpha\bar{z} - \bar{\alpha}z)$$

- For **infinitesimal transformation**

$$\delta_\Lambda \varphi = -\frac{1}{\sqrt{2}}\partial_z \Lambda = \frac{1}{\sqrt{2}}\bar{\alpha} f\,, \quad \delta_\Lambda \psi = q\Lambda\psi\,, \quad \delta_\Lambda \chi = -q\Lambda\chi$$

- **Combine** the two **transformations**

$$\delta = \delta_T + \delta_{\Lambda,\alpha=\epsilon}$$

- **Acts as shift of Wilson line** (symmetry of 6d action)

How does the symmetry act
in terms of 4d fields?
(relevant for perturbative corrections)

- **Action on 4d component fields**

$$
\begin{aligned}
\delta\psi_{n,j} &= \sqrt{2qf}(\epsilon\sqrt{n+1}\,\psi_{n+1,j} - \bar{\epsilon}\sqrt{n}\,\psi_{n-1,j})\,, \\
\delta\chi_{n,j} &= \sqrt{2qf}(-\epsilon\sqrt{n}\,\chi_{n-1,j} + \bar{\epsilon}\sqrt{n+1}\,\chi_{n+1,j})\,, \\
\delta\varphi_{l,m} &= (\epsilon M_{l,m} - \bar{\epsilon}\overline{M}_{l,m})\varphi_{l,m}\,, \quad M_{l,m} = \tfrac{1}{L}(m+il)\,, \\
\delta A_{\mu,l,m} &= (\epsilon M_{l,m} - \bar{\epsilon}\overline{M}_{l,m})A_{\mu,l,m}\,, \\
\delta\varphi_0 &= \sqrt{2}\bar{\epsilon}f
\end{aligned}
$$

- **Invariance guarantees no mass-correction and potential for massless scalar at perturbative level**

- **Transformation mixes the complete tower**

- **Descends from higher dimensions**

Summary

- **Many applications of flux compactifications** (SUSY breaking, moduli stabilization, hierarchies via localization of field profiles, multiplicity of generations, chirality)
- **Completely changes charged particle spectrum**
- Still has **mild behavior with respect to quantum corrections**
- **Leads to massless scalar** (“broken translational invariance”, mixing of whole tower, all-loop order)
- **Explicit form of (effective) action, transformations,…**

Outlook

- **Some problems:**
    - Scalar field only **couples** to **one zero-mode**
    - **Eaten up by graviphoton** (gravity switched on)
    - **Scalar field is uncharged**
- **Some solutions (?):**
    - **More than one flux** (several gauge fields, more dimensions) ➡ accidental shift symmetries, protection at 1-loop level
    - **Flux in non-Abelian** (tachyonic, stabilization)