Gravitational wave observations of binary black holes

Outline

- Gravitational waves
- Towards a model of binary black hole coalescence
- Detection and parameter estimation
    - detection
    - inference
- Results from GWTC-1
    - Physical parameters
    - Selected results on tests of general relativity
    - Some words on investigating the merger remnant
- Conclusions

Gravitational waves

- Einstein 1916-18: quadrupolar radiation in linear theory

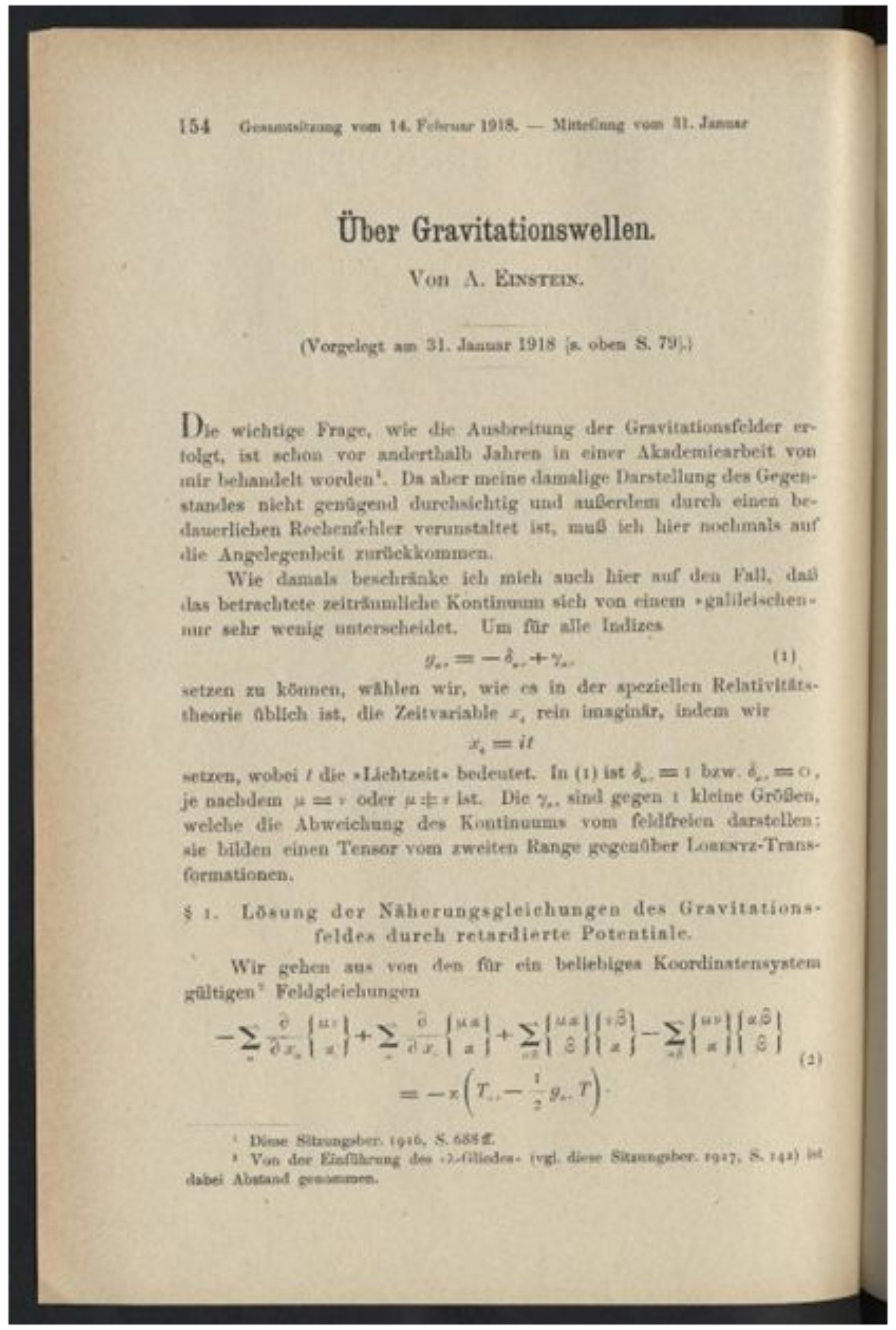
$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu} \qquad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

- Generated by time varying quadrupoles (at LO)

$$h \simeq \frac{1}{r}\frac{G}{c^4}\ddot{Q}$$

- Travel at the speed of light

$$v_{GW} = c$$



154 Gesamtsitzung vom 14. Februar 1918. — Mitteilung vom 31. Januar

Über Gravitationswellen.

Von A. Einstein.

(Vorgelegt am 31. Januar 1918 [s. oben S. 79].)

Die wichtige Frage, wie die Ausbreitung der Gravitationsfelder erfolgt, ist schon vor anderthalb Jahren in einer Akademiearbeit von mir behandelt worden[1]. Da aber meine damalige Darstellung des Gegenstandes nicht genügend durchsichtig und außerdem durch einen bedauerlichen Rechenfehler verunstaltet ist, muß ich hier nochmals auf die Angelegenheit zurückkommen.

Wie damals beschränke ich mich auch hier auf den Fall, daß das betrachtete zeiträumliche Kontinuum sich von einem »galileischen« nur sehr wenig unterscheidet. Um für alle Indizes

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} \qquad (1)$$

setzen zu können, wählen wir, wie es in der speziellen Relativitätstheorie üblich ist, die Zeitvariable x_4 rein imaginär, indem wir

$$x_4 = it$$

setzen, wobei t die »Lichtzeit« bedeutet. In (1) ist $\delta_{\mu\nu} = 1$ bzw. $\delta_{\mu\nu} = 0$, je nachdem $\mu = \nu$ oder $\mu \neq \nu$ ist. Die $\gamma_{\mu\nu}$ sind gegen 1 kleine Größen, welche die Abweichung des Kontinuums vom feldfreien darstellen; sie bilden einen Tensor vom zweiten Range gegenüber Lorentz-Transformationen.

§ 1. Lösung der Näherungsgleichungen des Gravitationsfeldes durch retardierte Potentiale.

Wir gehen aus von den für ein beliebiges Koordinatensystem gültigen[2] Feldgleichungen

$$-\sum_{\alpha} \frac{\partial}{\partial x_\alpha}\begin{Bmatrix}\mu\nu\\ \alpha\end{Bmatrix} + \sum_{\alpha} \frac{\partial}{\partial x_\nu}\begin{Bmatrix}\mu\alpha\\ \alpha\end{Bmatrix} + \sum_{\alpha\beta}\begin{Bmatrix}\mu\alpha\\ \beta\end{Bmatrix}\begin{Bmatrix}\nu\beta\\ \alpha\end{Bmatrix} - \sum_{\alpha\beta}\begin{Bmatrix}\mu\nu\\ \alpha\end{Bmatrix}\begin{Bmatrix}\alpha\beta\\ \beta\end{Bmatrix} = -\varkappa\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right) \qquad (2)$$

[1] Diese Sitzungsber. 1916, S. 688 ff.

[2] Von der Einführung des »λ-Gliedes« (vgl. diese Sitzungsber. 1917, S. 142) ist dabei Abstand genommen.

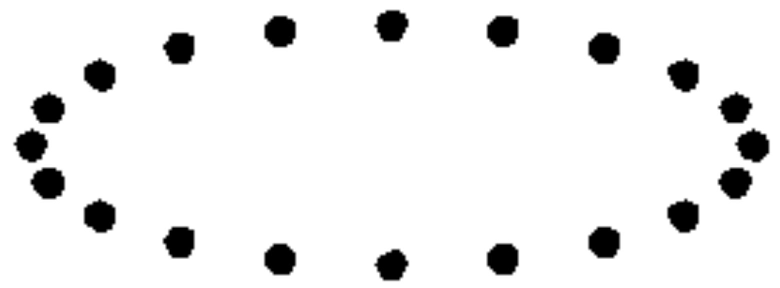
Polarisation basis

- In GR two transverse polarisations: + (plus), x (cross)

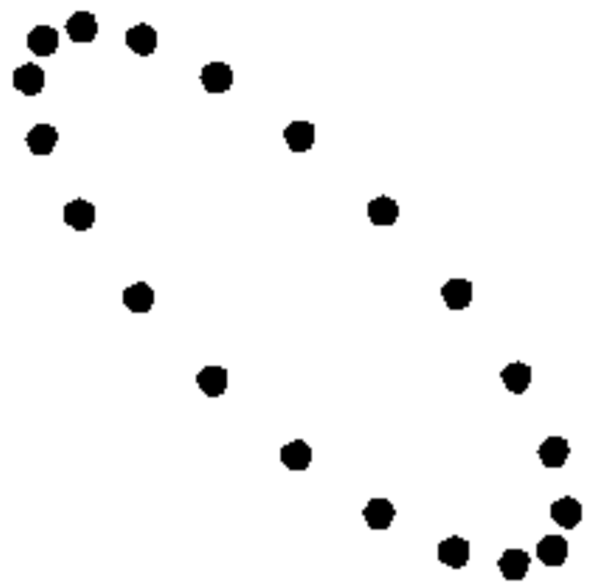
$$h = h_+ e^+ + h_\times e^\times$$

- Extensions of GR predict *more* polarisation states

+ (plus)



x (cross)



Incomplete history of the theory

- Physical reality unclear for many years
- Einstein & Rosen 1936: doubts existence in full theory
- Landau-Lifshiftz 1941: GW curiosity due to their tiny amplitude
- Dyson 1963: intense flash from coalescing binaries
- Weber ca 1960: search for GW
- Hulse & Taylor 1979: GW damping in binary pulsar
- Damour, Buonanno, Jarankowski, Blanchet, Damour, Will, Wiseman, Nagar, 1980s - today: PN theory, EOB, phenomenological models motivated by LIGO/Virgo experiments

Binary systems

- Binary systems emit GW because of their time varying quadrupole moment
- Radiation emitted at twice the orbital frequency
- Quadrupolar emission pattern
- Evolution of the frequency with time including radiation back-reaction

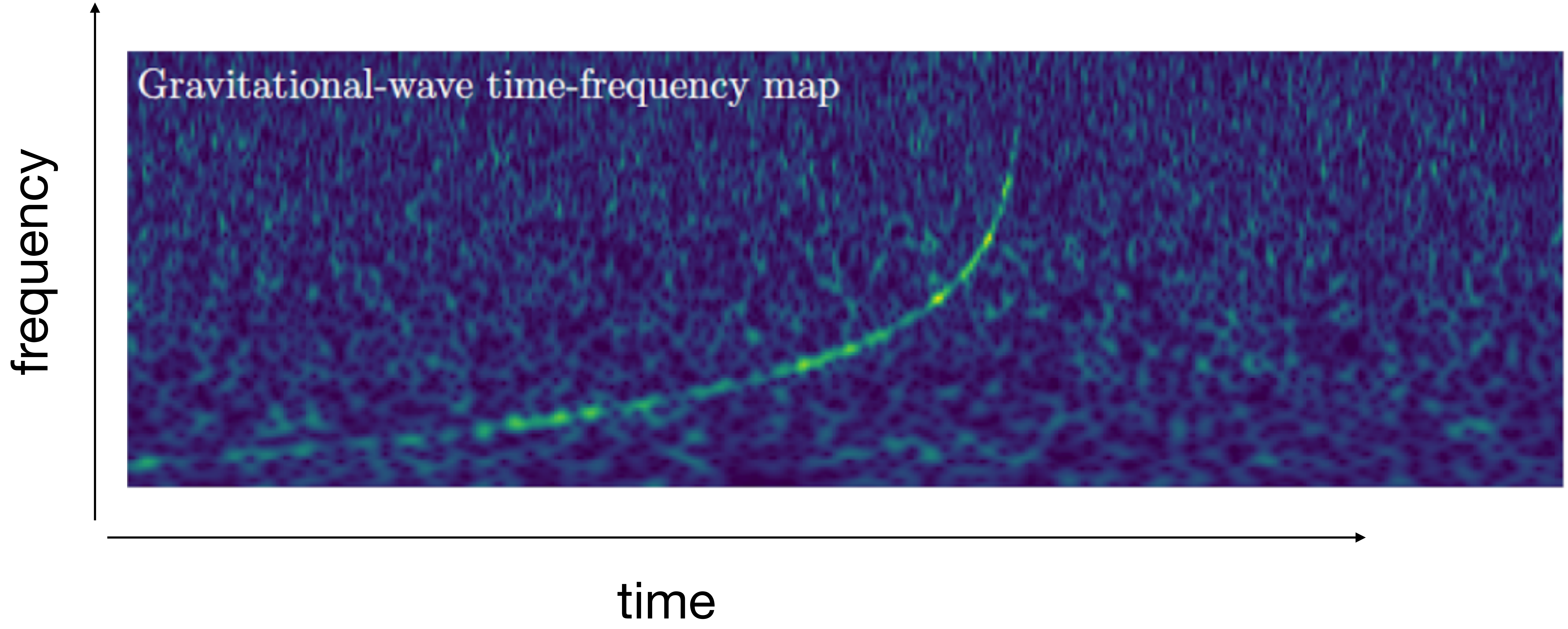
$$E = -\frac{Gm_1m_2}{2R}, F = \frac{32}{5}\frac{G^4}{c^5}\frac{m_1^2m_2^2(m_1+m_2)}{R^5}$$

$$\frac{dE}{dt} = -F$$

- From Kepler's law $\omega^2 = G(m_1+m_2)R^{-3}$, it is an equation for the frequency evolution

Chirps

- Solution gives typical “chirp” behaviour of GW



LVC, arXiv:1710.05832

Higher order corrections

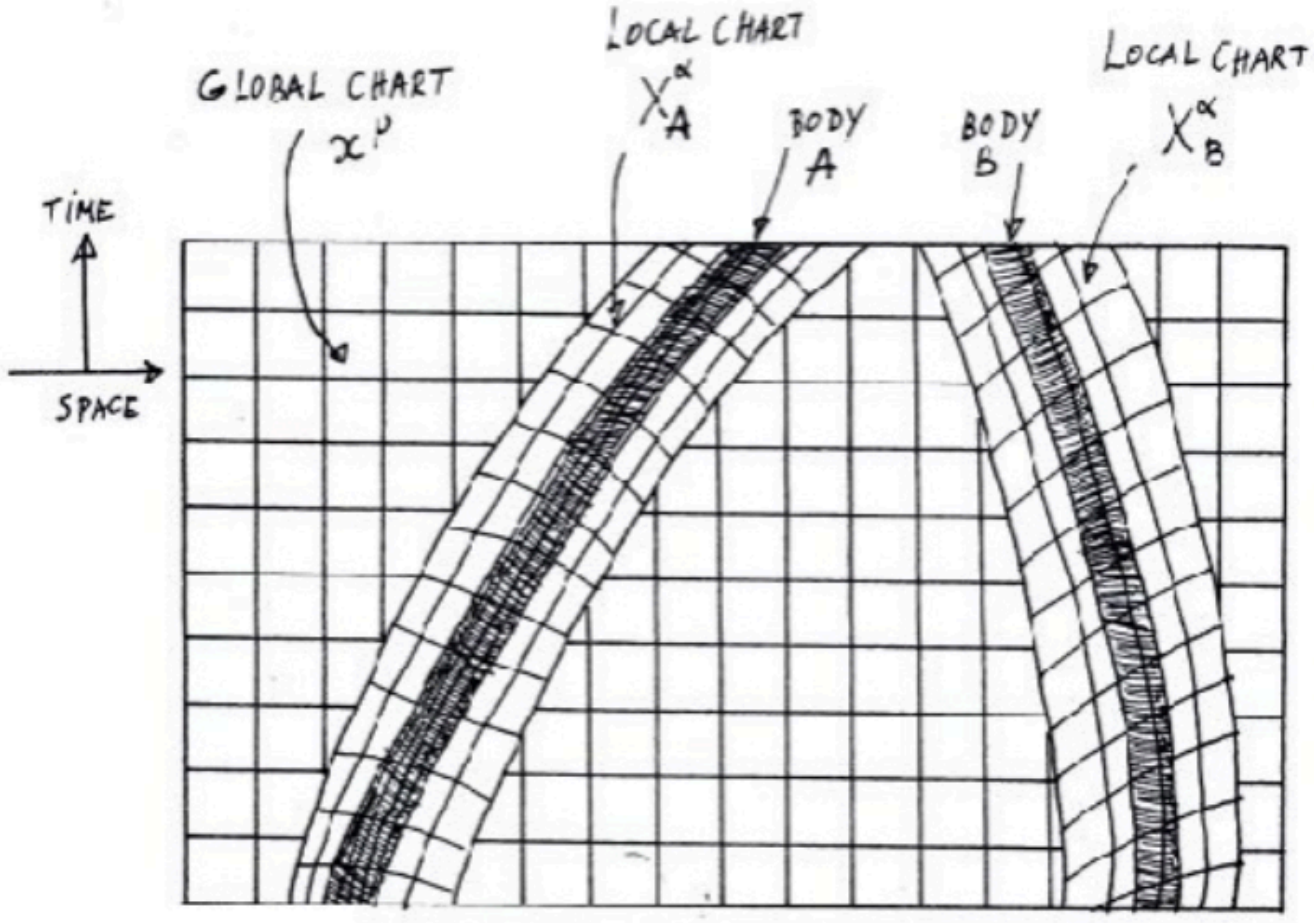
- Leading order treatment is appropriate at large separations ($F << 1$)
- Corrections:
  - higher source multipoles
  - velocity expansion in (v/c) of energy and flux
- Post-Newtonian theory (works by Damour, Blanchet, Will, Wiseman over past decades)

$$\begin{aligned} \frac{d\phi}{dt} - \frac{v^3}{M} &= 0 \\ \frac{dv}{dt} + \frac{\mathcal{F}(v)}{ME'(v)} &= 0 \end{aligned}$$

ϕ, v are wave phase and orbital velocity

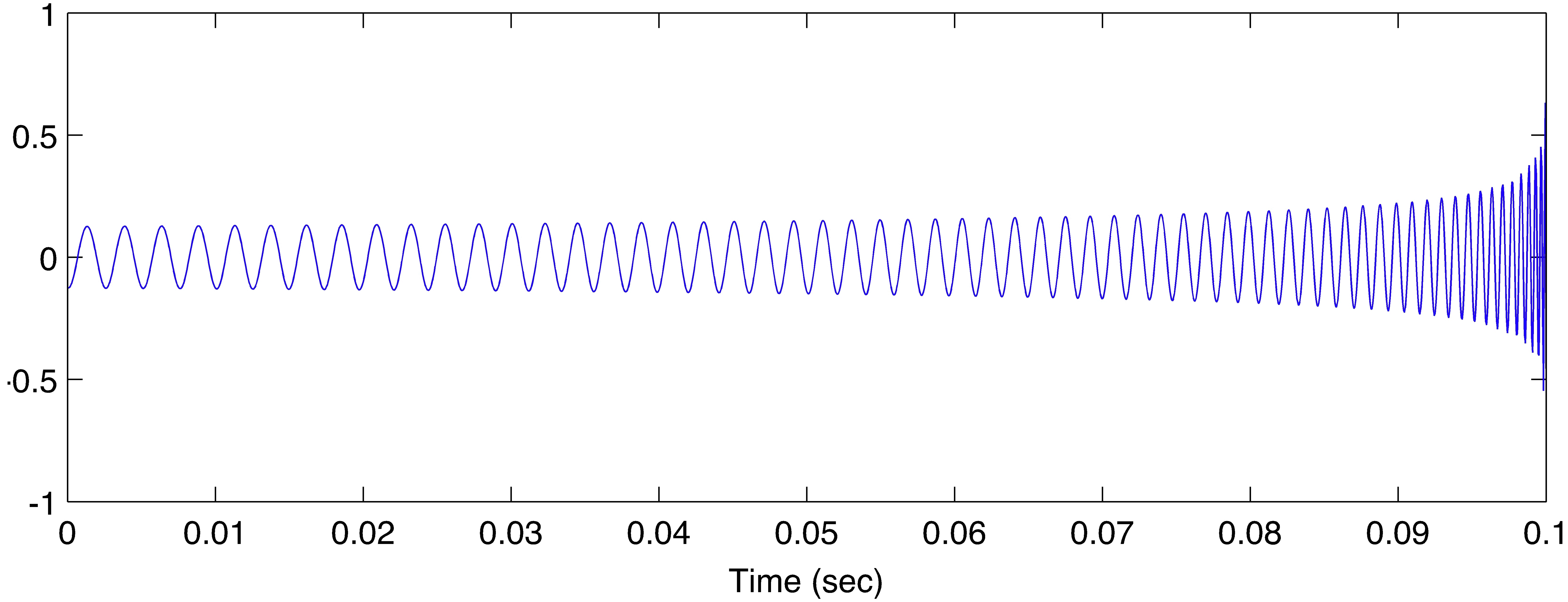
Matched asymptotic expansion

- Far from the source
  - radiation expanded in spin 2 spherical harmonics
- Near-zone expansion (PN) need to be matched with far-zone
- Multi-chart approach and matched asymptotic expansion (e.g Damour 1982)



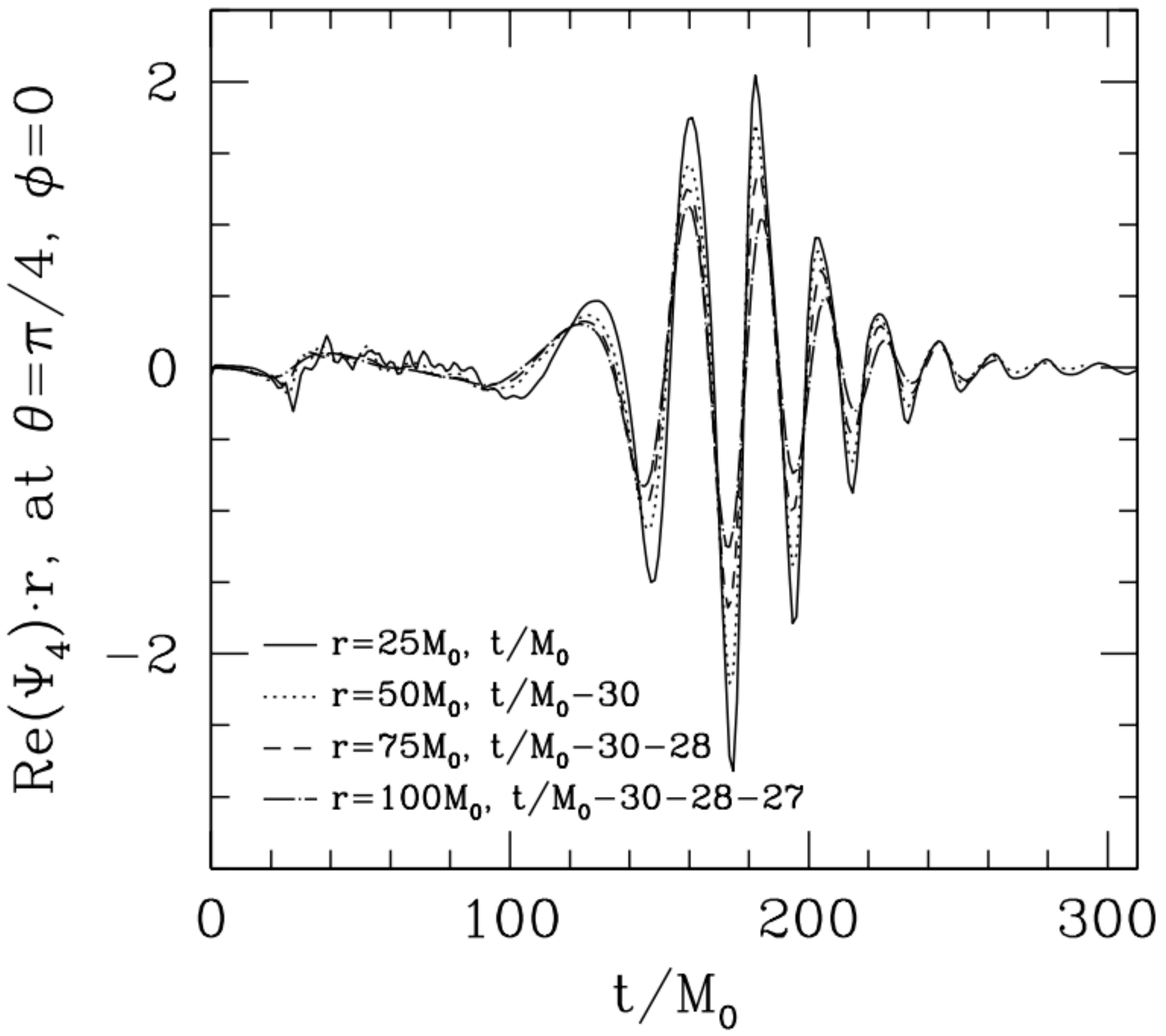
Inspiral waveforms

- Theory gives us waveforms for the inspiral phase of coalescence



Merger

- Understanding of merger needed development of numerical relativity
- Breakthrough: Pretorius 2005
- Remarkably:
  - Simple
  - Agreement with proposed pseudo-quasi-normal mode treatment from Buonanno & Damour 2000
- Remnant is a Kerr (rotating) black hole

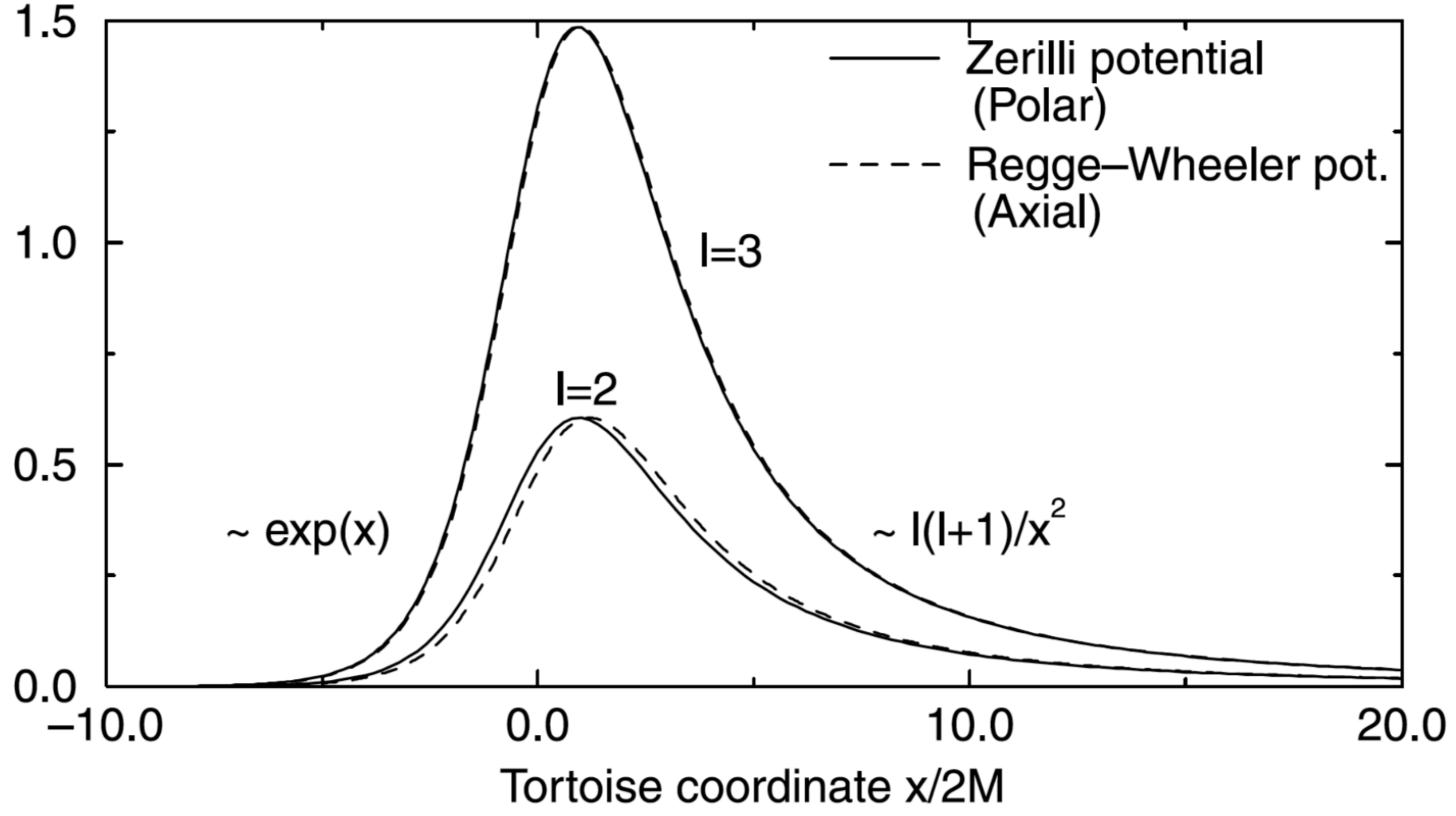


Pretorius, arXiv:0507014

Ringdown

- Remnant is an excited Kerr black hole
- GW emission studied in the context of black hole perturbation theory
  - Response to linear perturbations to Kerr metric
- Perturbations obey Schrodinger-like equation (Regge & Wheeler 56, Zerilli 70, Teukolski 72)
- Potential barrier around a BH

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} + V_{lm}(x)\right)\psi_{lm}(x,t) = 0$$



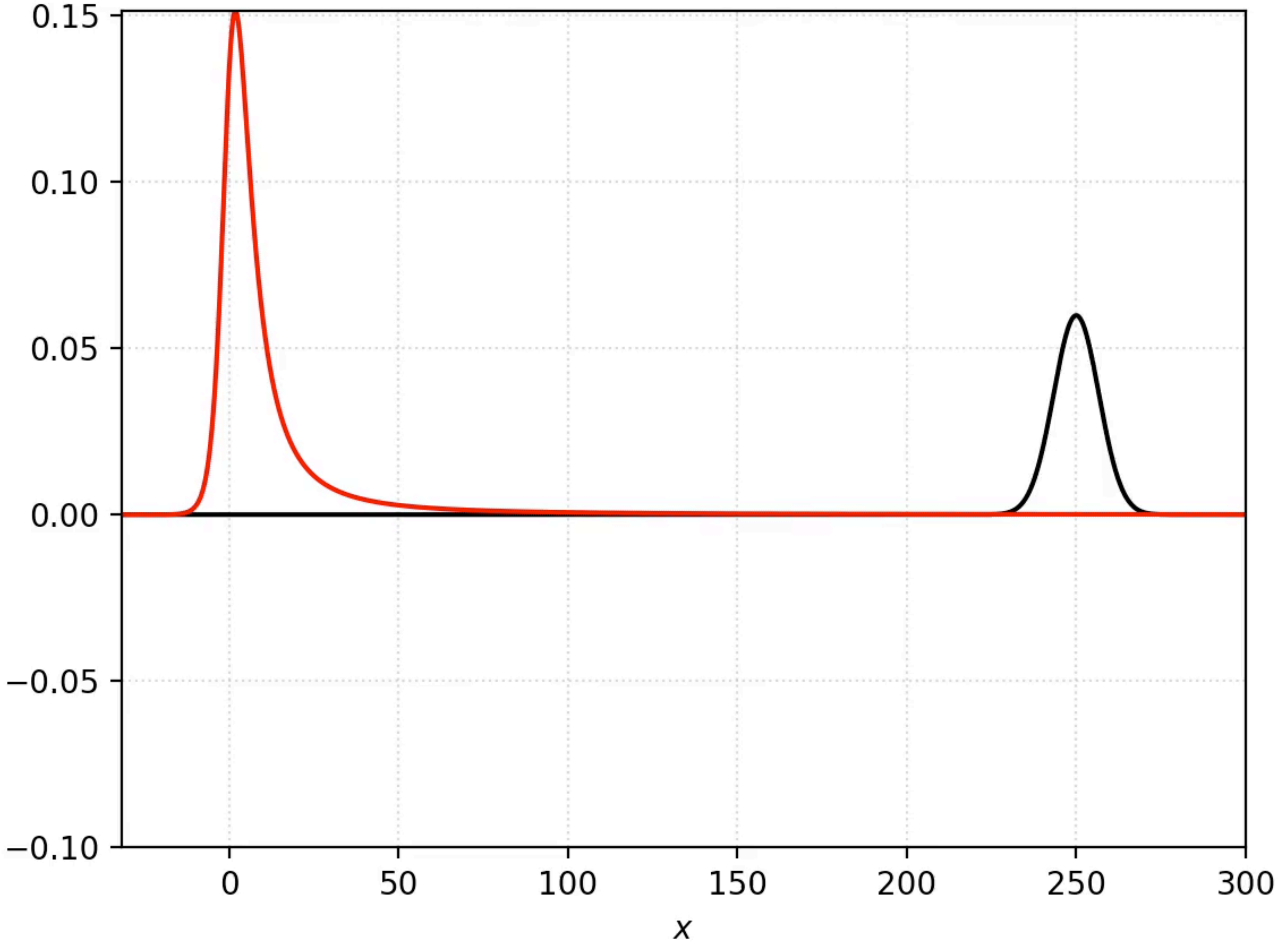
Nollert 99

Ringdown

- BH responds to perturbations by "ringing" (Vishveshawara 70, Press 71, Ruffini et al, 72, Chandrasekhar 75)
- Quasi-normal modes excited by light-ring crossing (Goebel 72)
- Ringdown waveform

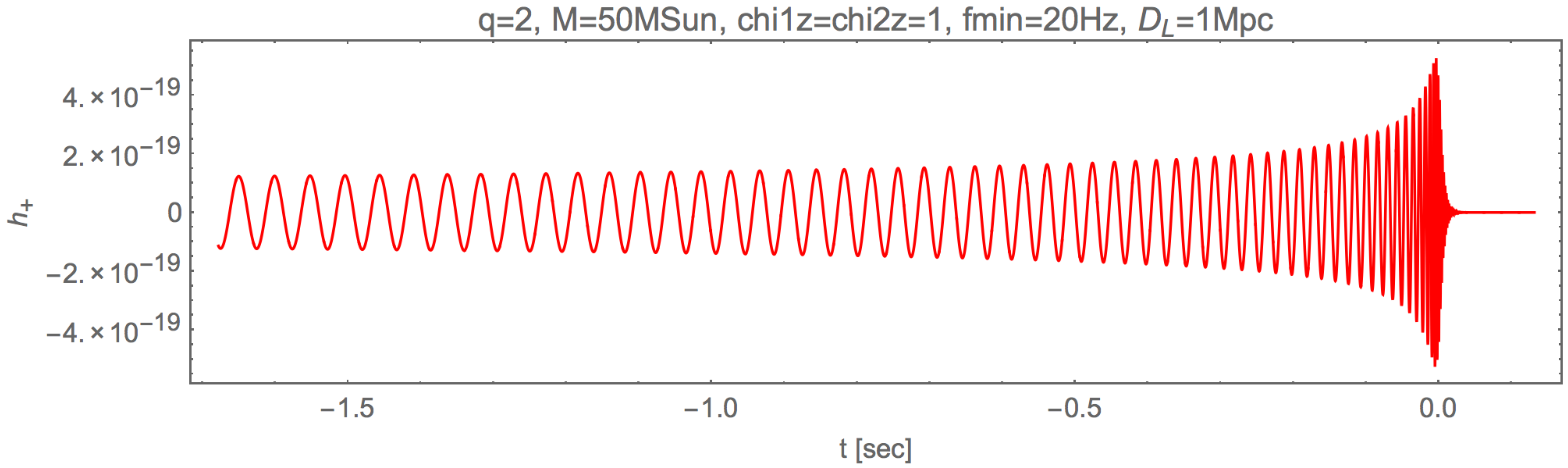
$$h(t) = \sum_{nlm} A_{nlm} e^{-\frac{t-t_0}{\tau_{nlm}}} \cos(\omega_{nlm}(t - t_0) + \varphi_{nlm})$$

- τ_{nlm}, ω_{nlm} depend only on BH mass and spin
    - Uniqueness theorems



Inspiral-merger-ringdown waveform

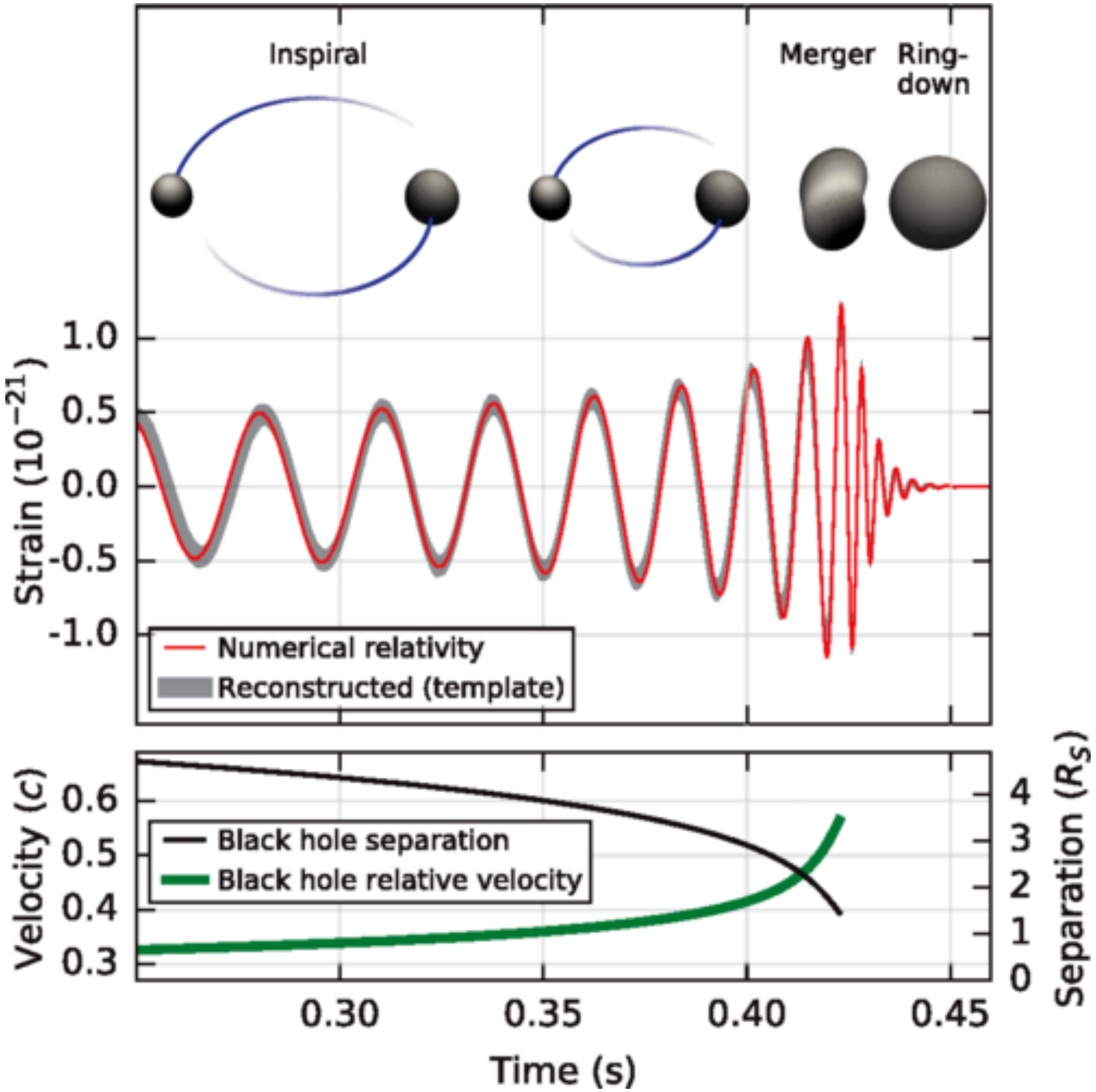
- We now have the main tools to predict a GW signal from the parameters of a binary black hole system



- We can thus infer the properties of observed signals
  - Enabling GW astronomy

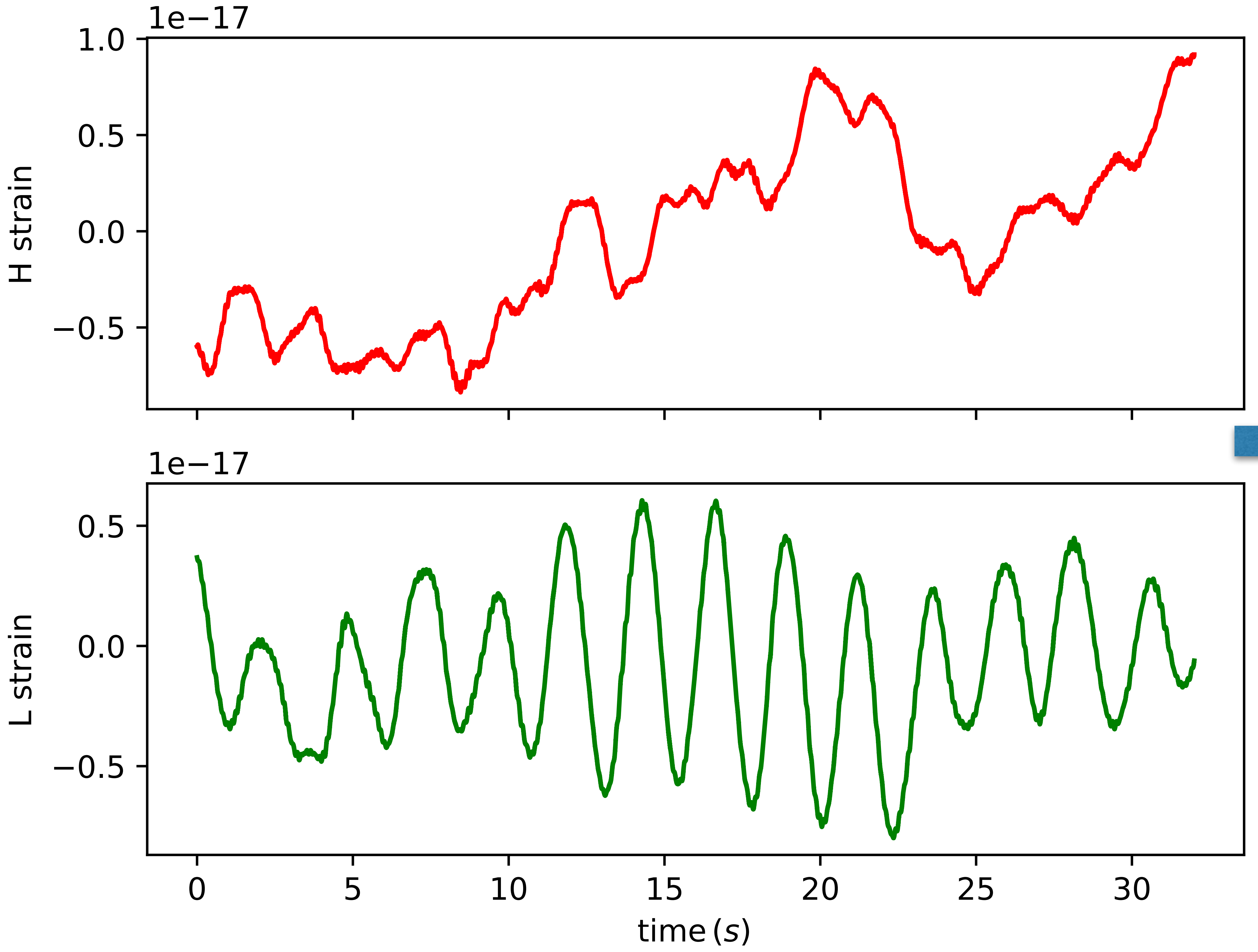
Summary

- Inspiral and merger:
  - Dynamics of space-time
  - Nature of component objects
- Ringdown:
  - Remnant
  - Uniqueness & no-hair conjectures
- Globally:
  - Propagation of GW



LVC, arXiv:1602.03837

Detection and inference



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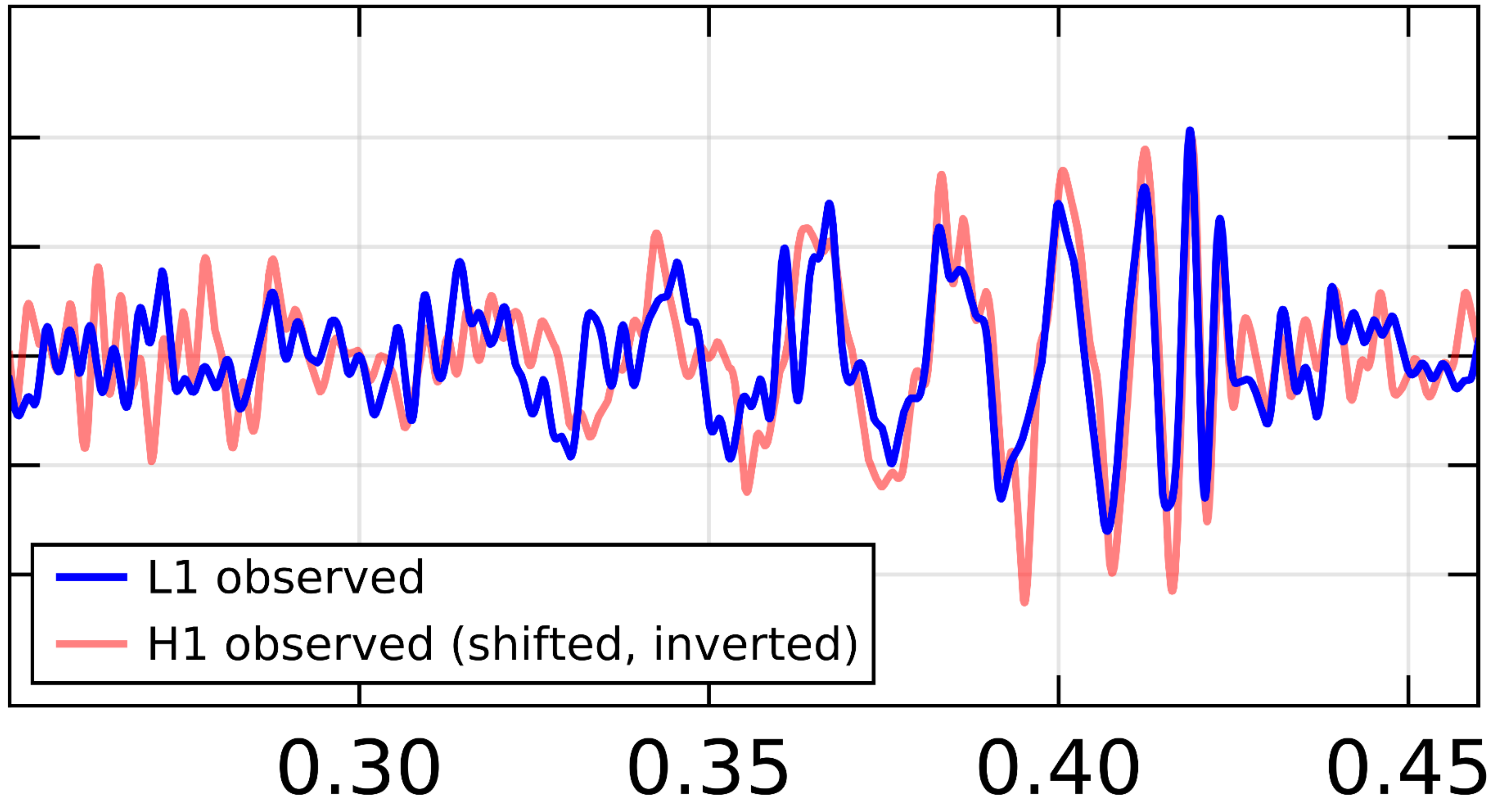
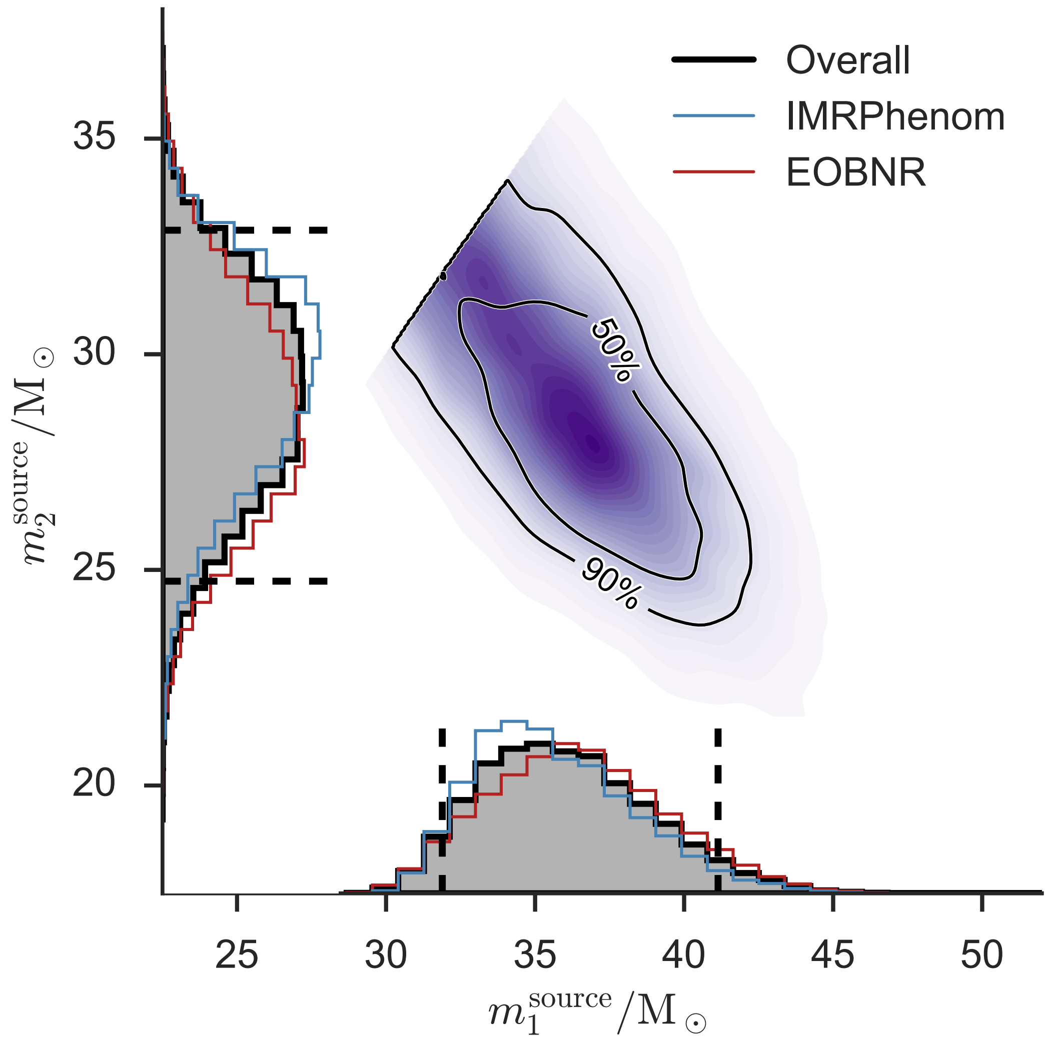




https://www.gw-openscience.org/about/

The detection problem

- Detector output $d(t) = h(t;\theta) + n(t)$
- Filter $A(t) = \int d\tau F(t+\tau)a(\tau) = \int df \tilde{F}^*(f)\tilde{a}(f)$
- Filtered output

$$\int df \tilde{F}^*(f)\tilde{d}(f) = \int df \tilde{F}^*(f)\tilde{h}(f) + \int df \tilde{F}^*(f)\tilde{n}(f)$$

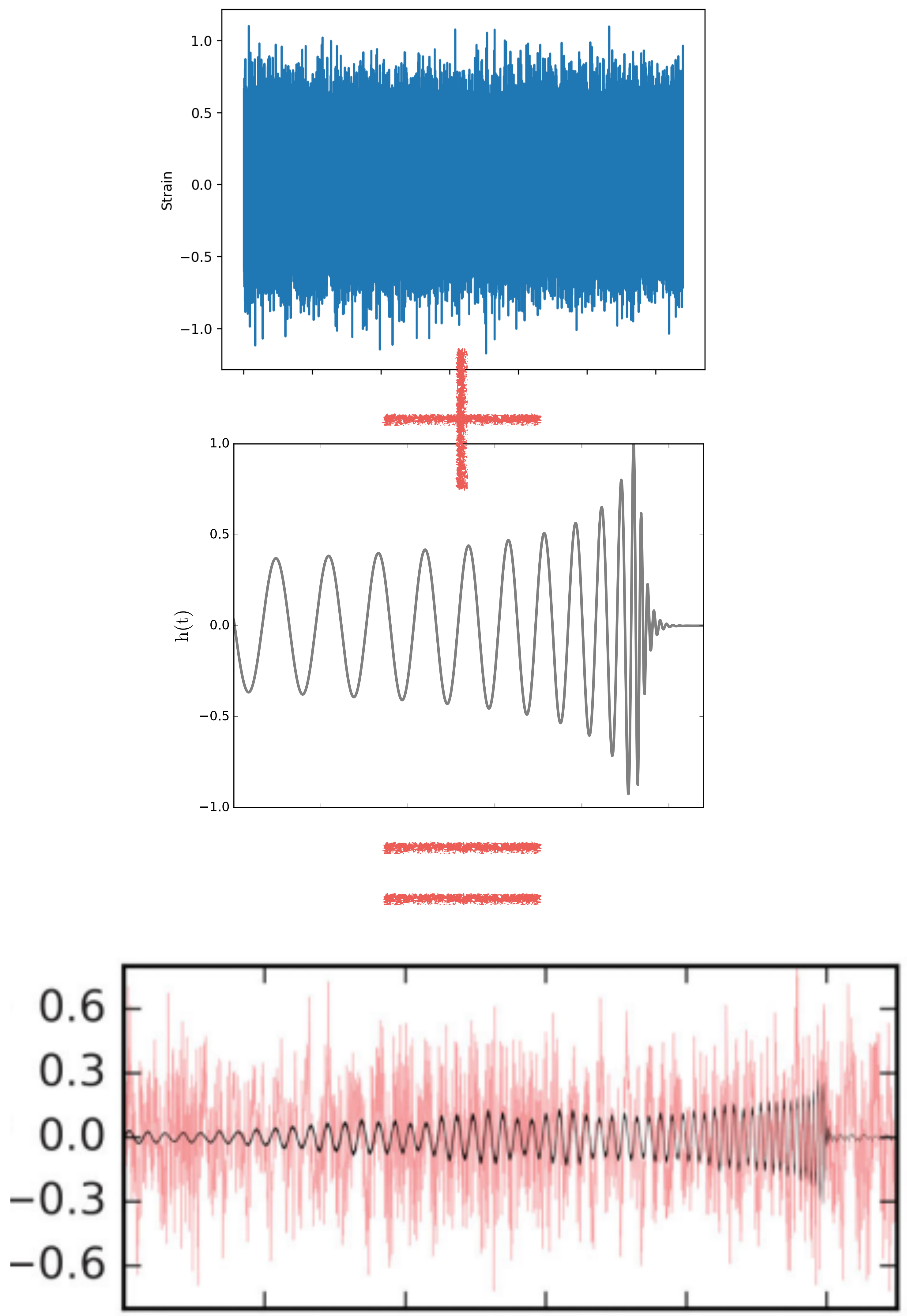
$$D = H + N$$

- signal-to-noise ratio

$$SNR \equiv \rho = \frac{H^2}{< N^2 >}$$

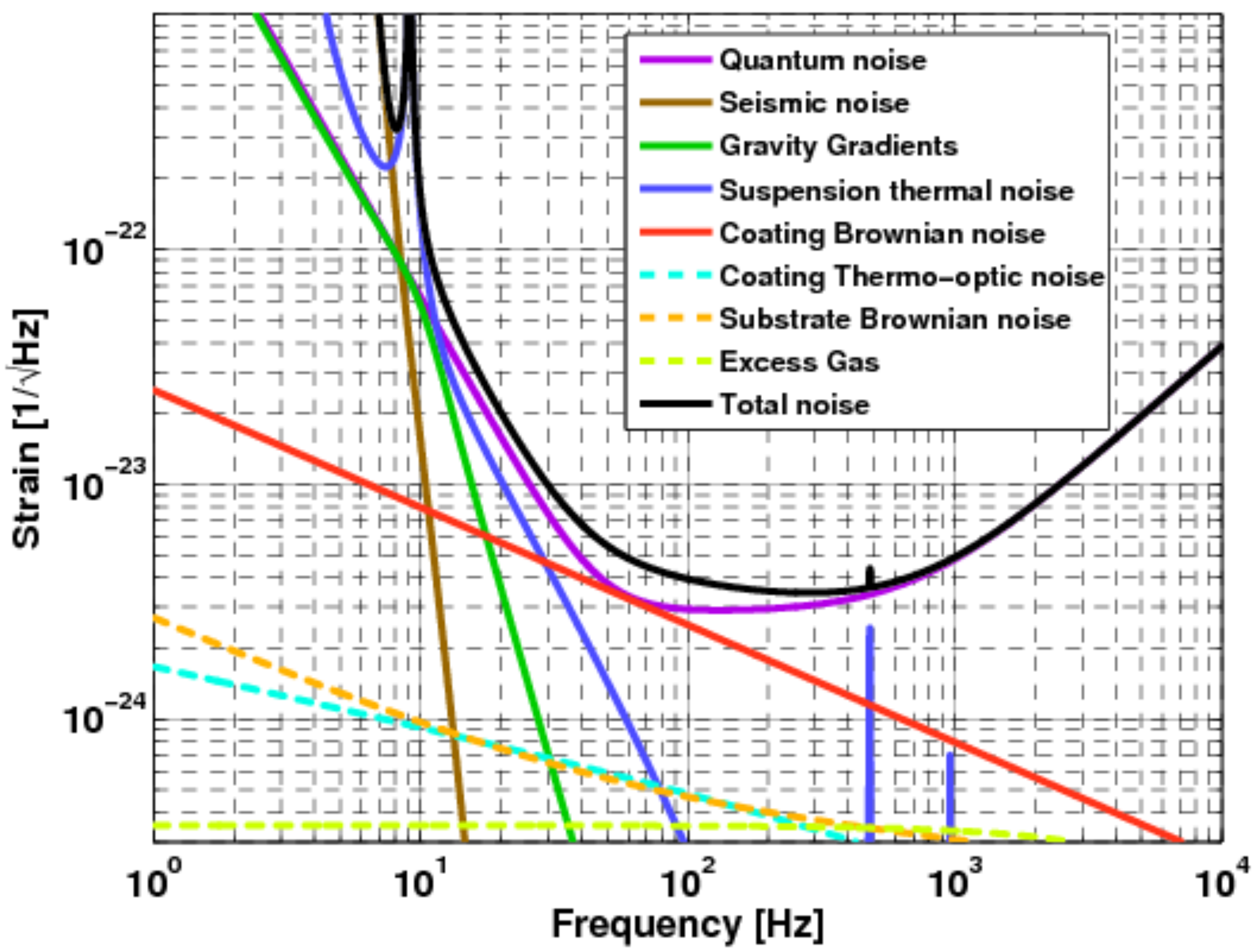
$$\tilde{F}^*(f) = C\frac{\tilde{h}(f)}{S_n(f)}$$ template that maximises the SNR,

Wiener filter



Power spectral density

- IFO noise is a superposition of several processes
  - Noise is a wide-sense stationary stochastic process
  - All information about $n(t)$ encoded in the power spectral density $S(f)$

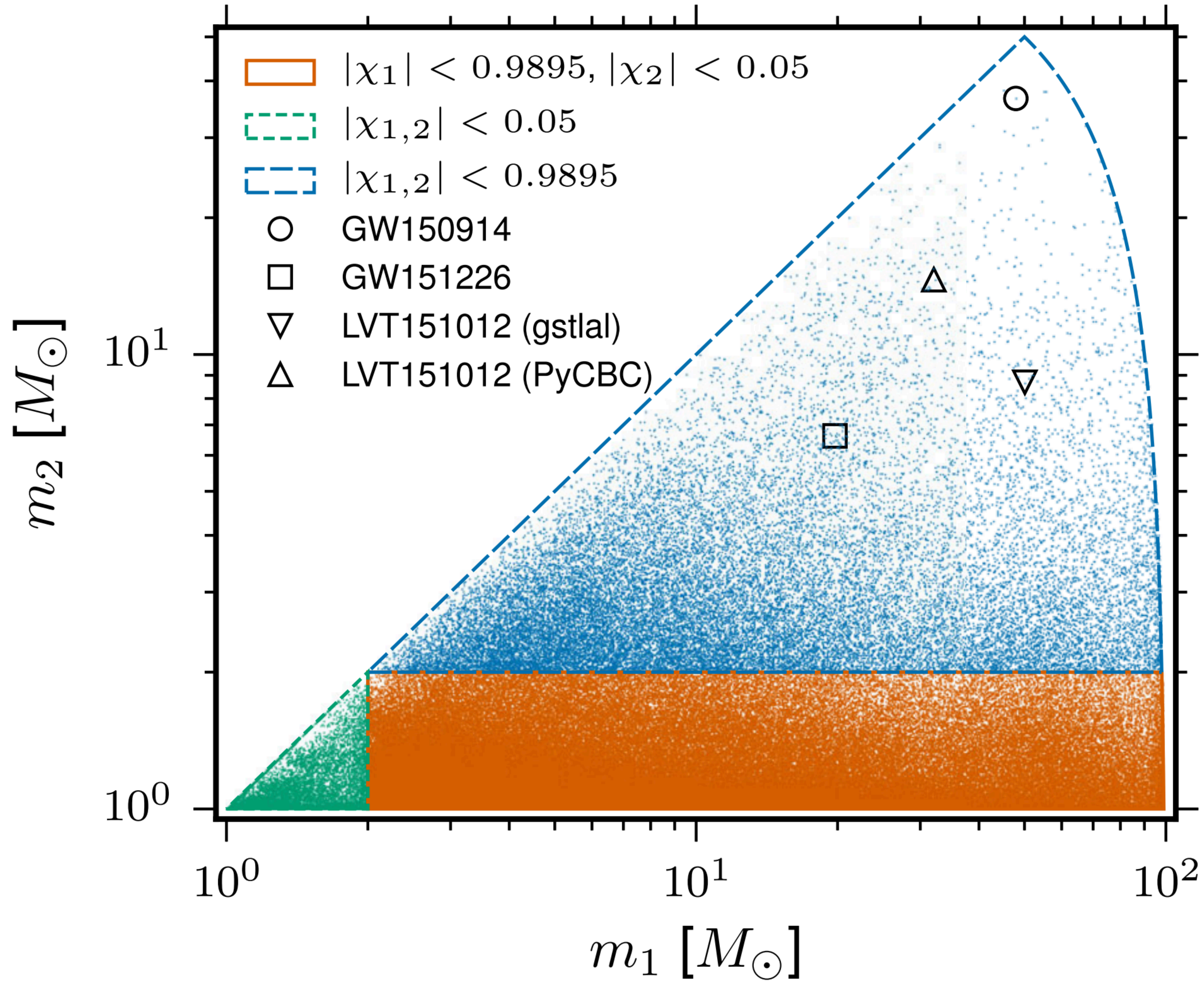


The need for templates

- Need to know the signal shape
  - Matched filtering
- large parameter space
  - the waveform models must be accurate and computationally efficient
- Development of models:
  - effective-one-body(EOB) (Buonanno & Damour, arXiv:9811091, Bohe+,arXiv:1611.03703)
  - Phenomenological models (e.g. Khan+,arXiv:1508.07253)

Template banks

- Construct template banks O(200000) templates
- Template distribution
  - Following some intrinsic metric
  - Stochastically
- Every template compared against the data
- Largest SNR registered gives detection candidate

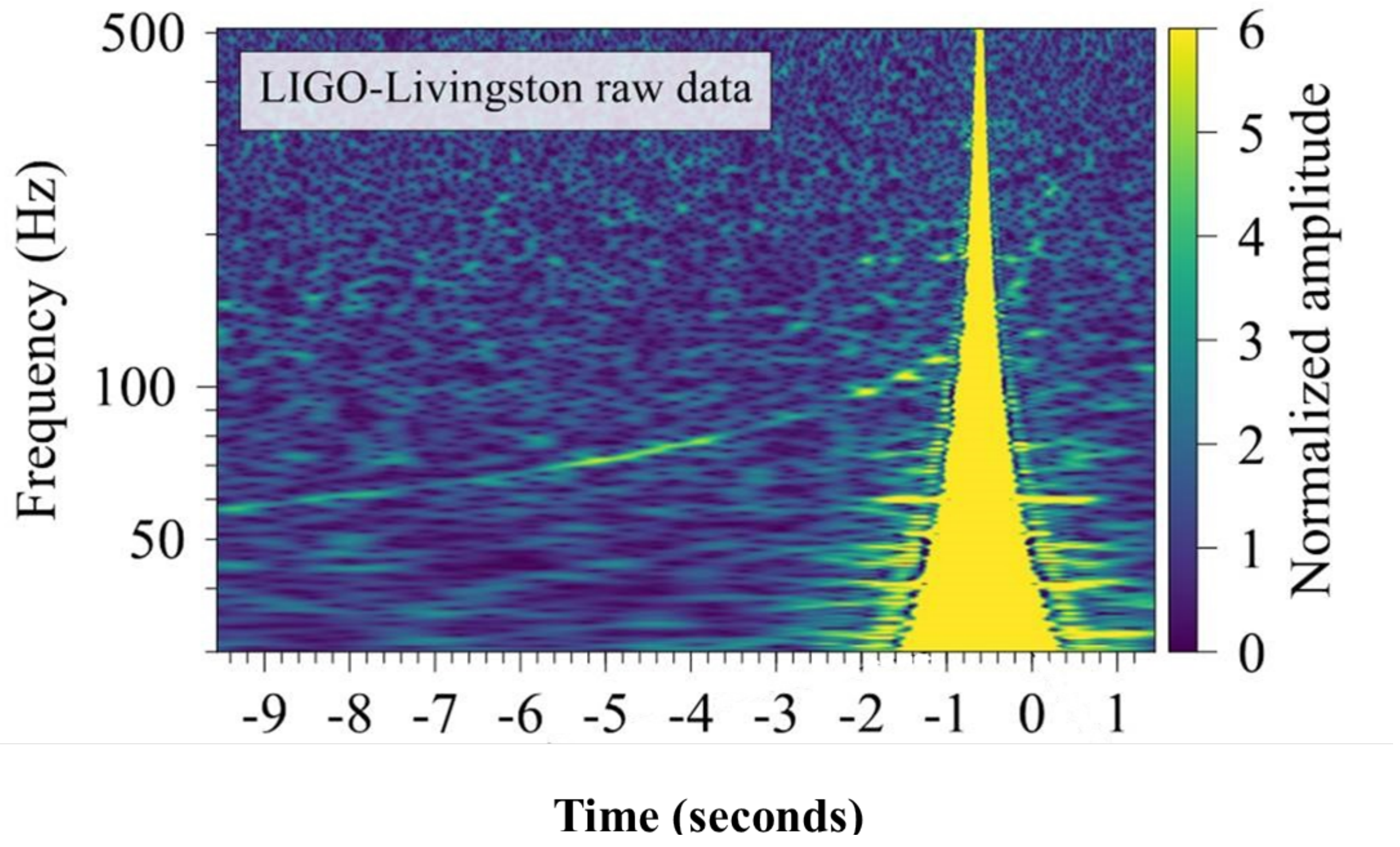


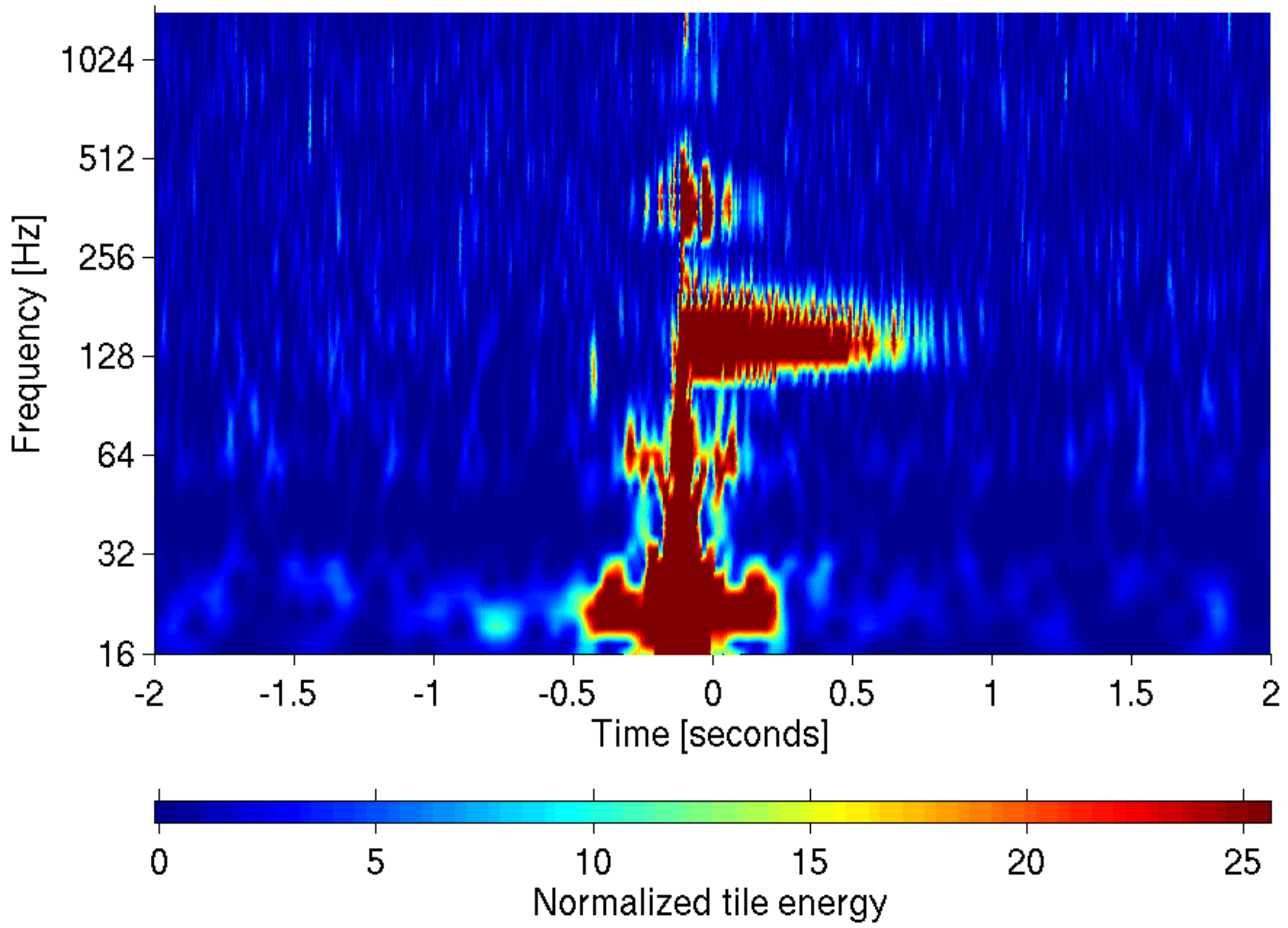
Significance estimation

- Significance of detection is evaluated against the hypothesis that noise could give raise to random trigger
- GW150914: 5.3 sigma, FAR < 6×10^{-7} yr^{-1}
- O1 had 51.5 days of data. How is it possible?
  - Timeslides

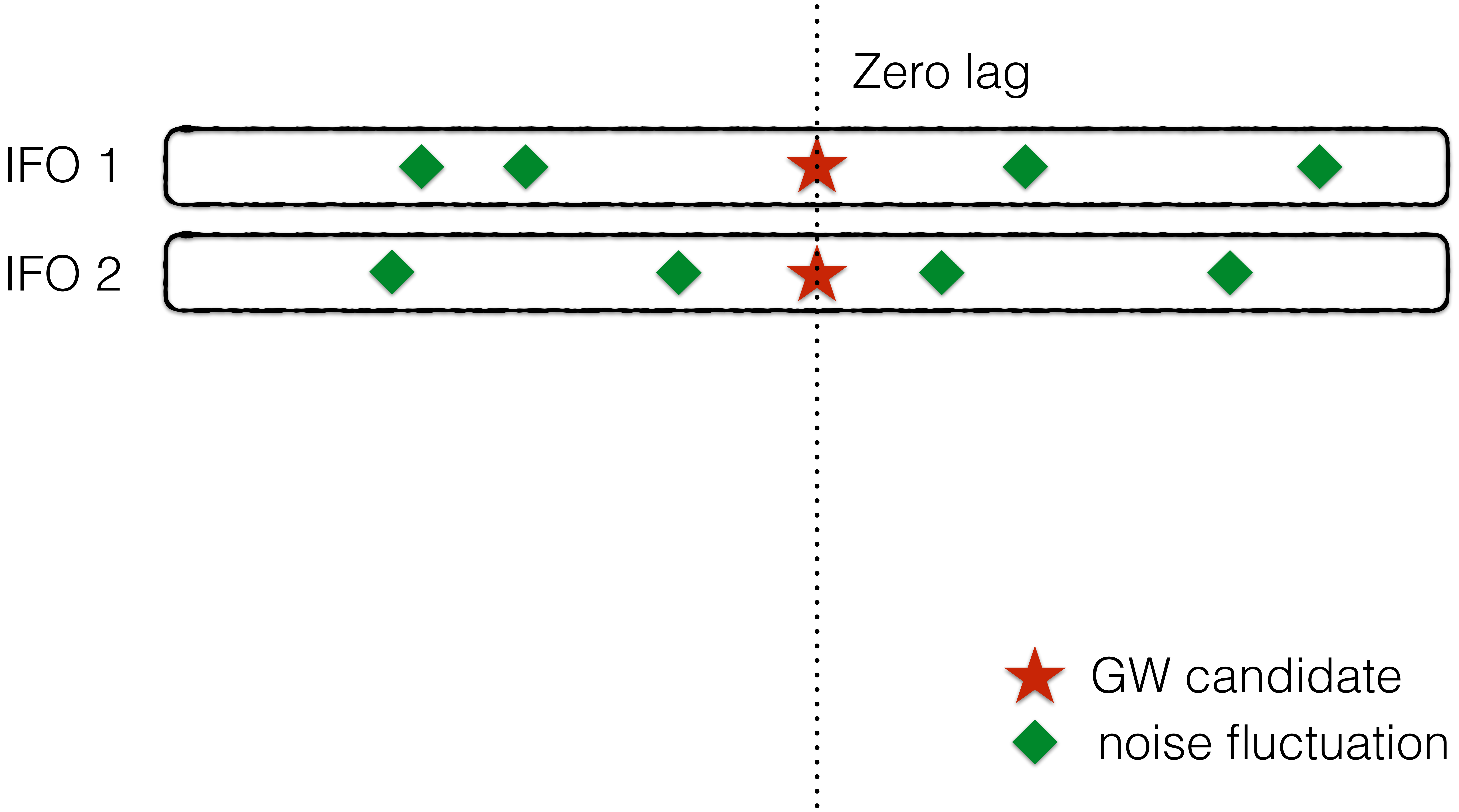
Timeslides

- Pure Gaussian noise $p(\rho|I) \propto e^{-\frac{1}{2}\rho^2}$
- The noise is not exactly Gaussian (glitches)
- Construct the empirical distribution of your detection statistics
- Chop the data in chunks and permute them in time
    - Synthetic, incoherent noise realisations
    - Background distribution

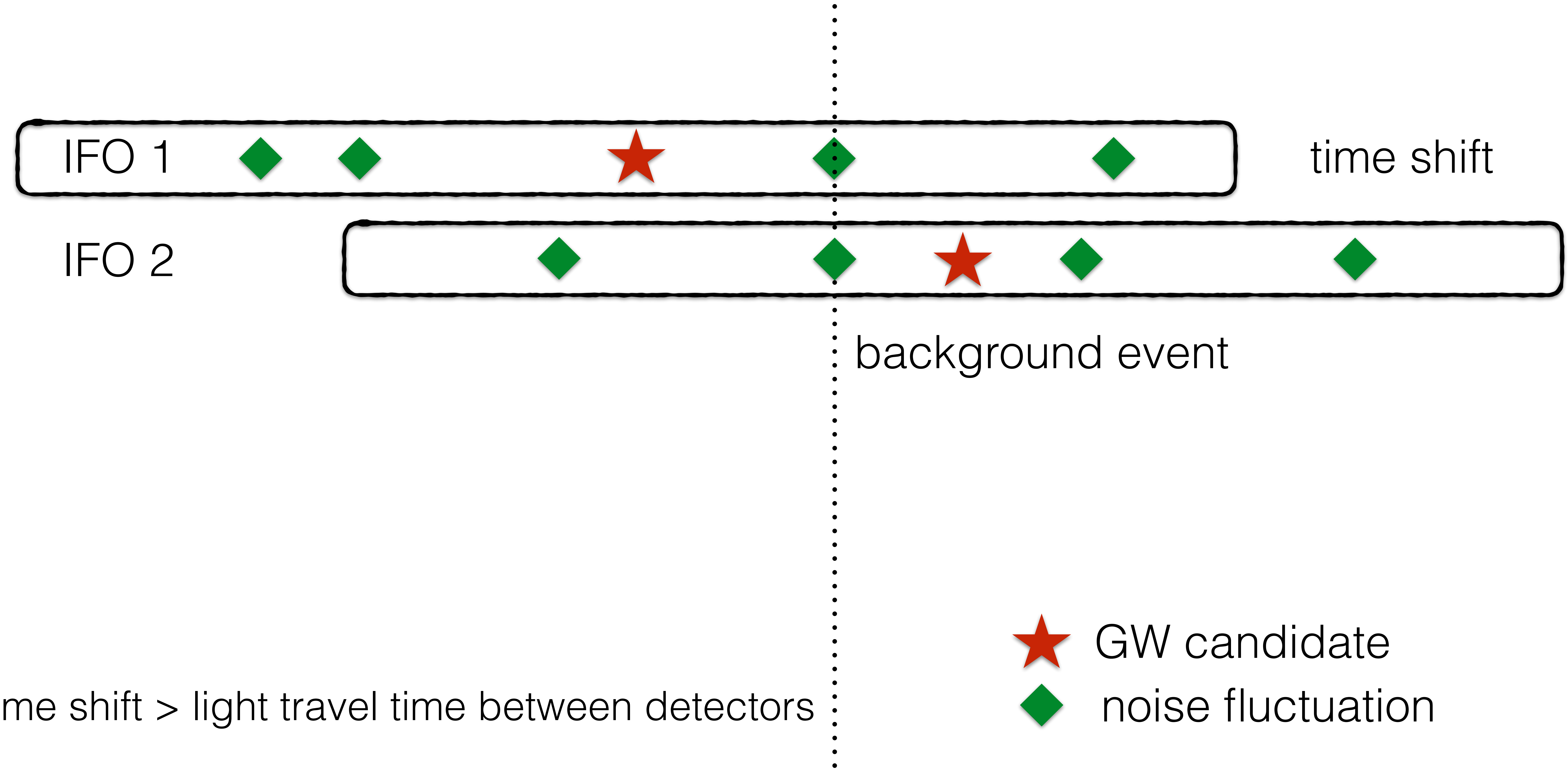




Timeslides

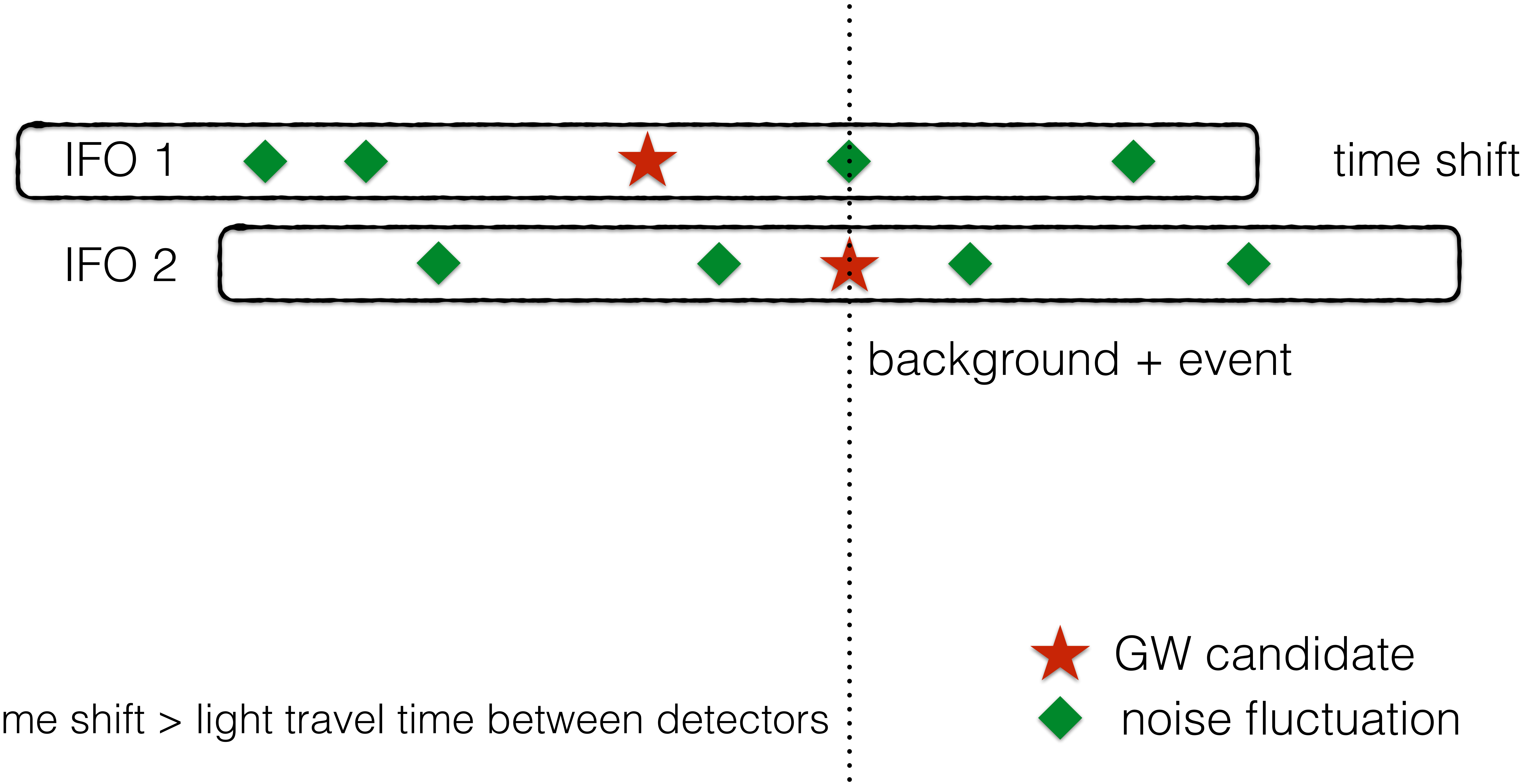


Timeslides

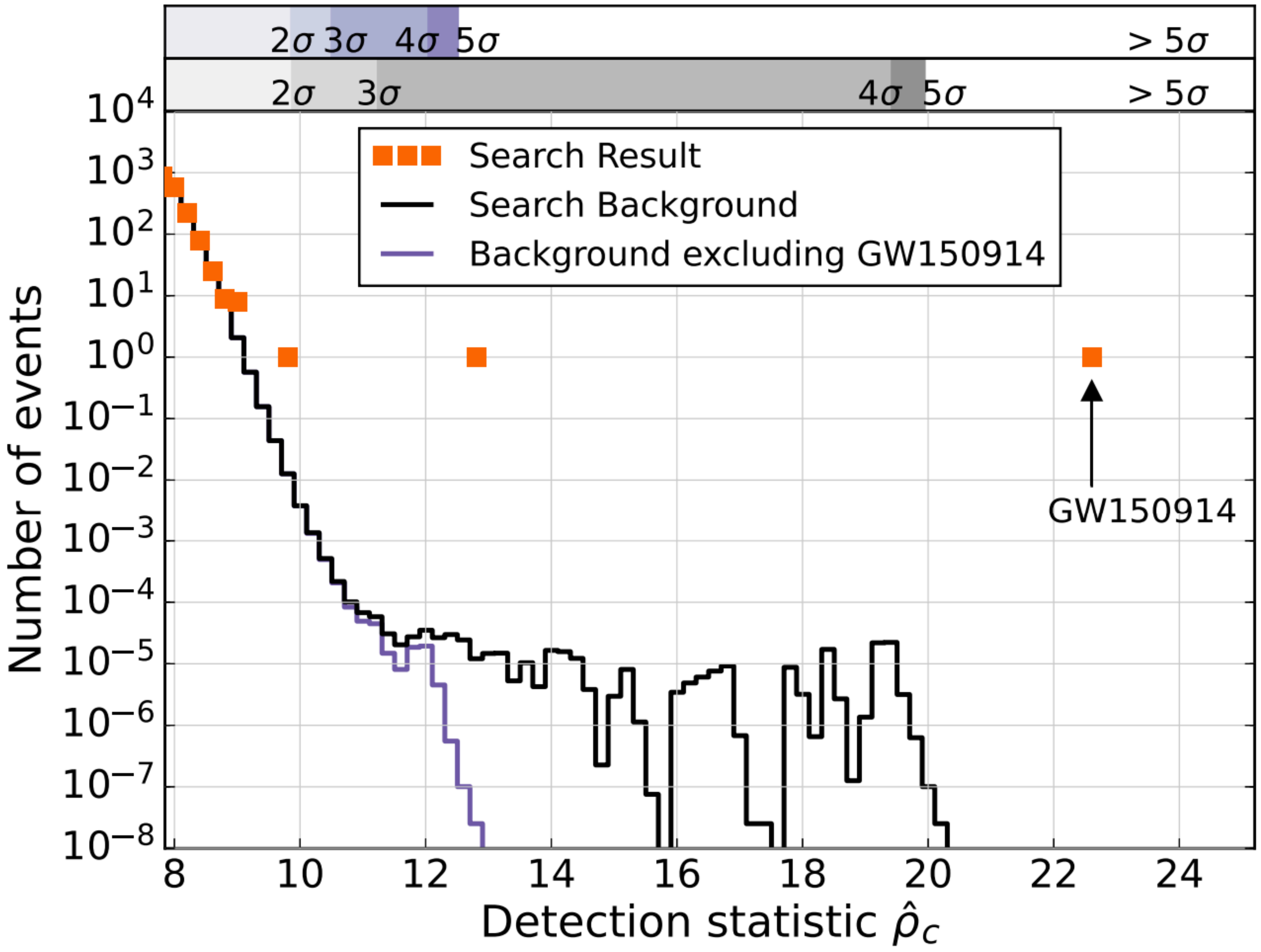


time shift > light travel time between detectors

Timeslides



time shift > light travel time between detectors

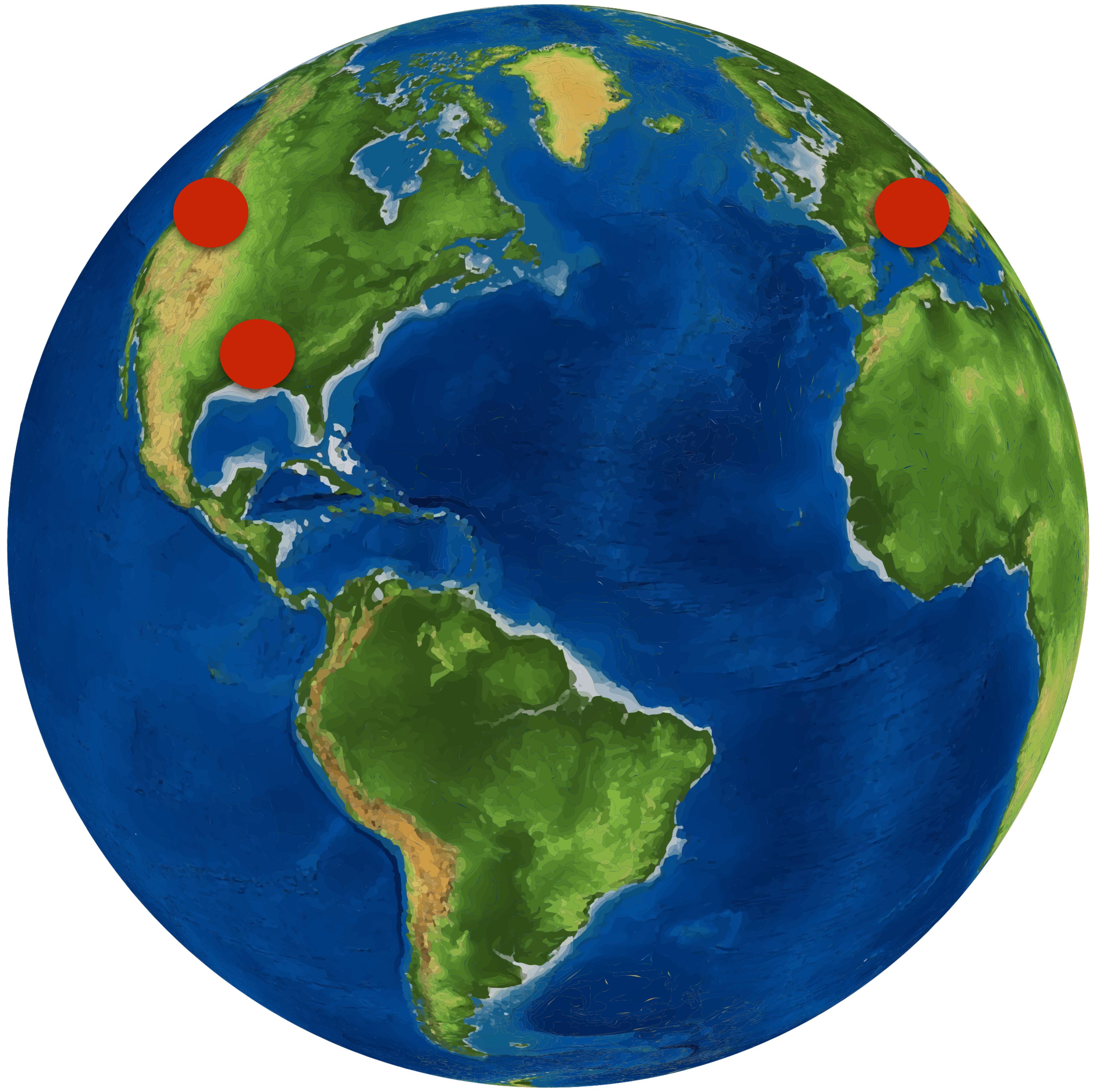


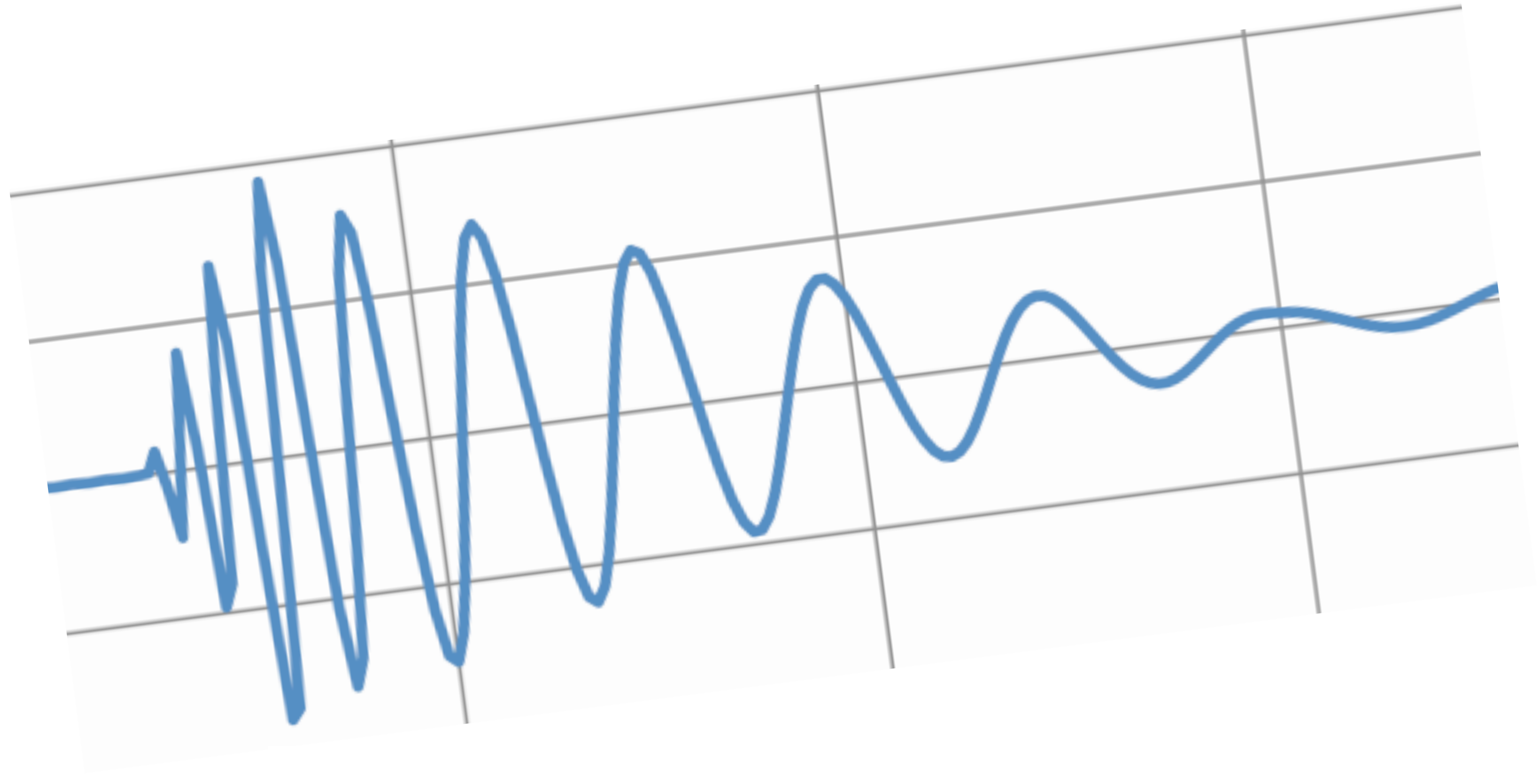
LVC, arXiv:1602.03837

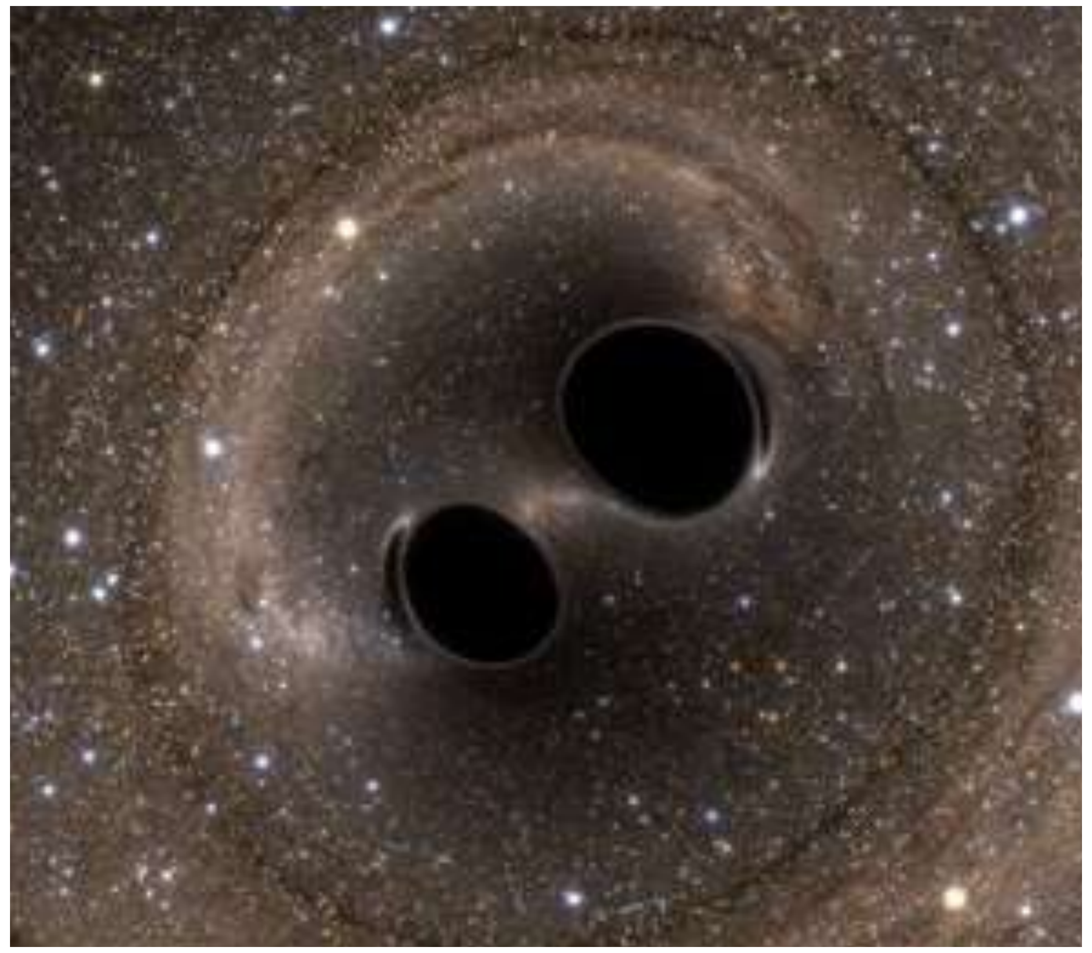
Sky localisation

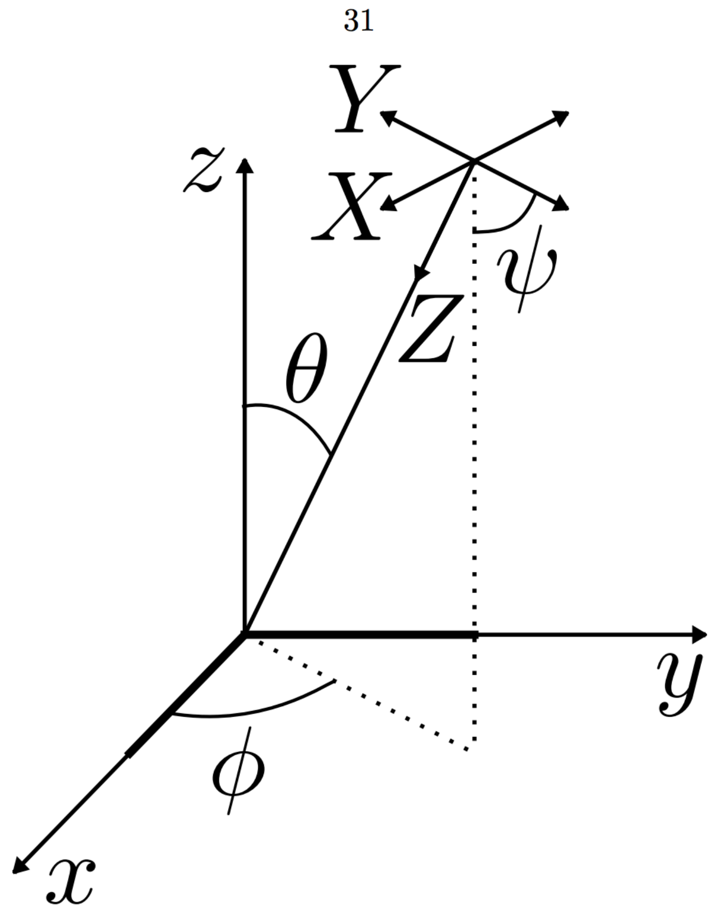
- Each detector sees a different signal

$$h = e^{2\pi i \vec{r}\cdot\vec{n} f}\left[F_+ h_+(\theta, f) + F_\times h_\times(\theta, f)\right]$$







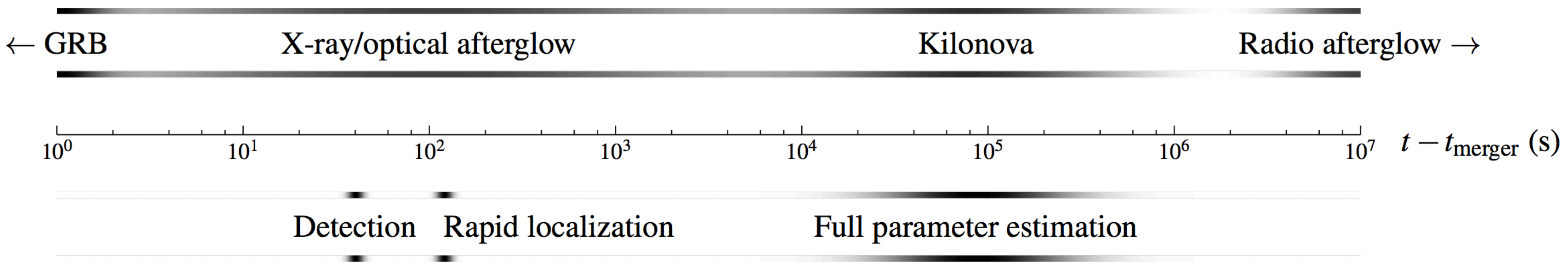


$$F_+ = \frac{1}{2}\left(1+\cos^2\theta\right)\cos 2\phi\cos 2\psi + \cos\theta\sin 2\phi\sin 2\psi$$

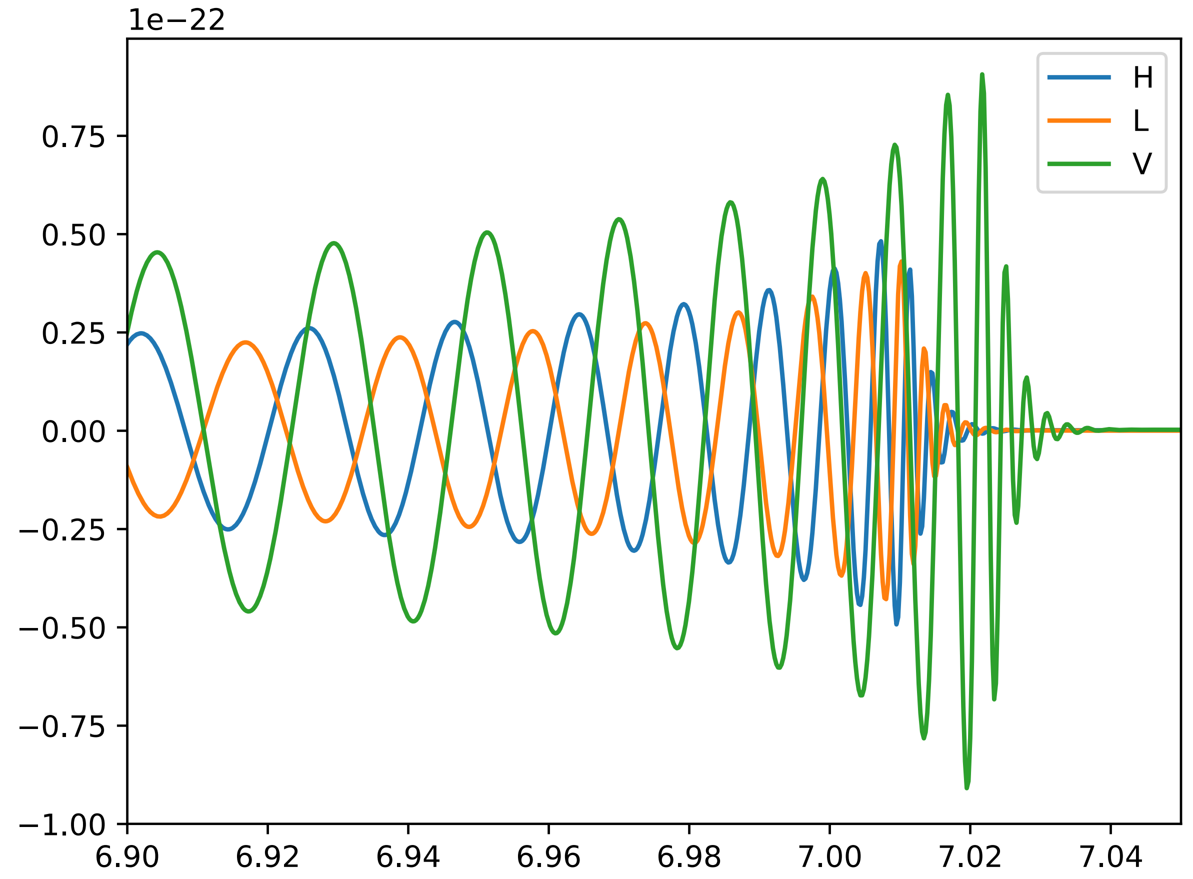
$$F_\times = \frac{1}{2}\left(1+\cos^2\theta\right)\cos 2\phi\sin 2\psi + \cos\theta\sin 2\phi\cos 2\psi$$

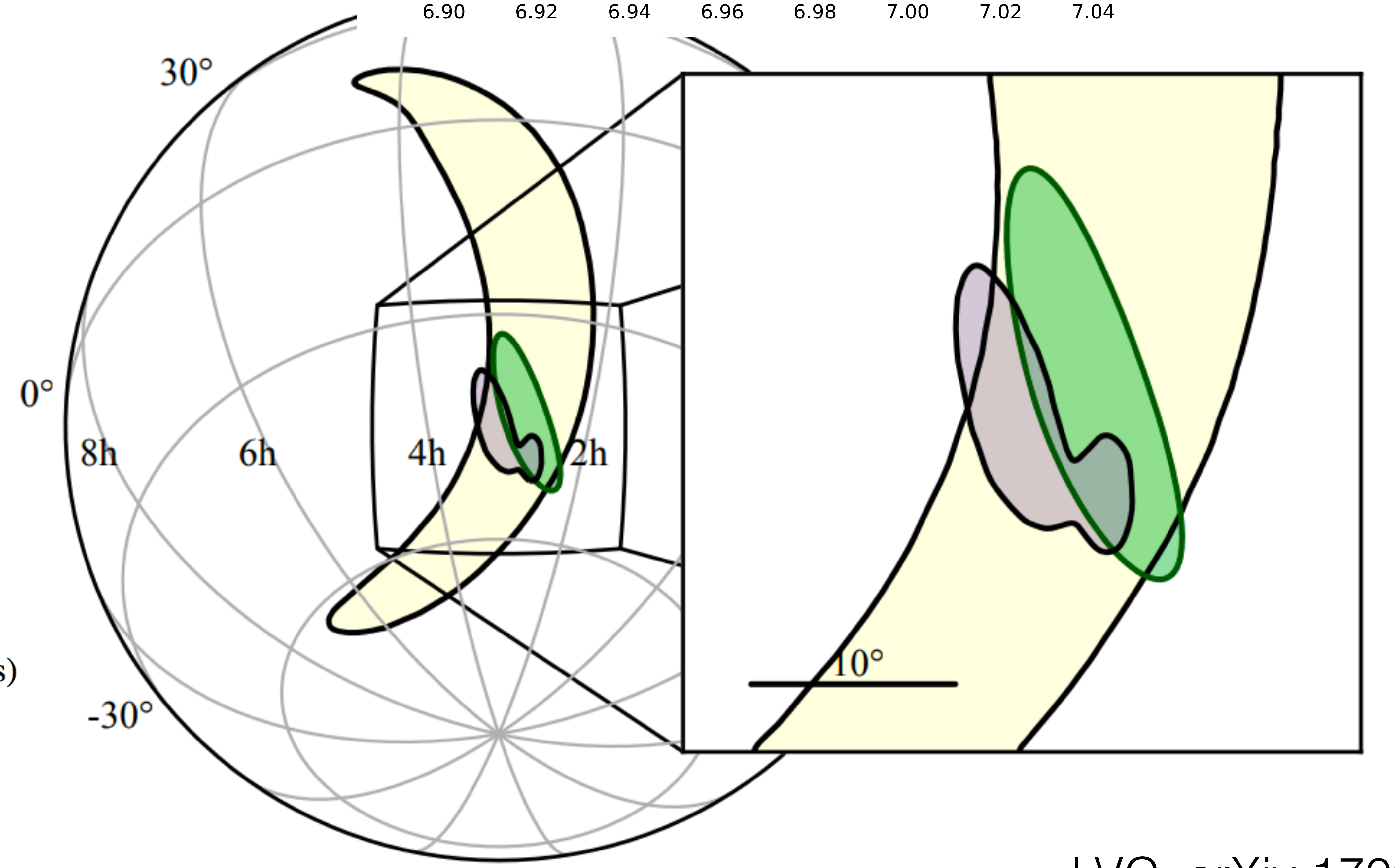
Sky localisation

- Relative amplitudes and time differences allow for rapid localisation O(100) s
- O3a: O(100) public alerts



Singer at al, arXiv:1404.5623

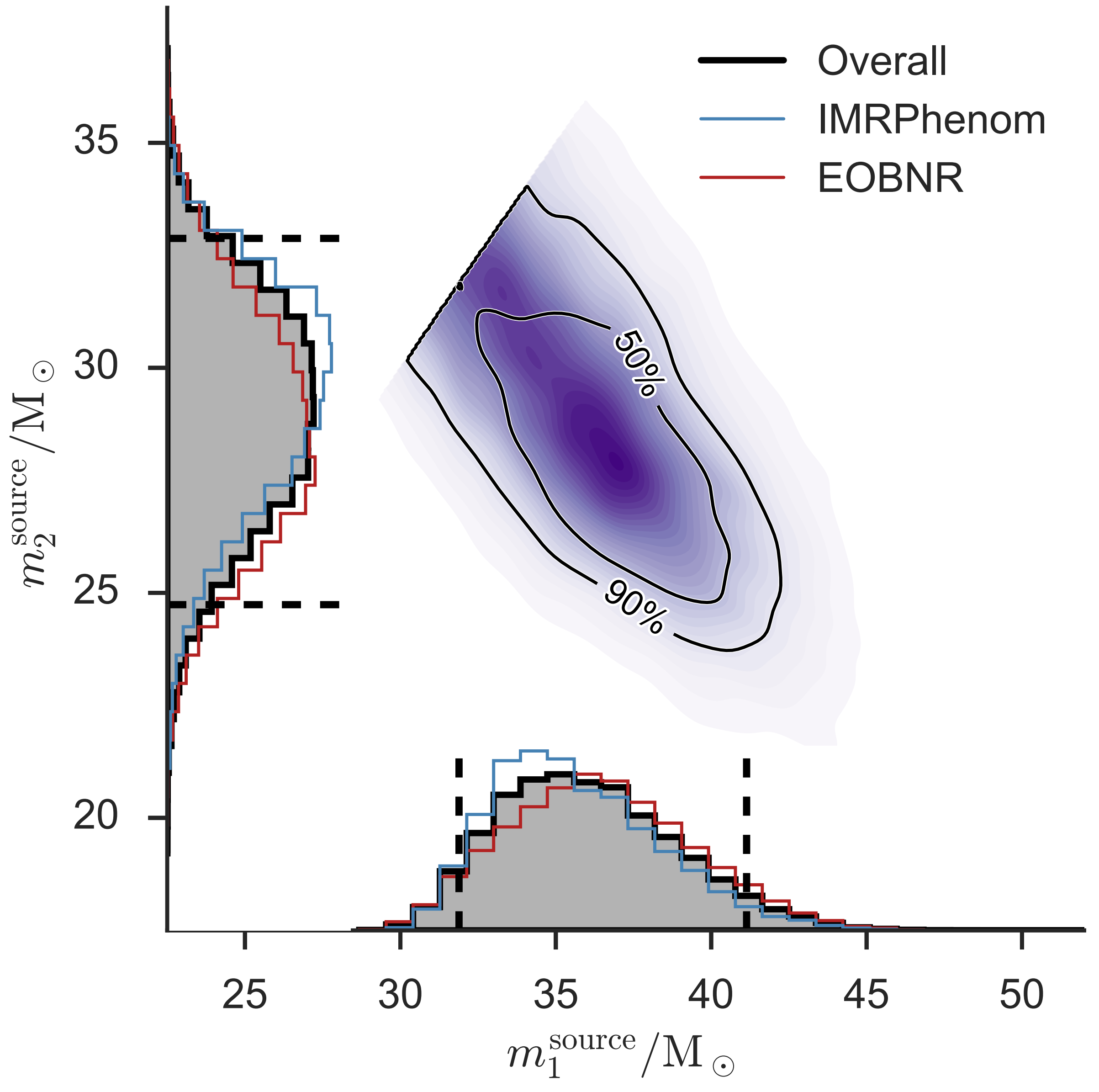


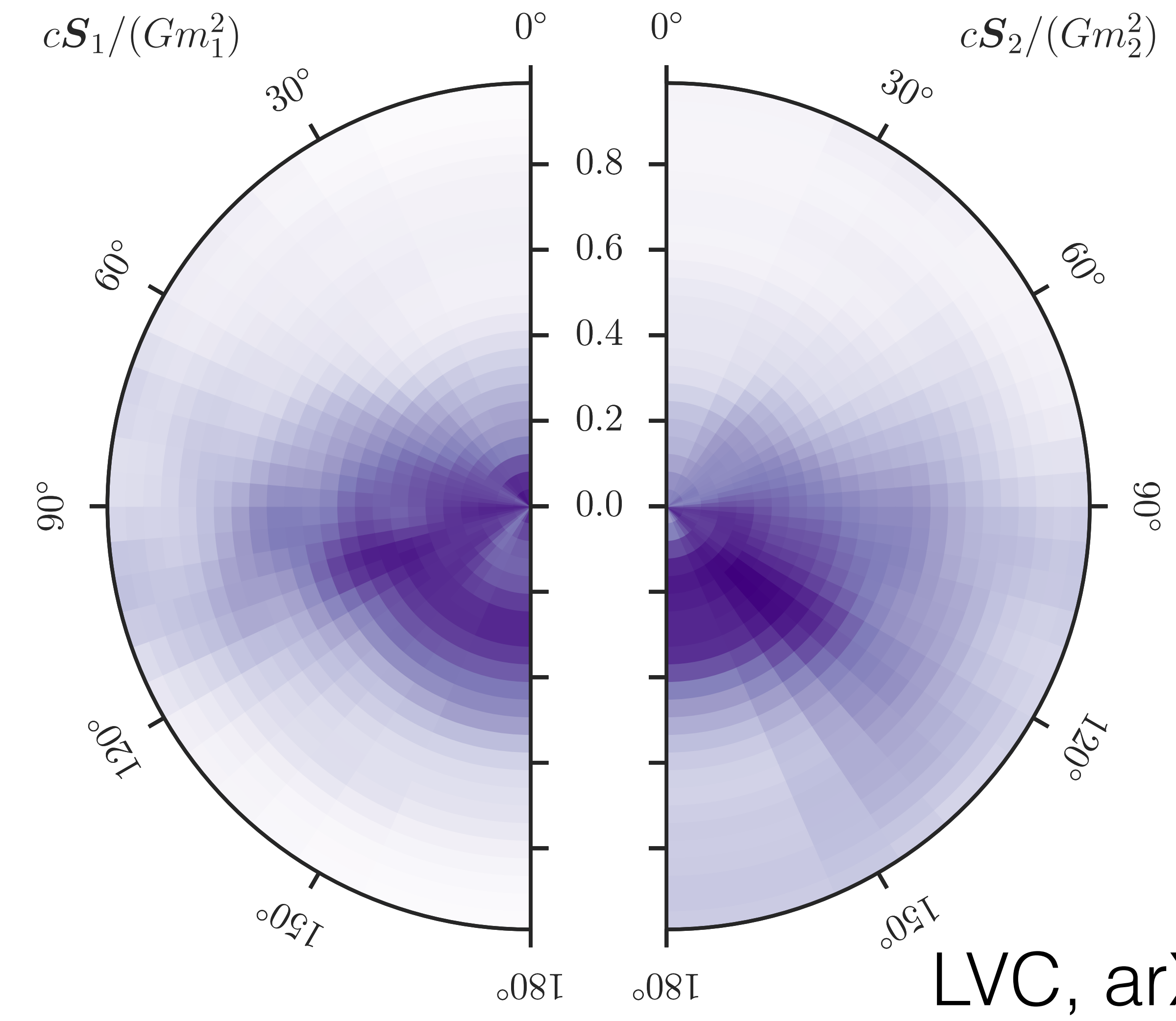


LVC, arXiv:1709.09660

Parameter estimation

- A detection only returns point estimate of physical parameters of the system
- Infer the source parameters to enable astrophysics and fundamental physics studies
  - Given a model for the GW signal, compute the probability distribution for the source parameters





LVC, arXiv:1602.03840

Parameter space

- $h(t;\theta)$ depends on a set of parameters θ
  - D=9 for non-spinning binaries: masses, orientation, sky location, reference time and phase, luminosity distance
  - D=15 in general: spin vectors
  - More parameters for extra physics (e.g. BH charges, tests of GR, tidal effects, etc…)

Bayes theorem

- Parameters are estimated computing the posterior distribution for all of them
  - joint posterior distribution

$$p(\theta|DSI) = \frac{p(\theta|SI)p(D|\theta SI)}{\int_\Theta p(\theta|SI)p(D|\theta SI)}$$

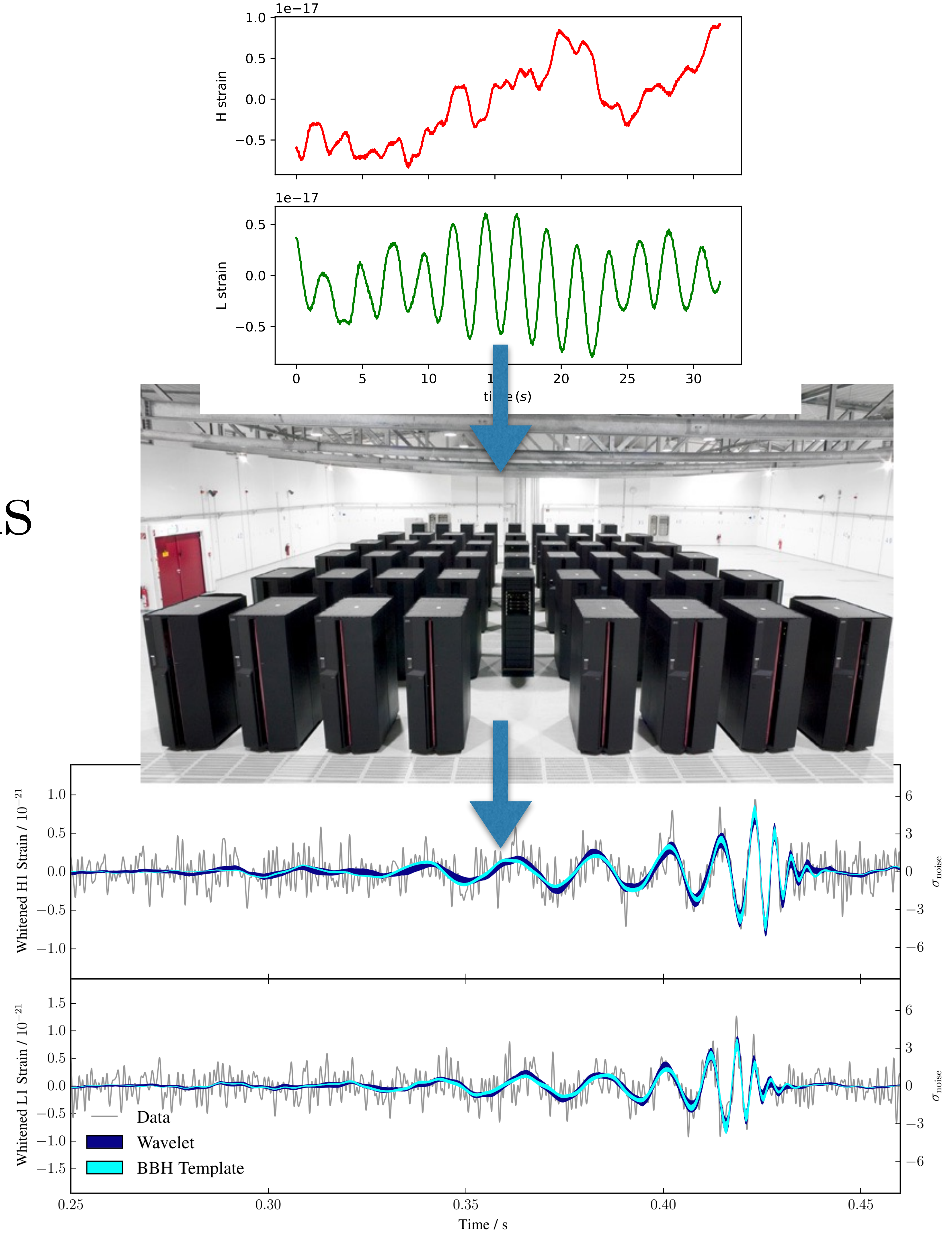
Large dimensional integral
numerical methods

  - Account for all correlations among parameters and all known physical information

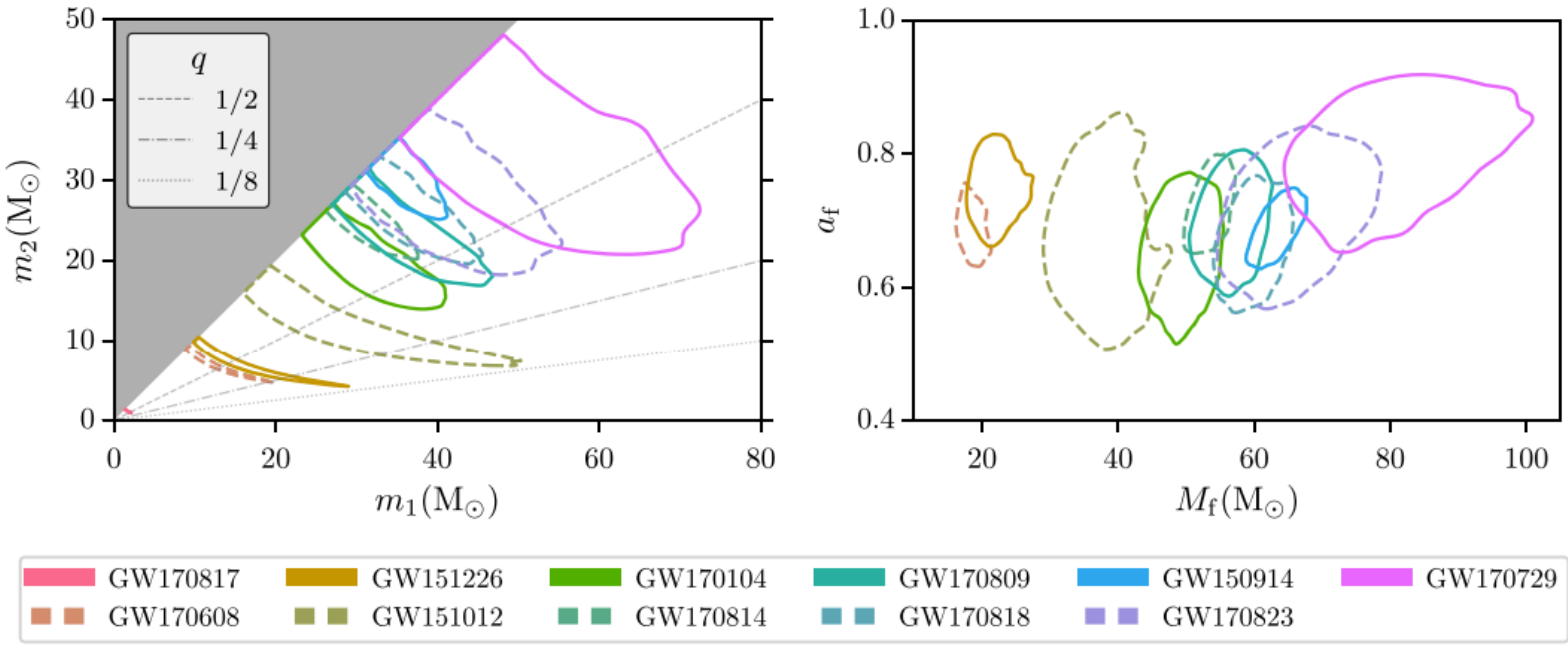
The challenge of parameter estimation

LVC: arXiv:1602.03840

- Parameter estimation is an expensive business
    - Generation of O(10^7) templates
    - If a template takes 1s to generate $\implies \tau \sim$ months
    - Need for fast AND accurate waveform models
- EOBNR(ROM) & Phenom
- Open access: https://github.com/lscsoft/lalsuite

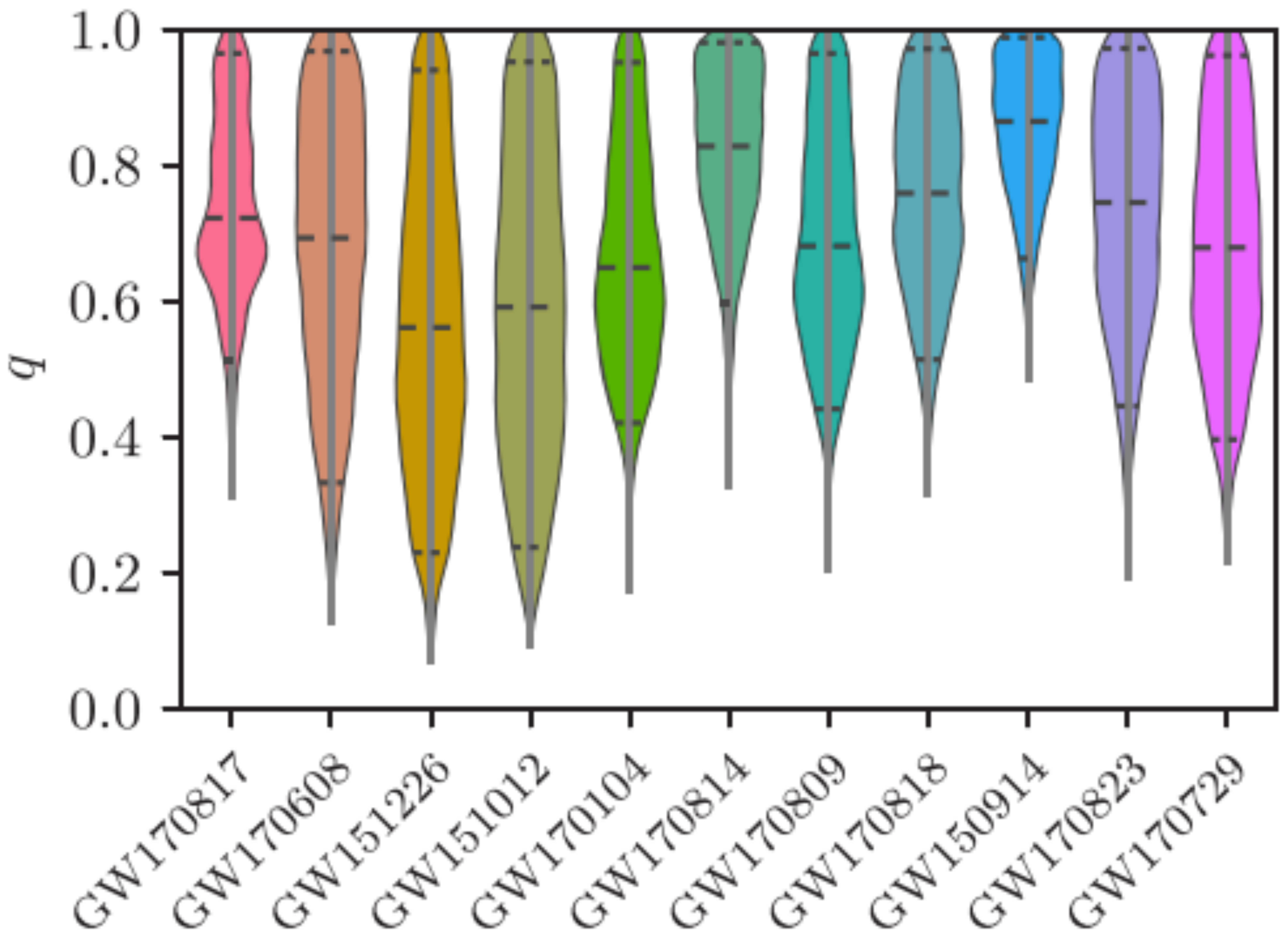


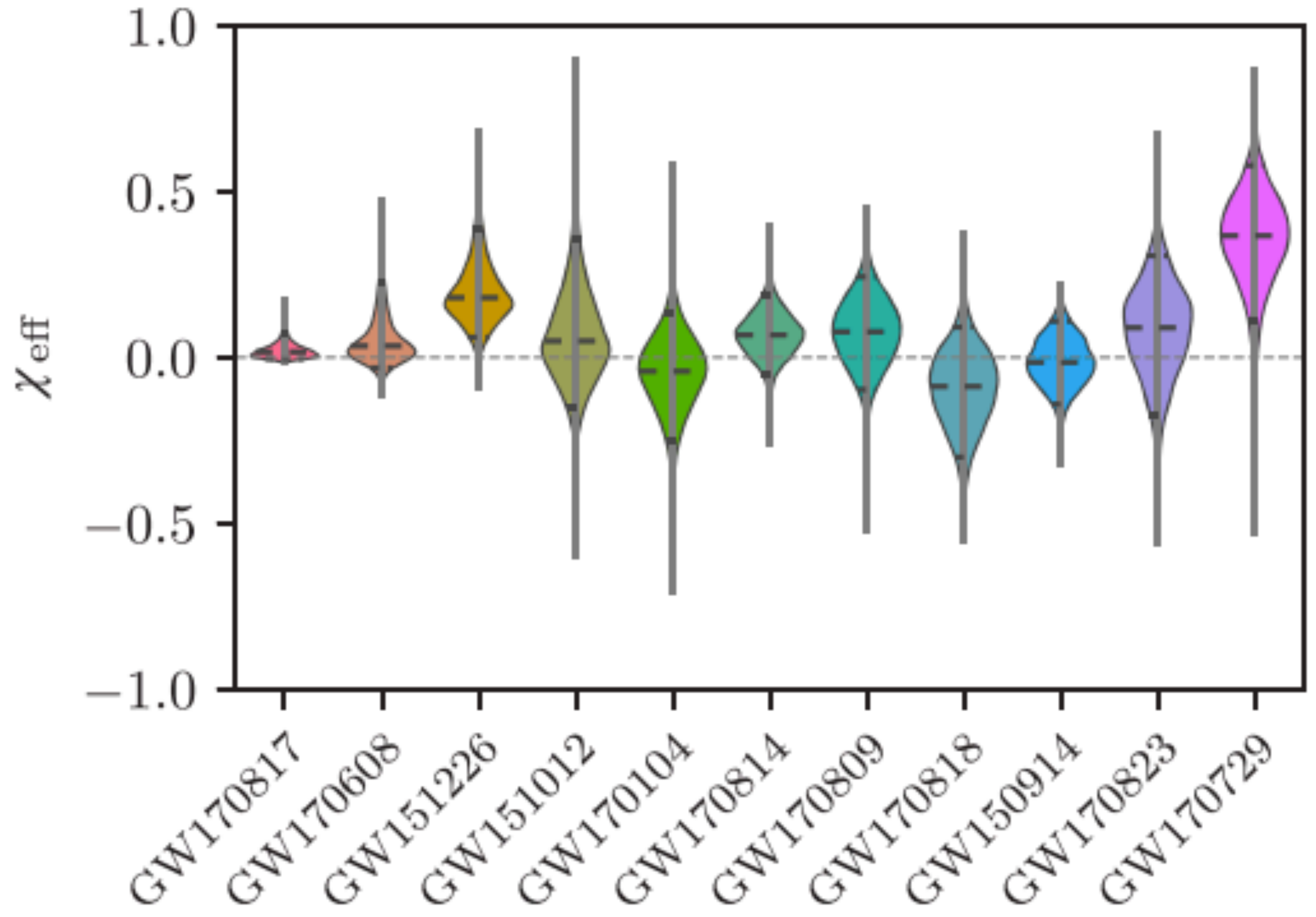
The known population of BBH - GWTC-1

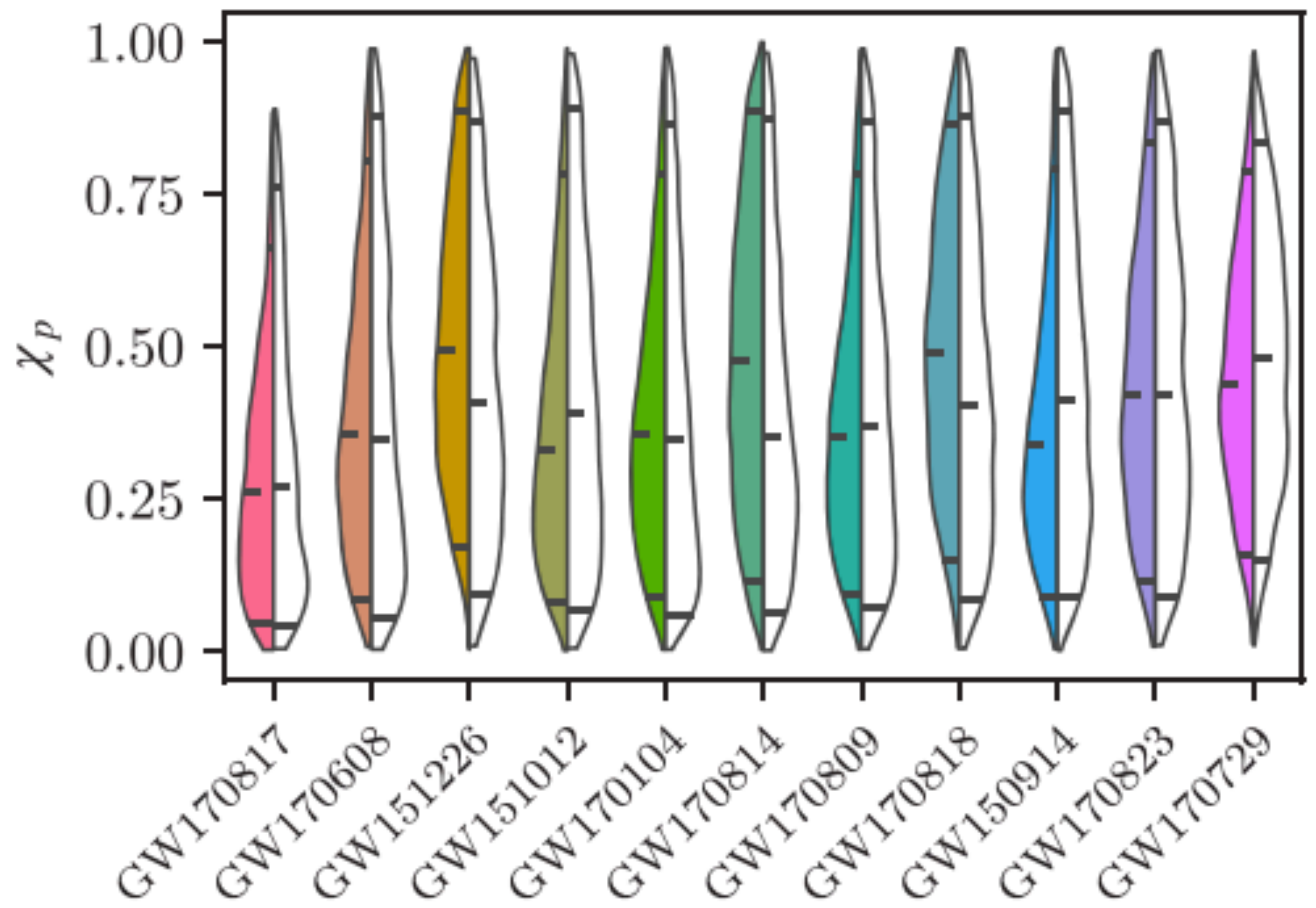


LVC, arXiv:1811.12907

The known population of BBH - GWTC-1







LVC, arXiv:1811.12907

Summary of GWTC-1

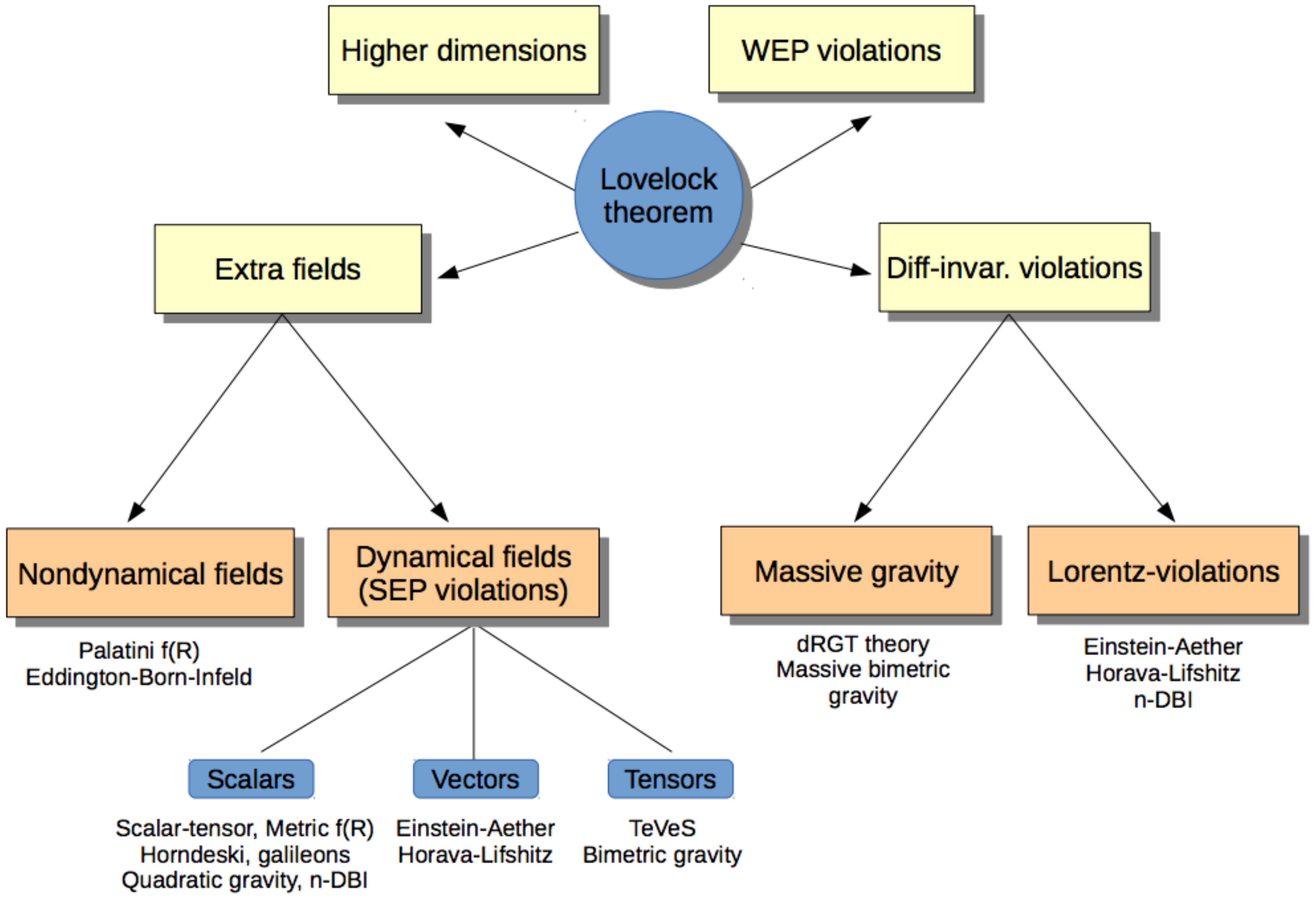
- Wide range of BH masses
  - progenitors $m \in (5,76)$
  - remnants $m \in (17,85)$
- Some evidence for spinning systems
  - GW151226
  - GW170729
- Remnant spin $\simeq 0.7$
- Merger rate $9.7 < R[\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}] < 101$

Event	$m_1/M_\odot$	$m_2/M_\odot$	$\mathcal{M}/M_\odot$	$\chi_{\rm eff}$	$M_f/M_\odot$	a_f
GW150914	$35.6^{+4.7}_{-3.1}$	$30.6^{+3.0}_{-4.4}$	$28.6^{+1.7}_{-1.5}$	$-0.01^{+0.12}_{-0.13}$	$63.1^{+3.4}_{-3.0}$	$0.69^{+0.05}_{-0.04}$
GW151012	$23.2^{+14.9}_{-5.5}$	$13.6^{+4.1}_{-4.8}$	$15.2^{+2.1}_{-1.2}$	$0.05^{+0.31}_{-0.20}$	$35.6^{+10.8}_{-3.8}$	$0.67^{+0.13}_{-0.11}$
GW151226	$13.7^{+8.8}_{-3.2}$	$7.7^{+2.2}_{-2.5}$	$8.9^{+0.3}_{-0.3}$	$0.18^{+0.20}_{-0.12}$	$20.5^{+6.4}_{-1.5}$	$0.74^{+0.07}_{-0.05}$
GW170104	$30.8^{+7.3}_{-5.6}$	$20.0^{+4.9}_{-4.6}$	$21.4^{+2.2}_{-1.8}$	$-0.04^{+0.17}_{-0.21}$	$48.9^{+5.1}_{-4.0}$	$0.66^{+0.08}_{-0.11}$
GW170608	$11.0^{+5.5}_{-1.7}$	$7.6^{+1.4}_{-2.2}$	$7.9^{+0.2}_{-0.2}$	$0.03^{+0.19}_{-0.07}$	$17.8^{+3.4}_{-0.7}$	$0.69^{+0.04}_{-0.04}$
GW170729	$50.2^{+16.2}_{-10.2}$	$34.0^{+9.1}_{-10.1}$	$35.4^{+6.5}_{-4.8}$	$0.37^{+0.21}_{-0.25}$	$79.5^{+14.7}_{-10.2}$	$0.81^{+0.07}_{-0.13}$
GW170809	$35.0^{+8.3}_{-5.9}$	$23.8^{+5.1}_{-5.2}$	$24.9^{+2.1}_{-1.7}$	$0.08^{+0.17}_{-0.17}$	$56.3^{+5.2}_{-3.8}$	$0.70^{+0.08}_{-0.09}$
GW170814	$30.6^{+5.6}_{-3.0}$	$25.2^{+2.8}_{-4.0}$	$24.1^{+1.4}_{-1.1}$	$0.07^{+0.12}_{-0.12}$	$53.2^{+3.2}_{-2.4}$	$0.72^{+0.07}_{-0.05}$
GW170817	$1.46^{+0.12}_{-0.10}$	$1.27^{+0.09}_{-0.09}$	$1.186^{+0.001}_{-0.001}$	$0.00^{+0.02}_{-0.01}$	≤ 2.8	≤ 0.89
GW170818	$35.4^{+7.5}_{-4.7}$	$26.7^{+4.3}_{-5.2}$	$26.5^{+2.1}_{-1.7}$	$-0.09^{+0.18}_{-0.21}$	$59.4^{+4.9}_{-3.8}$	$0.67^{+0.07}_{-0.08}$
GW170823	$39.5^{+11.2}_{-6.7}$	$29.0^{+6.7}_{-7.8}$	$29.2^{+4.6}_{-3.6}$	$0.09^{+0.22}_{-0.26}$	$65.4^{+10.1}_{-7.4}$	$0.72^{+0.09}_{-0.12}$

Extensions of GR

- Alternative theories
  - Introduce extra degrees of freedom:
    - additional fields
    - higher-curvature terms
  - Challenge GR assumptions:
    - Lorentz invariance
    - Equivalence principle
- High curvature and dynamics ideal setting

Lovelock theorem: In 4D, the only divergence free symmetric rank-2 tensor constructed only by the metric and its derivatives up to 2nd order and preserving diffeomorphism invariance is the Einstein tensor plus a constant.



Berti et al, arXiv:1501.07274

Effects on the waveform

- Alternative theories of gravity modify the waveform
  - change the φ coefficients by introducing additional parameters
    - e.g. "massive gravity"
  - add extra orders not present in the GR waveform
    - e.g. Brans-Dicke
- Non-BHs or hairy BHs would show different ringdown spectra
- Preferred approach is "un-modelled"
  - Perturb around GR, e.g. $\varphi_{eff} = \varphi_{GR}(1 + \delta\varphi)$

$$h(f) = A(f)e^{i\Phi(f)}$$

$$\Phi(f) = \sum_{k=1}^{7}(\varphi_k + \varphi_k^l \log(f))f^{(5-k)/3} + \sum_{i \neq k} \varphi_i f^i$$

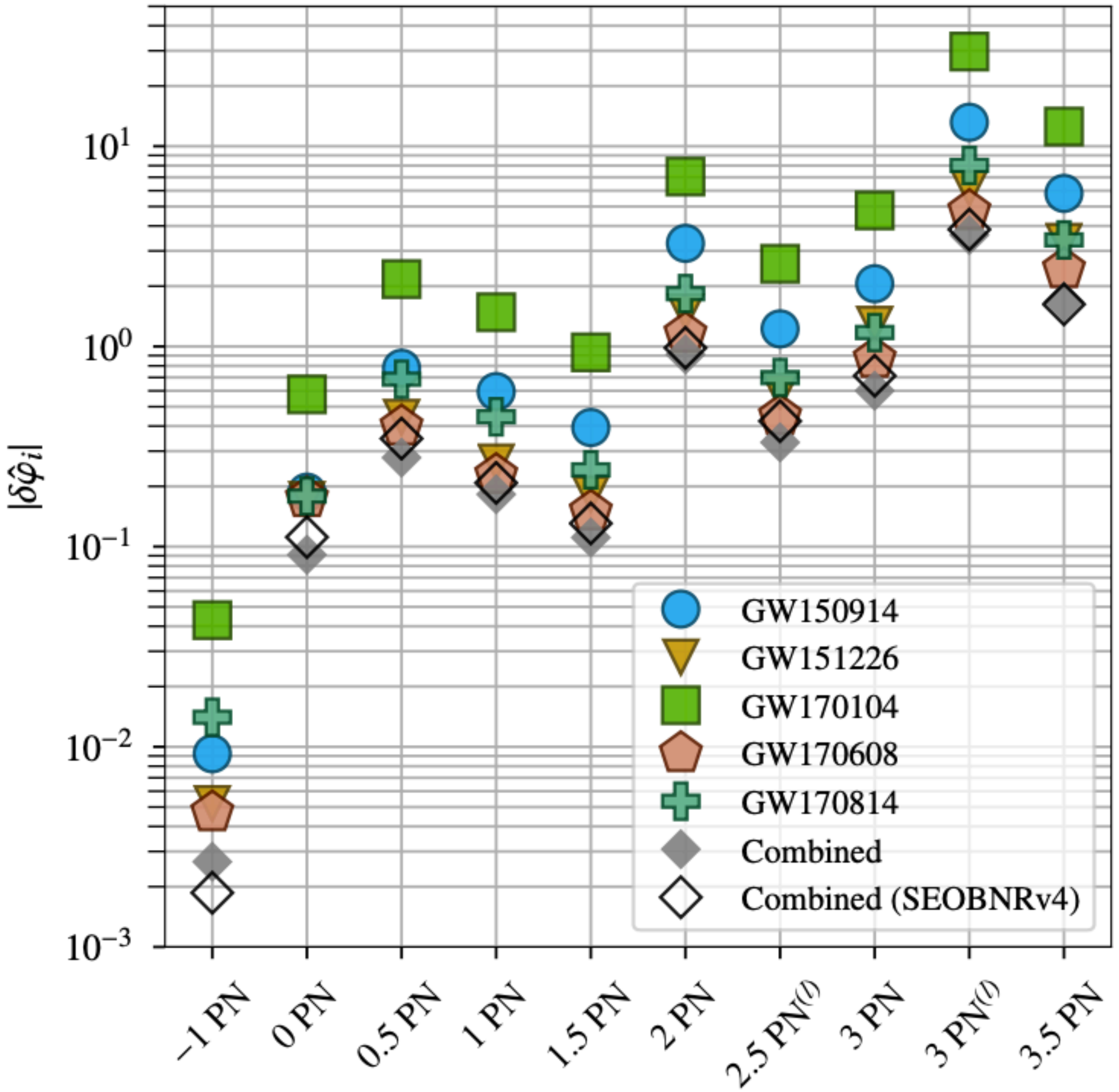
$$\varphi_j \equiv \varphi_j(m_1, m_2, \vec{s}_1, \vec{s}_2)\ \forall\, j = k, i$$

$$\Phi_{MG}(v) = \Phi_{GR}(v) - \frac{\pi^2 DM}{\lambda_g^2(1+z)} v^{-1}$$

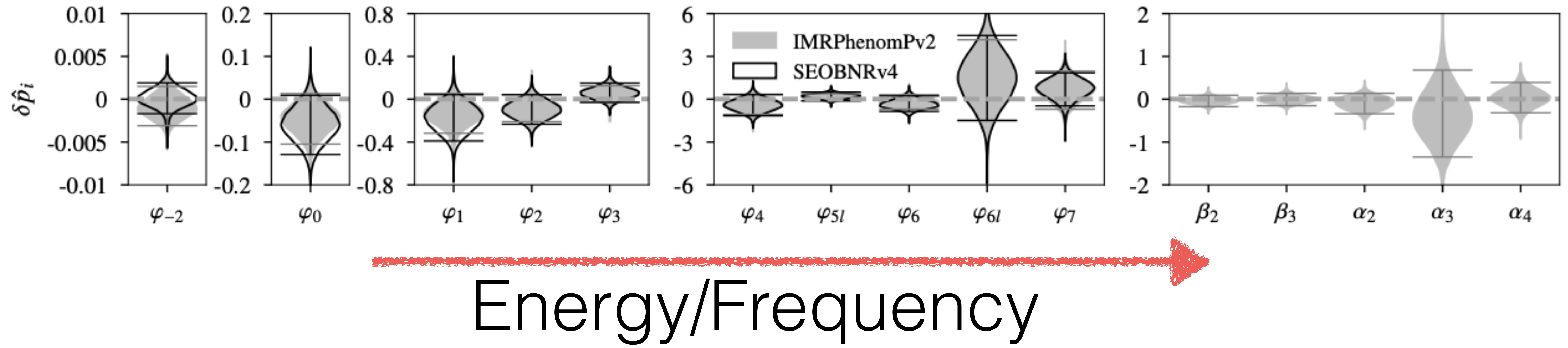
$$\Phi_{\mathrm{BD}}(v) = \Phi_{\mathrm{GR}}(v) - \frac{5S^2}{84\omega_{BD}} v^{-2}$$

post-Newtonian series

- Test of the dynamics of space-time from the inspiral $\hat{\varphi}_j \equiv \varphi^{\mathrm{GR}}(1+\delta\hat{\varphi}_j)$



effective merger and post-merger coefficients



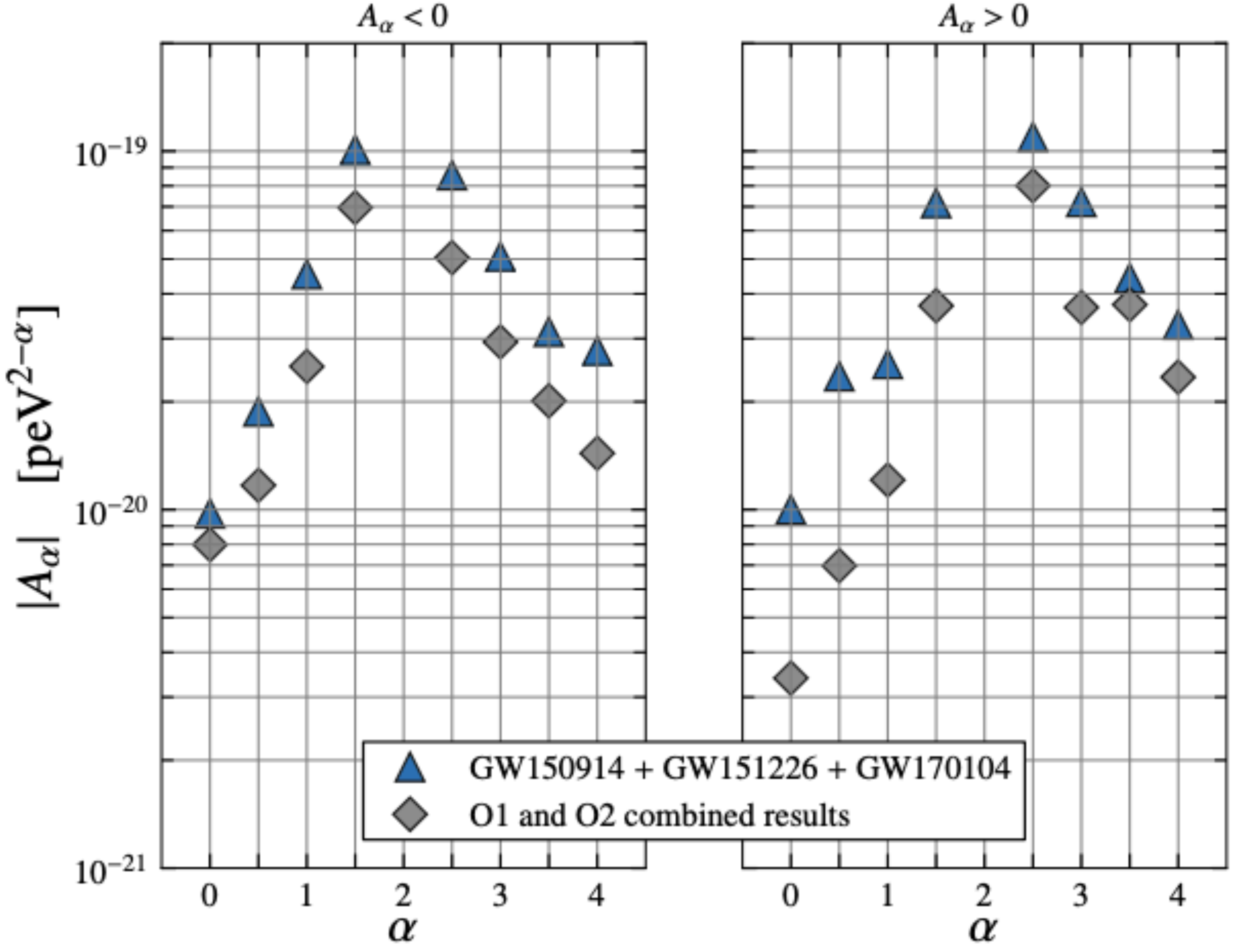
LVC, arXiv:1903.04467

Propagation tests

- Lorentz invariance violations
  - modification to the dispersion relation

    $$E^2 = p^2c^2 + A_\alpha p^\alpha c^\alpha$$

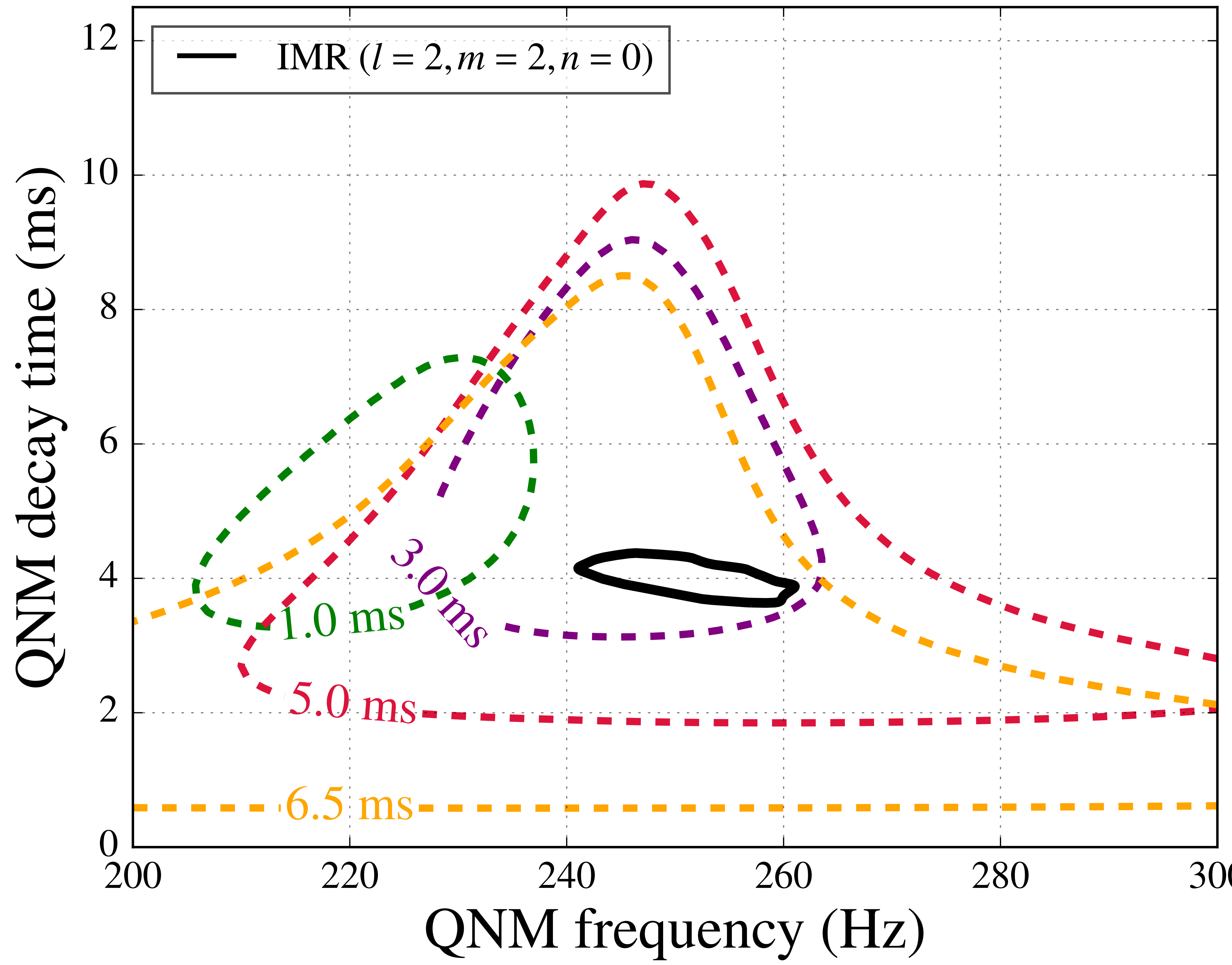
  - Induce a frequency(energy) dephasing
- Provide constraints at each value of α



LVC, arXiv:1903.04467

Remnant properties

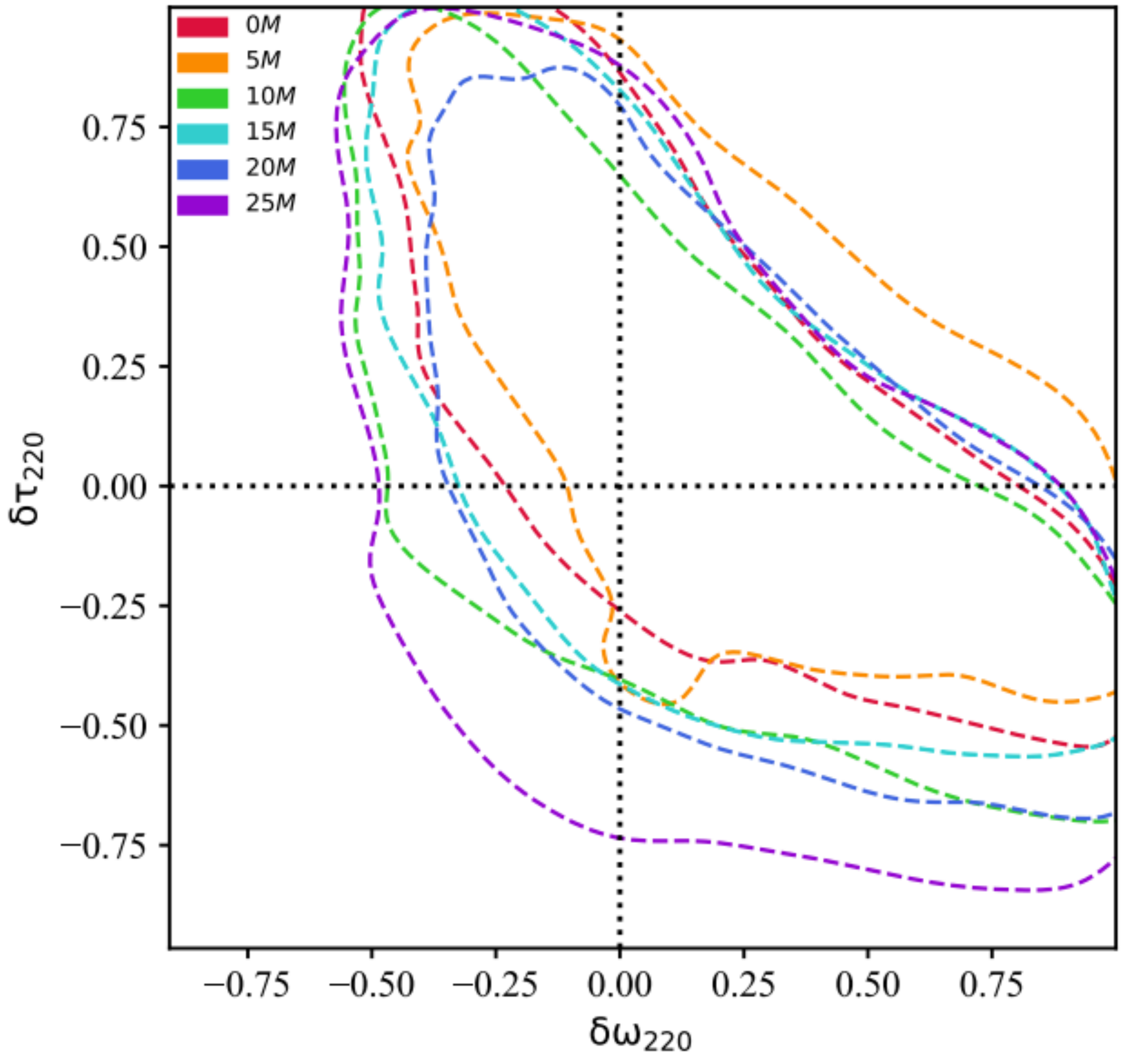
- Studying the remnant properties requires to isolate the ringdown
  - Time of transition between non-linear and linear regime is an open problem (Carullo et al 2018, Bhagwat et al 2018)
- Measure QNM frequency and time
- Infer BH mass and spin
- Tests of no-hair conjecture

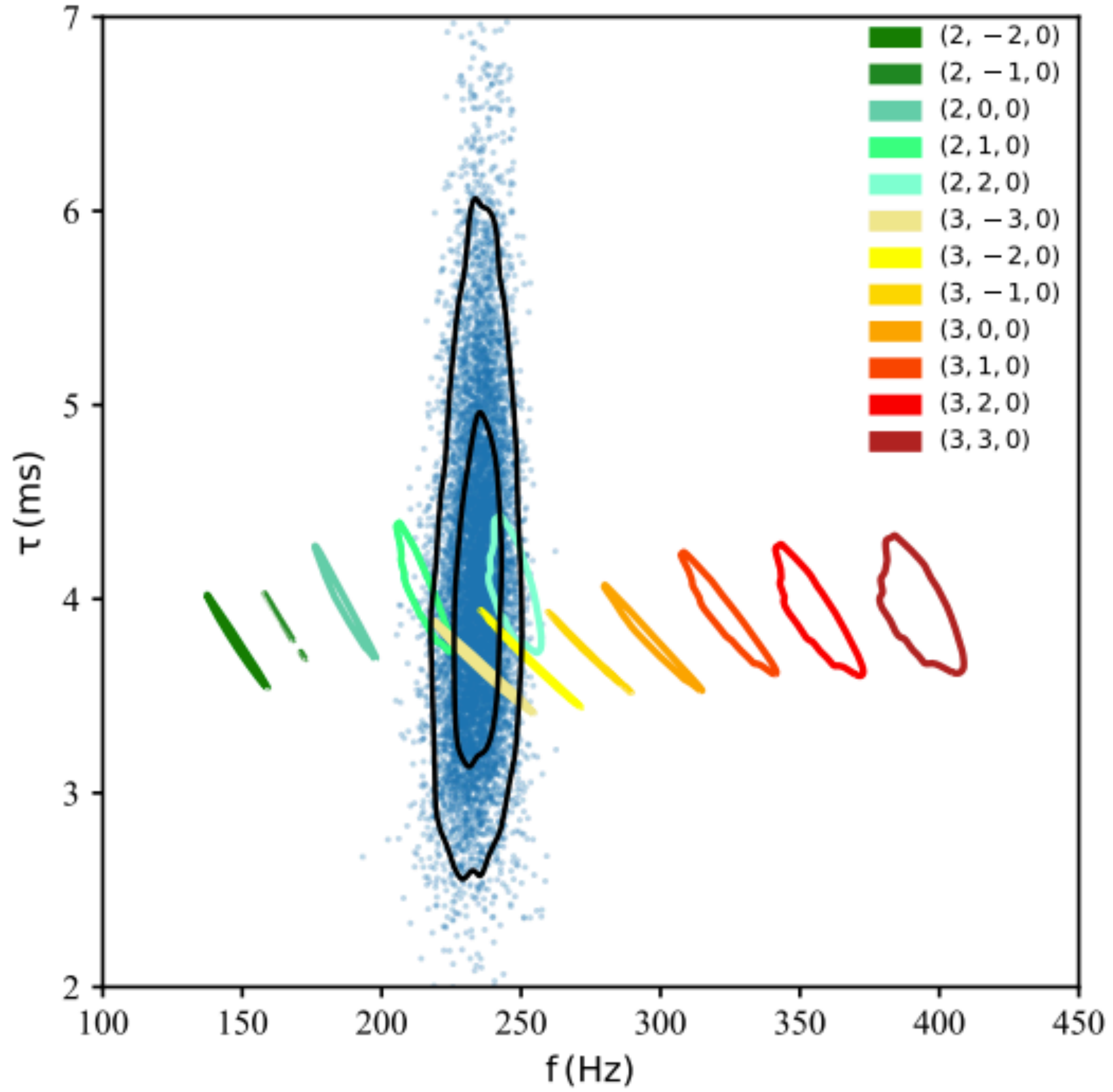


LVC,arXiv:1602.03841

Black hole spectroscopy

- Concept introduced in Dreyer et al 2004, Berti et al 2006
- Carullo et al 2019:
  - use posterior distribution to infer ringdown modes
  - Constrain deviations from BH uniqueness





Carullo et al, arXiv:1902.07527

Conclusions

- Gravitational waves from binary black holes are becoming routine
  - Increase in sensitivity will lead to hundreds of BBH in coming years
    - Technical challenge
    - Astrophysics
      - Constraints on stellar physics from mass and spin distributions
    - Cosmology (LVC, arXiv:1710.05835 and references there in)
    - Fundamental and black hole physics
      - Insights in quantum gravity?
- Look forward to results from O3a, and check GW190412 out