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14/11/19

# The EW Phase Transition

→ Yield Precise Understanding of EWSB in Early Universe



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- ➔ Yield Precise Understanding of EWSB in Early Universe
- → (Possible) Answer to Open Mysteries at Interface of Particle Physics & Cosmology

Origin of Matter-Antimatter Asymmetry EW-scale Baryogenesis



# The EW Phase Transition

- → Yield Precise Understanding of EWSB in Early Universe
- → (Possible) Answer to Open Mysteries at Interface of Particle Physics & Cosmology
- → (Possible) Cosmological Relics from the EW Epoch

Gravitational Wave Signal



Courtesy of D. Weir (Helsinki) Hindmarsh, Huber, Rummukainen, Weir, PRD **92** (2015) 123009

Sourced by Collisions of Higgs bubbles from a first order EW phase transition & subsequent plasma motions







- **O** Phases separated by potential barrier
- O Broken phase bubbles nucleate, expand, merge





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O Broken phase **bubbles** nucleate, **expand**, merge



(if in plasma  $\rightarrow$  create fluid waves)



- **O** Phases separated by potential barrier
- Broken phase bubbles nucleate, expand, merge/collide <u>Anisotropic Stress</u>
  ↓
  ↓
  Sources Gravitational Wave Production



Physics Beyond the SM can induce a 1<sup>st</sup> Order EW Phase Transition



Two "Types" of Cosmological 1<sup>st</sup> Order PTs

#### O "Vacuum" Transitions

Fluid/plasma effects negligible (either plasma is very diluted or coupling between transition field and plasma small/non-existent)

Bubble walls accelerate until collision

Energy of PT stored in bubble walls

**O** Thermal Transitions

Energy of PT transferred to plasma

Plasma exerts friction on bubble wall

Terminal bubble wall velocity (steady state)



**O** Decay rate 
$$\Gamma(T) \approx T^4 \exp\left(-\frac{S_3(T)}{T}\right)$$

**O** O(3) symmetric action

$$S_3(T) = 4\pi \int dr r^2 \left[ \frac{1}{2} \left( \frac{d\phi}{dr} \right)^2 + V(\phi, T) \right]$$

**O** Bubble profile (bounce)

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} - \frac{\partial V(\phi, T)}{\partial \phi} = 0$$
  
$$\phi(r \to \infty) = 0 \text{ and } \dot{\phi}(r=0) = 0$$





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#### Nucleation temperature:

One Higgs bubble per Horizon volume (on average)

$$N(T_n) = \int_{t_c}^{t_n} dt \frac{\Gamma(t)}{H(t)^3} = \int_{T_n}^{T_c} \frac{dT}{T} \frac{\Gamma(T)}{H(T)^4} = 1$$

Linde, Phys. Lett. B100 (1981) 37; Nucl. Phys. B216 (1983) 421



**Two KEY Phase Transition Quantities:** 

**O** (Available) Transition Energy (normalized)

$$\alpha = \frac{\epsilon}{a_+ T^4}$$





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• Duration of the Transition (-1)

$$\frac{\beta}{H} \equiv -\frac{dS_3}{dt}\Big|_{t=t_n} \approx T \frac{d(S_3/T)}{dT}\Big|_{T=T_n}$$

(Related to the change of the Decay Rate)





GW frequency ~ size of bubbles @ collision

For  $T_* \sim 100 \text{ GeV}$  and  $\frac{\beta}{H_*} \sim 100, \text{GW}$  frequency

(redshifted to today!)  $\sim mHz$ 

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it the time of Ave buł

erage number of bubbles per horizon a bble coalescence/percolation (Transition Completes, 
$$T_*$$
)  
 $H_*$ 

### 1<sup>st</sup> Order (EW) Phase Transition



Figueroa et al., PoS GRASS2018 (2018) 036

### 1<sup>st</sup> Order (EW) Phase Transition



LISA can probe the EW epoch of the early Universe



# The LISA Mission

(Laser Interferometer Space Antenna)

### A brief status report

Thanks to G. Nardini

### 2017: LISA proposal to ESA



LISA Collaboration, 1702.00786

Launch date 2030-2034 LISA Mission selected by ESA (Summer 2017) + (On Jan 22 2018, LISA passed ESA's Mission Definition Review)



From the proposal: Audley et al, arXiv:1702.00786

# SI7.2 : Measure, or set upper limits on, the spectral shape of the cosmological stochastic GW background

OR7.2: Probe a broken power-law stochastic background from the early Universe as predicted, for example, by first order phase transitions [21] (other spectral shapes are expected, for example, for cosmic strings [22] and inflation [23]). Therefore, we need the ability to measure  $\Omega = 1.3 \times 10^{-11} (f/10^{-4} \text{ Hz})^{-1}$  in the frequency ranges 0.1 mHz < f < 2 mHz and 2 mHz < f <20 mHz, and  $\Omega = 4.5 \times 10^{-12} (f/10^{-2} \text{ Hz})^3$  in the frequency ranges 2 mHz < f < 20 mHz and 0.02 < f <0.2 Hz.

# GW – Collider complementarity

Timeline: LISA GW Observatory in the Context of High-Energy Colliders



After LHC, LISA is next step in exploration of EW scale physics



### GW from the EW Phase Transition with LISA



Assess the capability of LISA to probe GW signal from EW epoch ⇒ BSM physics

### GW from the EW Phase Transition with LISA



Assess the capability of LISA to probe GW signal from EW epoch  $\Rightarrow$  BSM physics Need to predict GW signal as robustly as possible

### Thermal EW Phase Transition

#### Energy liberated from phase change transferred (mostly) to plasma

□ Kinetic energy  $\Rightarrow$  Thermal plasma bulk motion

 $\Box$  Thermal energy  $\Rightarrow$  Thermal plasma gets heated up

Depending on Higgs bubble wall velocity, energy transfer to plasma creates different types of **expanding fluid shells** 

Laine, Phys. Rev. D**49** (1994) 3847 Espinosa, Konstandin, No, Servant, JCAP **1006** (2010) 028



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Courtesy of D. Cutting (Sussex)



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### Fluid shell Profiles

$$\partial^{\mu}T^{\rm plasma}_{\mu
u} = 0$$
 (with appropriate boundary conditions on bubble wall)

Local Thermal Equilibrium  

$$T^{plasma}_{\mu\nu} = w \ u_{\mu}u_{\nu} - g_{\mu\nu} \ p$$

$$w = e + p$$

$$u_{\mu} = \frac{(1, \mathbf{v})}{\sqrt{1 - \mathbf{v}^2}} = (\gamma, \gamma \mathbf{v})$$

Self-similarity 
$$v(r,t) = v(\xi = r/t)$$

Estimate of Energy available for GW production (fluid bulk motion for one bubble)

$$\int \overline{U}_{\mathbf{f}}^2 = \frac{3}{e \, v_{\mathbf{w}}^3} \int w(\xi) \, v^2 \gamma^2 \xi^2 d\xi = \frac{\kappa \alpha}{1+\alpha}$$

(enthalpy weighted) plasma RMS four velocity

Hindmarsh, Huber, Rummukainen, Weir, PRD 96 (2017) 103520

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Hindmarsh, Huber, Rummukainen, Weir, PRD 96 (2017) 103520



Efficiency coefficient (PT Energy Budget)

Kamionkowski, Kosowsky, Turner, PRD **49** (1994) 2837 Espinosa, Konstandin, No, Servant, JCAP **1006** (2010) 028

□ Gravitational waves (GWs) produced by several sources in a PT:

$$h^2 \Omega_{\rm gw} = h^2 \Omega_{\phi} + h^2 \Omega_{\rm sw} + h^2 \Omega_{\rm turb}$$

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LISA Cosmology Working Group effort to provide state-of-art:

2015 CosWG Review

Caprini et al, JCAP 1604 (2016) 001

#### + very recent update

Caprini et al, arXiv:1910.13125



#### Science with the space-based interferometer eLISA. II: Gravitational waves from cosmological phase transitions

Chiara Caprini<sup>a</sup>, Mark Hindmarsh<sup>b,c</sup>, Stephan Huber<sup>b</sup>, Thomas Konstandin<sup>d</sup>, Jonathan Kozaczuk<sup>e</sup>, Germano Nardini<sup>f</sup>, Jose Miguel No<sup>b</sup>, Antoine Petiteau<sup>g</sup>, Pedro Schwaller<sup>d</sup>, Géraldine Servant<sup>d,h</sup>, David J. Weir<sup>i</sup>

#### Detecting gravitational waves from cosmological phase transitions with LISA: an update

Chiara Caprini<sup>a</sup>, Mikael Chala<sup>b,c,†</sup>, Glauber C. Dorsch<sup>d</sup>, Mark Hindmarsh<sup>e,f</sup>, Stephan J. Huber<sup>f</sup>, Thomas Konstandin<sup>g,‡</sup>, Jonathan Kozaczuk<sup>h,i,j,§</sup>, Germano Nardini<sup>k</sup>, Jose Miguel No<sup>l,m</sup>, Kari Rummukainen<sup>e</sup>, Pedro Schwaller<sup>n</sup>, Geraldine Servant<sup>g,o</sup>, Anders Tranberg<sup>k</sup>, David J. Weir<sup>e,p,¶</sup> For the LISA Cosmology Working Group

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 $\square$   $h^2\Omega_{\phi}$  sourced by collisions of bubble walls

Kosowsky, Turner, Watkins, PRL **69** (1992) 2026; PRD **45** (1992) 4514 Huber, Konstandin, JCAP **0809** (2008) 022 Weir, PRD **93** (2016) 124037 Cutting, Hindmarsh, Weir, PRD **97** (2018) 123513

In general, negligible expect for very strong supercooling  $\Rightarrow \alpha >> 1$ 

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In general, negligible expect for very strong supercooling  $\Rightarrow \alpha >> 1$ 

Such amount of supercooling incompatible with PT completion... Ellis, Lewicki, No, JCAP **1904** (2019) 003

... except for conformal scalar potentials!

### Iason Baldes talk coming up!

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Hindmarsh, Huber, Rummukainen, Weir, PRL 112 (2014) 041301; PRD 92 (2015) 123009; PRD 96 (2017) 103520
Hindmarsh, PRL 120 (2018) 071301
Konstandin, JCAP 1803 (2018) 047
Hindmarsh, Hijazi, arXiv:1909.10040

#### Typically dominant signal

GW power spectrum (numerical simulations)



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 $10^{-8}$ 

\_\_\_\_\_Ω<sub>sw</sub>

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Typically dominant signal

After  $\tau_{\rm sh} \sim L_{\rm f}/\overline{U}_{\rm f}$ , fluid becomes nonlinear (shock formation) the characteristic fluid length scale Sound wave GW source shuts-off

$$\frac{d\Omega_{\rm gw,0}}{d\ln(f)} = 0.687 F_{\rm gw,0} K^2 (H_* R_*/c_s) \tilde{\Omega}_{\rm gw} C \left(\frac{f}{f_{\rm p,0}}\right) \left(\times H_* \tau_{\rm sh}\right)$$

 $H_*\tau_{\rm sh} = H_*R_*/K^{1/2} < 1$ 

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characteristic fluid length scale







Cutting, Hindmarsh, Weir, arXiv:1906.00480

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Caprini, Durrer, Servant, JCAP **0912** (2009) 024 Roper Pol, Mandal, Brandenburg, Kahniashvili, Kosowsky, arXiv:1903.08585

- → Turbulent flow expected to develop when sound waves shut-off
- $\rightarrow$  Vorticity can also coexist with sound waves for deflagrations and  $\alpha$  > 0.1

Cutting, Hindmarsh, Weir, arXiv:1906.00480

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Hindmarsh, Huber, Rummukainen, Weir, PRL **112** (2014) 041301; PRD **92** (2015) 123009; PRD **96** (2017) 103520 Hindmarsh, PRL **120** (2018) 071301 Konstandin, JCAP **1803** (2018) 047 Hindmarsh, Hijazi, arXiv:1909.10040

#### Typically dominant signal

After  $\tau_{\rm sh} \sim L_{\rm f}/\overline{U}_{\rm f}$  , fluid becomes nonlinear (shock formation)  $$\Box_{\rm f}$$  Sound wave GW source shuts-off

### $\square$ $h^2 \Omega_{\mathrm{turb}}$ sourced by plasma turbulence (vortical modes)

Gogoberidze, Kahniashvili, Kosowsky, PRD **76** (2007) 083002 Caprini, Durrer, Servant, JCAP **0912** (2009) 024 Roper Pol, Mandal, Brandenburg, Kahniashvili, Kosowsky, arXiv:1903.08585

→ Turbul Mutimerican to develop when sound waves shut-off
 → Vorticity can also coexist with sound to be fortiging and α > 0.1
 Cutting, Hindmarsh, Weir, arXiv:1906.00480

#### Duration of sound wave GW source

Initially assumed linear fluid regime lasts approx. a Hubble time

$$\tau_{\rm sh} \gtrsim H_*^1$$



LISA signal to noise

$$SNR = \sqrt{\mathcal{T} \int_{f_{\min}}^{f_{\max}} df \left[\frac{h^2 \Omega_{GW}(f)}{h^2 \Omega_{Sens}(f)}\right]^2}$$

$$h^2 \Omega_{\text{Sens}}(f) = \frac{2\pi^2}{3H_0^2} f^3 S_h(f)$$



PTPlot Tool - D. Weir

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PTPlot Tool - D. Weir

### Understanding of vorticity generation is ongoing...

Cutting, Hindmarsh, Weir, arXiv:1906.00480



#### Detonations ( $\alpha > 0.1$ )







#### Deflagrations ( $\alpha > 0.1$ )





### Understanding of vorticity generation is ongoing...

Cutting, Hindmarsh, Weir, arXiv:1906.00480



Deflagrations with large  $\alpha$  (> 0.1) generate significant vorticity coexisting with sound waves!

GW generation vs EW Baryogenesis in 1<sup>st</sup> Order EW Phase Transition

GW generation vs EW Baryogenesis in 1<sup>st</sup> Order EW Phase Transition

GWs: Sizable plasma bulk motion ⇒ Sizable v EWBG: Velocities ~ 0.05 - 0.1 preferred (efficient transport)

Incompatible?

GW generation vs EW Baryogenesis in 1<sup>st</sup> Order EW Phase Transition



GW generation vs EW Baryogenesis in 1<sup>st</sup> Order EW Phase Transition

For detonations:EWBG would notwork(inefficient transport)



GW generation vs EW Baryogenesis in 1<sup>st</sup> Order EW Phase Transition



GW generation vs EW Baryogenesis in 1<sup>st</sup> Order EW Phase Transition



**"Supersonic EWBG"** 







**BSM**: New Physics sizeably coupled to Higgs can drastically change the EWPT nature

▶ New Physics should induce deviations in Higgs couplings

▶ New Physics needed close to EW scale

### Some further aspects of the EW Phase Transition

Effective Potential (finite T)



$$V_{\text{eff}}(h,T) = V_0(h) + V_0^{\text{loop}}(h) + V_T(h,T)$$
$$V_1^{T}(h,T) = \frac{T^4}{2\pi^2} \left[ \sum_i \pm n_i J_{\pm} \left( \frac{m_i^2(-h_{-})}{T^2} \right) \right]$$

High-T expansion:  

$$T^{4}J_{+}\left(\frac{m^{2}}{T^{2}}\right) = -\frac{\pi^{4}T^{4}}{45} + \frac{\pi^{2}m^{2}T^{2}}{12} - \underbrace{\left(\frac{T\pi(m^{2})^{3/2}}{6} - \frac{(m^{4})}{32}\log\frac{m^{2}}{a_{b}T^{2}}\right)}_{G} - \underbrace{\left(\frac{m^{2}}{32}\right)}_{T^{4}J_{-}}\left(\frac{m^{2}}{T^{2}}\right) = \frac{7\pi^{4}T^{4}}{360} - \frac{\pi^{2}m^{2}T^{2}}{24} - \frac{(m^{4})}{32}\log\frac{m^{2}}{a_{f}T^{2}},$$

 $V_{eff}(h,T) \approx (a T^2 - \mu^2) h^2 - E(T) h^3 + \lambda_{eff}(T) h^4$