A Complete Theoretical Framework for Antarctica based ANITA experiment and its Implication for the ANITA Anomalous Events

HEP Seminar The Service de Physique des Particules Élémentaires of the Université Libre de Bruxelles (ULB) Inter-University Institute for High Energies IIHE (ULB-VUB) Belgium, Europe

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The ANITA Experiment (ANtarctic Impulsive Transient Antenna)

NASA long-duration balloon payload with an array of radio antennas.





He balloon flying at a height 37Km above Antarctica, carries array of radio antennas to detect radio signals from Cosmic Neutrinos/UHECRs 1/52

Goals of ANITA Mission

- ANITA is the First Experiment to Implement Radio Technique in the EeV Energy regime (1 EeV = 10¹⁸ eV)
- UHECR induced radio emission in Air detected by balloon-borne radio antennas

Why Antarctica?





Lots of ice ~ 1 million Km³ **Coverage Radius ~ 700 Km Detector at height ~ 37 Km** Ice has Excellent RF clarity **RF attenuation length: ~ 1km** Low Population, Less noise

IceCube (Energy ~ PeV), ARA, ARIANNA are other experiments in Antarctica

Journal of Astronomical Instrumentation VOL. 06, NO. 02, P. Gorham, D.Z. Besson, P. Dasgupta, P. Jain et. al.

UHE Neutrinos: The Ideal Astronomical Messengers



Experimentalists During the Launch of ANITA





28-32 days at float



Primary Goal of ANITA :

UHE v Detection based on Askaryan Mechanism in dense medium (Antarctic ice)



Gurgen Askaryan

v + ice (H₂O) → knock-on collisions → excess e⁻

Charge excess moving with velocity greater than the velocity of light in the traversed medium — Cherenkov Radiation



Working Principle of ANITA

- Detect UHE Neutrinos via Askaryan effect in ice

 Main Goal
- Coherent Geo-Synchrotron Radiation at Radio Frequencies from highly relativistic e⁺-e⁻ pair gyrating in Earth's mag. field



P. W. Gorham et al. arXiv:1710.11175



How to Distinguish between Neutrino Induced signals and UHECR induced signals ?

Neutrino Signal vs. Air Shower Signal



Contribution to ANITA Mission ?

Using First Principle Calculation

We developed

"First Complete Theoretical Framework" for ANITA

&

its successor Neutrino detection experiments

- Development of First Complete & General theoretical framework for reflected RF signal simulation
 P. Dasgupta and P. Jain (Thesis supervisor)
- Incorporated Antarctic Surface features
 & Curvature of Earth using First Principle Calculation
- Investigated the mystery of ANITA Anomalous Events
- Prof. David Besson (at the Dept. Of Physics, Univ. of Kansas, USA) & I investigated Antarctic subsurface Reflection and Transmission
- Extension of this formalism for in-ice (such as RADAR, RNO-G) or under water neutrino detection





Development of Complete Theoretical Framework for NASA sponsored ANITA experiment

Decomposition of Spherical Waves into plane waves: Weyl Formalism (Year 1919)

A dipole radiator on the z axis

- Extensive Air Shower (EAS)
- Geosynchroton radiation (200- 650) MHz



Start with the simplest case

Assume Earth Surface is FLAT

Flat +Smooth Flat +Rough



Decomposition of Spherical Waves into plane waves: Weyl Formalism



S. Prohira, A. Novikov, P. Dasgupta, P. Jain et al. Phys. Rev. D 98, 042004

Compute the Incident Field associated with each plane wave

$$\vec{E} = \vec{\nabla}(\vec{\nabla} \cdot \vec{\Pi}) + k^2 \vec{\Pi}$$
$$\vec{H} = \frac{k^2}{i\omega\mu} (\vec{\nabla} \times \vec{\Pi})$$

Electric and Magnetic field associated with each incident plane wave

$$\vec{E}_{inc} = \frac{ik^3}{8\epsilon\pi^2} \tilde{\Pi}[-\sin^2\alpha\cos\beta\sin\beta\hat{x} + (1 - \sin^2\alpha\sin^2\beta)\hat{y} + (\sin\alpha\sin\beta\cos\alpha)\hat{z}]$$
$$\vec{H}_{inc} = \frac{ik^2\omega}{8\pi^2} \tilde{\Pi}[\cos\alpha\hat{x} + (\cos\beta\sin\alpha)\hat{z}].$$
Decompose into s and p components
$$\vec{E}_{inc} = \vec{E}_{inc}^{s} + \vec{E}_{inc}^{p}$$
$$\vec{H}_{inc} = \vec{H}_{inc}^{s} + \vec{H}_{inc}^{p}$$

Reflection and Transmission: Flat Surface



Transmitted Field Components E^s_{trans}, E^p_{trans}, H^s_{trans}, H^p_{trans}





Impose boundary conditions at z=0

$$f_r^s = \frac{k\cos\alpha - k_1\cos\alpha_t}{k\cos\alpha + k_1\cos\alpha_t}$$

$$f_r^p = \frac{k_1\cos\alpha - k\cos\alpha_t}{k_1\cos\alpha + k\cos\alpha_t}$$

$$f_r^s = \left(\frac{k}{k_1}\right)^2 \frac{2k_1\cos\alpha}{k_1\cos\alpha_t + k\cos\alpha}$$

$$f_t^p = \left(\frac{k}{k_1}\right)^2 \left(\frac{1}{\cos\alpha_t}\right) \frac{2k_1\cos^2\alpha}{k_1\cos\alpha + k\cos\alpha_t}$$

Transmitted Field: Flat Reflecting Surface

$$E_{(trans),y} = \frac{ik_1^3}{8\epsilon_1 \pi^2} \int_0^{2\pi} \int_0^{\frac{\pi}{2} - i\infty} \tilde{\Pi}_t (f_t^s \cos^2 \beta_t + f_t^p \cos^2 \alpha_t \sin^2 \beta_t) \sin \alpha d\alpha d\beta$$

Transmitted Electric field component perpendicular to the Plane of incidence (H- Pol)

Reflected Field (y component) for H-Pol : Flat Refleting Surface

$$E_{(ref),y} = \frac{ik^3}{8\epsilon\pi^2} \int_0^{2\pi} \int_0^{\frac{\pi}{2}-i\infty} \tilde{\Pi}_{ref} (f_r^s \cos^2\beta - f_r^p \cos^2\alpha \sin^2\beta) \sin\alpha d\alpha d\beta$$

 Electric fieid component perpendicular to Plane of incidence (H- Pol)

Compute this Integral for different Elevation Angles in the Frequeny range (200-650) MHz

Also change R.I of ice from 1.25 to 1.75

Compare the simulated field Ref/Direct amplitude with Fresnel Coeffcients 22/52

H-Pol (reflected/direct) amplitude Ratio: Flat Surface calculation using our theoretical framework

<pre>#angle(°)</pre>	<pre>#amp(flat)</pre>	#fresnel
4	0.862801	0.867387
5	0.835699	0.837217
6	0.806814	0.808184
7	0.779242	0.780260
8	0.753162	0.753415
9	0.726390	0.727618
10	0.702397	0.702839
12	0.654847	0.656209
15	0.590581	0.593123
20	0.502614	0.504245
25	0.432435	0.432603
30	0.374581	0.375000
35	0.327991	0.328724
40	0.290877	0.291543
45	0.261001	0.261666
50	0.237114	0.237675
55	0.216842	0.218464
60	0.202191	0.203177
65	0.190783	0.191156
70	0.180672	0.181906
75	0.174257	0.175055
78	0.171902	0.171982
80	0.168961	0.170338
85	0.167334	0.167576

$$E_{(ref),y} = \frac{ik^3}{8\epsilon\pi^2} \int_0^{2\pi} \int_0^{\frac{\pi}{2} - i\infty} \tilde{\Pi}_{ref} (f_r^s \cos^2\beta - f_r^p \cos^2\alpha \sin^2\beta) \sin\alpha d\alpha d\beta$$

S. Prohira, A. Novikov, P. Dasgupta, P. Jain et al. PHYS. REV. D 98, 042004 (2018) 23/52

This framework works perfectly for a flat diectric medium

Our result matches with Fresnel Reflection and Transmission coefficients



Now the realistic case of Spherical +Uneven Earth Surface

Complexity starts from here

A Rigorous Formalism to study the Radio Signals Reflecting off a Spherical Surface



- We extend our Formalism for a Spherical Reflecting Surface
- We do not make any approximation
- Surface topography data for Roughness Model
- S. Prohira, A. Novikov, P. Dasgupta, P. Jain et al. Phys. Rev. D 98, 042004



For each plane wave with angle alpha and beta we need to find the rotation matrix to transform the coordinate sytem from x-y-z to x'-y'-z'

$$Rot = \begin{pmatrix} \cos(\alpha' - \alpha)\cos\beta & \cos(\alpha' - \alpha)\sin\beta & -\sin(\alpha' - \alpha) \\ -\sin\beta & \cos\beta & 0 \\ \sin(\alpha' - \alpha)\cos\beta & \sin(\alpha' - \alpha)\sin\beta & \cos(\alpha' - \alpha) \end{pmatrix}$$

We then translate the coordinate system to change o to o'



S. Prohira, A. Novikov, P. Dasgupta, P. Jain et al. Phys. Rev. D 98, 042004

Antarctic Surface Topography from The Reference Elevation Model of Antarctica (REMA)

We receive this data from National Geospatial-Intelligence Agency (NGA), USA



Antarctic Surface



Reflection and Transmission: Spherical Surface

- Reflected Field components $E'^{(s)}_{ref} E'^{(p)}_{ref} \& H'^{(s)}_{ref} H'^{(p)}_{ref}$
- Transmitted Field Components E^{'(s)}_{trans}, E^{'(p)}_{trans} & H^{'(s)}_{trans}, H^{'(p)}_{trans}

$$E'_{ref} = E'^{(s)}_{ref} + E'^{(p)}_{ref}$$

$$H'_{ref} = H'^{(s)}_{ref} + H'^{(p)}_{ref}$$

$$H'_{ref} = H'^{(s)}_{ref} + H'^{(p)}_{ref}$$

$$H'_{trans} = H'^{(s)}_{trans} + H'^{(p)}_{trans}$$

Impose boundary conditions at Z' = 0

$$f_r'^s = \frac{k\cos\alpha' - k_1\cos\alpha'_t}{k\cos\alpha' + k_1\cos\alpha'_t},$$

$$f_r'^p = \frac{k_1\cos\alpha' - k\cos\alpha'_t}{k_1\cos\alpha' + k_1\cos\alpha'_t},$$

$$f_r'^s = \left(\frac{k}{k_1}\right)^2 \frac{2k_1\cos\alpha'}{k\cos\alpha' + k_1\cos\alpha'_t}.$$

$$f_t'^p = \left(\frac{k}{k_1}\right)^2 \left(\frac{1}{\cos\alpha_t}\right) \frac{2k_1\cos\alpha\cos\alpha'}{k_1\cos\alpha' + k\cos\alpha'_t}$$

Antarctic Surface Roughness Model by Peter Gorham

Gorham's Roughness Model is an average roughness model

We started with simple model to compute reflected fields



L is the radius we compute around the specular angle theta_z For $L_0 = 150$ m, $Sigma_h(L) = 0.041$ m H = Hurst Parameter = 0.65

$$\sigma_h(L) = \sigma_h(L_0) \left(\frac{L}{L_0}\right)^H$$

$$F(k, \rho, \theta) = \exp[-2k^2\sigma_h(\rho_\perp)^2\cos^2\theta_z]$$

k (wave number) is dependent on the Frequency of Incoming wave 30/52

After Computing Electric and Magnetic fields in x'-y'z'

We transform back to the original (x-y-z) coordinate system by inverse rotation



Reflected fields for Spherical + Rough Reflecting Surface

32/52



Take Ratio with the amplitude of Direct pulse propagated from Dipole to Detector

Compare this amplitude ratio r/d with the HiCal Experimental data

S. Prohira, A. Novikov, P. Dasgupta, P. Jain et al. PHYS. REV. D 98, 042004 (2018)

r/d power ratio compared with HiCal data



Testing our Framework

- We Compute Reflected H-Pol Fields and compared with HiCal-2 Data
- This Formalism works very well for elevation angle >10°
- → For elevation angle < 10° are off from HiCal-2 data</p>
- → Results shown in August 2018 in Physical Review D. Volume 98, 042004
- Refinement of the Framework is necessary to apply this model for all elevation angles applicable to ANITA payload"

Refinement of our theory

 Next, we developed a formalism called "Local Plane Wave Approximation" which is a modified version of our theory developed in Physical Review D, volume 98, 042004

 This theoretical development is currently under review in Astroparticle Physics Journal (arXiv:1811.00900v2)

Refinement of Framework : "Local Plane Wave Approximation"



$$Rot_{1} = \begin{pmatrix} \cos\xi\cos\tilde{\beta} & \sin\tilde{\beta} - \sin\xi\cos\tilde{\beta} \\ -\cos\xi\sin\tilde{\beta} & \cos\tilde{\beta} & \sin\xi\sin\tilde{\beta} \\ \sin\xi & 0 & \cos\xi \end{pmatrix}$$

By Rotatiing the coordinate system and subsequent translation we make sure k_{inc}^{m} , k_{ref}^{m} & local normal lie in the same plane

$$\begin{split} \vec{E}_{i}^{\prime\prime\prime} &= R_{y^{\prime}}(\psi)\vec{E}_{i}^{\prime} \\ &= \frac{ik^{3}}{8\epsilon\pi^{2}}\tilde{\Pi}_{S,i}[(\sin\tilde{\beta}\cos^{2}\tilde{\alpha}\cos\psi + \sin\alpha\cos\tilde{\alpha}\sin\beta\sin\psi)\hat{x}^{\prime\prime\prime} + \cos\tilde{\beta}\hat{y}^{\prime\prime\prime} \\ &+ (\sin\alpha\cos\tilde{\alpha}\sin\beta\cos\psi - \cos^{2}\tilde{\alpha}\sin\tilde{\beta}\sin\psi)\hat{z}^{\prime\prime\prime}] \end{split}$$

$$\begin{split} \vec{H}_{i}^{\prime\prime\prime\prime} &= R_{y'}(\psi) \vec{H}_{i}^{\prime} \\ &= \frac{ik^{2}\omega}{8\pi^{2}} \tilde{\Pi}_{S,i} [\cos \tilde{\beta} \cos(\tilde{\alpha} - \psi) \hat{x}^{\prime\prime\prime} - \cos \tilde{\alpha} \sin \tilde{\beta} \hat{y}^{\prime\prime\prime} \\ &+ \cos \tilde{\beta} \sin(\tilde{\alpha} - \psi) \hat{z}^{\prime\prime\prime}] \end{split}$$

P. Dasgupta and P Jain arXiv:1811.00900, April 2020

Reflection and Transmission: Spherical Surface with Local Plane Wave Analysis

- Reflected Field components E^{'''(s)}_{ref} E^{'''(p)}_{ref} & H^{'''(s)}_{ref} H^{'''(p)}_{ref}
- Transmitted Field components $E^{\prime\prime\prime(s)}_{trans}$, $E^{\prime\prime\prime(p)}_{trans}$ & $H^{\prime\prime\prime(s)}_{trans}$, $H^{\prime\prime\prime(p)}_{trans}$

 $H'''_{ref} = H'''^{(s)}_{ref} + H'''^{(p)}_{ref}$ $H'''_{trans} = H'''^{(s)}_{trans} + H'''^{(p)}_{trans}$

Impose boundary conditions at z=0

 $\mathbf{E}_{ref}^{""} = \mathbf{E}_{ref}^{""(s)} + \mathbf{E}_{ref}^{""(p)}$

 $\mathbf{E}_{\text{trans}}^{\text{mass}} = \mathbf{E}_{\text{trans}}^{\text{mass}} + \mathbf{E}_{\text{trans}}^{\text{mass}}$

$$f_{r}^{\prime(s)} = \frac{k\cos(\tilde{\alpha} - \psi) - k_{1}\cos(\tilde{\alpha}_{t} - \psi)}{k\cos(\tilde{\alpha} - \psi) + k_{1}\cos(\tilde{\alpha}_{t} - \psi)},$$

$$f_{r}^{\prime(s)} = \left(\frac{k}{k_{1}}\right)^{2} \frac{2k_{1}\cos(\tilde{\alpha} - \psi) + k_{1}\cos(\tilde{\alpha} - \psi)}{k\cos(\tilde{\alpha} - \psi) + k_{1}\cos(\tilde{\alpha}_{t} - \psi)},$$

$$f_{t}^{\prime(s)} = \left(\frac{k}{k_{1}}\right)^{2} \frac{2k_{1}\cos(\tilde{\alpha} - \psi) + k_{1}\cos(\tilde{\alpha}_{t} - \psi)}{k\cos(\tilde{\alpha} - \psi) + k_{1}\cos(\tilde{\alpha}_{t} - \psi)},$$

Local Plane Wave Approximation: Electric and Magnetic Field (HPol &VPol)

Peter Gorham's roughness model

$$F(k,\rho,\theta) = \exp[-2k^2\sigma_h(\rho_\perp)^2\cos^2\theta_z]$$
$$\sigma_h(L) = \sigma_h(L_0) \left(\frac{L}{L_0}\right)^H$$

Reflected fields for a Spherical + Rough Reflecting Surface

$$E_{r,y} = \frac{ik^3}{8\epsilon\pi^2} \tilde{\Pi}_{S,r} \left[f_r^{\prime(s)} \cos^2 \tilde{\beta} - f_r^{\prime(p)} \cos \tilde{\alpha} \cos(\tilde{\alpha} - 2\psi) \sin^2 \tilde{\beta} \right]$$
$$E_{(r,total),y} = \int_0^{2\pi} \int_0^{\frac{\pi}{2} - i\infty} F_{rough} E_{r,y} \sin \alpha d\alpha d\beta.$$

Take Ratio with the amplitude of Direct pulse propagated from Dipole to Detector Compare this amplitude ratio r/d with the HiCal Experimental data

r/d power ratio for a spherical-rough surface: Local Plane Wave Approximation



P. Dasgupta and P Jain (Under Review: Astroparticle Physics Journal), arXiv:1811.00900v2

Using First Principle Calculation

We developed

"First Complete Theoretical Framework" for ANITA

&

its successor neutrino detection experiments

"Anomalous Events" Detected by ANITA



ANITA Anomalous Events

Two Unusual steeply pointed up-going air showers with E ~ EeV scale



Dominantly H-Pol

No Phase Inversion !

Tau-neutrino interaction in ice?

Our Formalism Explain these anomalous events??

Next we try to unfold this mystery

HiCal Pulse



Generalizing Framework for Electromagnetic Pulses (Applicable to ANITA events)

- monochromatic spherical electromagnetic waves.
- Reflection from ice as a function of frequency, angle of reflection.

We need to consider Electromagnetic pulses to study the ANITA signals

$$ilde{F}(n) = \sum_{p=0}^{N-1} f(p) e^{irac{2\pi n}{N}p}$$

$$E'_{ref,y} = \int_0^{\frac{\pi}{2} - i\infty} \int_0^{2\pi} \int_{\omega} \frac{ik}{2\pi} \tilde{\Pi}_{S,r} F_{rough}(\omega, \alpha, \beta, \theta_z) \eta(\alpha, \beta, \omega) (\tilde{F}(\omega) e^{-i\omega(t+t_0)}) d\omega d\Omega.$$

Superposition of dipole radiation of different frequencies

Incorporating our formalism to compute Direct and Reflected pulse profile (using HiCal 2 pulses)

Polarity of Reflected signals



P. Dasgupta, P. Jain arXiv: 1811.00900

New Roughness Models

- Antarctic topographic data one side is quite smooth but the other side has lots of variation in altitude.
- In order to account for this we allowed our incident angle to vary over a range of about 3 Km by about 1 degree.
- We also simply increase the curvature. This will model a hill like structure in the region of mystery events



Non-Inversion of phase using new roughness model



We see non-inversion of phase in simulated reflected pulses

Possible explanation for ANITA mystery events ?



Time in ns

Sub-Surface Reflection: Another Possible Explanation for Mystery Event



I am at the finishing stage of this work with Prof. Dave Besson, PI of HiCal Expt

(work in progress)

Pic Courtesy: Dave Z Besson



Conclusion

We developed complete theoretical framework for UHECR induced radio detection at Balloon payload for a spherical, rough reflecting surface.

Future ground based and Balloon-borne Cosmic Ray detection experiments can use our formalism.

Incorporation of actual Antarctic surface topographical data in the ANITA Monte Carlo (icemc)

Explanation for ANITA mystery events ? We show that surface topography can cause additional phase inversion in pulses.

Our framework can be implemented for the in ice propagation of the radio signals applicable to RADAR, RNO-G project

Thank You

I PHAN

BACK UP

Recent results with random surface roughness

Random surface
 No polarity Inversion !!
 roughness profile



A Rigorous Framework for ANITA-HiCal

Reflection off a Flat Surface

Reflection off a Spherical Surface





Extensive Air Shower (EAS)

geosynchroton radiation

A dipole radiator on the z axis



Our formalism might be able to unfold the mystery of ANITA anomalous events.

We will submit these new results soon.

Future Scope in UHE Neutrino Study



Huge ANITA Data to be analysed

Cosmic Ray and Tau neutrino analysis

Unfolding Mystery events & the Sources

ANITA 5 Proposal !! ARA, ARIANA Experiment !

Frequency dependence of r/d power ratio





P. Dasgupta and P Jain arXiv:1811.00900, November 2018



Reflecting Surface is Curved

Reflection of Spherical waves from Spherical-Rough surface

We also studied the case of non-uniform Roughness

Fourier-Bessel Integral: Expansion of Spherical Wave into plane waves

$$h_0^{(1)}(kR) = \frac{e^{ikR}}{ikR} = \int_{i\infty}^1 e^{ikR\eta} d\eta.$$

Using k.R= k R cos y

$$\frac{e^{ikR}}{ikR} = \int_0^{\frac{\pi}{2} - i\infty} e^{ikR\cos\gamma} \sin\gamma \,d\gamma$$

Since value of the integral is invarient to a rotation of the reference axes, Hence we get the desired Weyl representation of Spherical Waves into plane waves

$$\frac{e^{ikR}}{R} = \frac{ik}{2\pi} \int_0^{2\pi} \int_0^{\pi} \int_0^{\pi} e^{ikR \cos \gamma} \sin \alpha \, d\alpha \, d\beta.$$

Why to study Ultra High Energy Neutrinos ??



- Ideal Astronomical Messengers
- Uncharged, not affected by B_{earth}
- New information on UHE (> 10¹⁹ eV) CR
- GZK effect : CR interaction with Y_{CMB} produce neutrinos.
- UHE neutrino Sources !!

$$p_{CR}^{+} + \gamma_{CMB} \rightarrow \Delta^* \rightarrow n + \pi^+$$

$$\pi^+ \rightarrow \mu^+ + \nu_{\mu}$$

$$\mu^+ \to e^+ + \nu_e + \overline{\nu}_\mu$$

ANITA and HiCal Working Principle

• Antarctic Impulsive Transient Antenna is a balloon-borne RF (200-650)

MHz Receiver Array.

- High Altitude Calibration (HiCal) is a Balloon-Borne RF Transmitter, in concert with the ANITA RF receiver array.
- ANITA-HiCal Measures Antarctic Surface Reflectivity in RF regime.
- Main Goal of ANITA is to detect Highest Energy Particles via the Radio signals produced by the UHECR interacrion with Earth's atmosphere.
- Down Coming Charged Particles produce Mainly H-Pol radiation.



P. W. Gorham et al. arXiv:1710.11175

ANITA Detector





He balloon flying at a height 37Km above Antarctica, carries array of radio antennas to detect radio signals from Cosmic Neutrinos/UHECRs

Weyl Formalism : Decomposition of Spherical Waves into Plane Waves



 $\vec{k}_I = k(\sin \alpha \cos \beta \hat{x} + \sin \alpha \sin \beta \hat{y} - \cos \alpha \hat{z})$ Incident wave vector k(α , β)

 $\vec{k}_t = k_1 [\sin \alpha_t \cos \beta_t \hat{x} + \sin \alpha_t \sin \beta_t \hat{y} - \cos \alpha_t \hat{z}]$ Transmitted wave vector k(α_t, β_t)

* Weyl Formalism (H. Weyl, Ann. Physik, 60,481,1919) Stratton(1941), Born & Wolf (1980)

Spherical Earth Surface: Reflection and Refraction of radio signals



S. Prohira, A. Novikov, P. Dasgupta, P. Jain et al. Phys. Rev. D 98, 042004

Include Non Uniform Roughness Parameter

Previously we incorporated Roughness Factor that assumed a circular region around the specular point

$$F(k,\rho,\theta) = \exp[-2k^2\sigma_h(\rho_\perp)^2\cos^2\theta_z] \qquad \sigma_h(L) = \sigma_h(L_0)\left(\frac{L}{L_0}\right)^H$$

where, sigma(L_0)= 0.041, L_0 = 150 m, H= Hurst Parameter=0.65,

Now, I use an elliptical region around the specular pt $X^2 + (aY)^2 = L^2$ with a<1

We compute reflected pulses using this assymetric roughness parameter "a" choosing a=0.1, 0.25, 0.5 and a=1 (which is the symmetric case, as given by Peter Gorham's model) 0.041 < =sigma(L₀)<=0.071 and L₀ changed accordingly between 150m to 80 m RESULTS with different assymetric factors "a", and slightly changed sigma(L₀), and L₀ values are obtained.