Introduction to Quantum Field Theory and QCD

Lectures 1&2

Darren Forde CERN & NIKHEF

BND Summer School 2010 Oostende, Belgium, Sept 6-17th 2010

Overview

- Aim of these lectures is to explain the basics of how computations are performed in quantum field theories.
- We will restrict ourselves to:
  - Perturbative computations.
  - Re-normalisable theories such as QED and QCD.
- This is still leaves a large amount to cover.

Lecture 1

- Why we need quantum field theory?
- The basics of a quantum field theory.
- The simplest example; the Klein-Gordon Field.
- The simple harmonic oscillator and ladder operators.
- The Hilbert and Fock spaces of a theory.

The Standard Model

- The Standard Model (SM) encompasses our Knowledge of particle physics.
- There are the fundamental particles,

				Charge (e)	Gauge Bosons
Leptons	ν_e	ν_μ	ν_τ	0	γ
	e	μ	τ	-1	$W^\pm, Z$
Quark Flavours, 3 colours	u	c	t	2/3	g
	d	s	b	-1/3	
					Spin1

- Those related to QCD we never see unconfined in Nature.

QCD

- Different quarks are bound together
  - Mesons - quark and anti-quark doublet.
  - Baryons - quark, quark, quark triplet.
- There is a whole spectrum of particles with different charges and spin depending upon which quarks are bound.
- For example,
  - Mesons - π, K, ρ, ...
  - Baryons - p, n, Λ, Δ, ...

Relativistic QM

- To understand the SM we need to understand Quantum Field Theory.
- Why is it needed? we want to combine Special Relativity (SR) and Quantum Mechanics (QM).
- Try to create a relativistic Schrodinger Eq. Create using the correspondence principle,

$$E^2 = p^2 + m_0^2$$

$$\Rightarrow \quad -\partial_t^2 \Psi = (-\nabla^2 + m_0^2)\Psi$$

$$\Rightarrow \quad (\Box + m_0^2)\Psi = 0$$

The Klein-Gordon Eq.

Correspondence Principle

$$E \rightarrow i\partial_t$$

$$\vec{p} \rightarrow -i\vec{\partial}$$

$$\Box = \partial_t^2 - \nabla^2$$

Negative Energy Solutions

- The solution to the KG eq can be written as a plane-wave,

$$\psi(\vec{x}) = e^{-ik_\mu \cdot x^\mu}$$

- Inserting this into the KG equation we get,

$$(\partial_\mu \partial^\mu + m_0^2) e^{-ik_\mu \cdot x^\mu} = ((-ik_\mu)(-ik^\mu) + m_0^2) e^{-ik_\mu \cdot x^\mu} = 0$$

- Which means that $k^2 = m_0^2$ and so we have a negative energy,

$$E = \pm \sqrt{\vec{k}^2 + m_0^2}$$

Causality

- The Hamiltonian for a single relativistic particle is given by,

$$H = \sqrt{\vec{p}^2 + m_0^2}$$

- Compute the amplitude for a particle to travel between x and x' in time t.

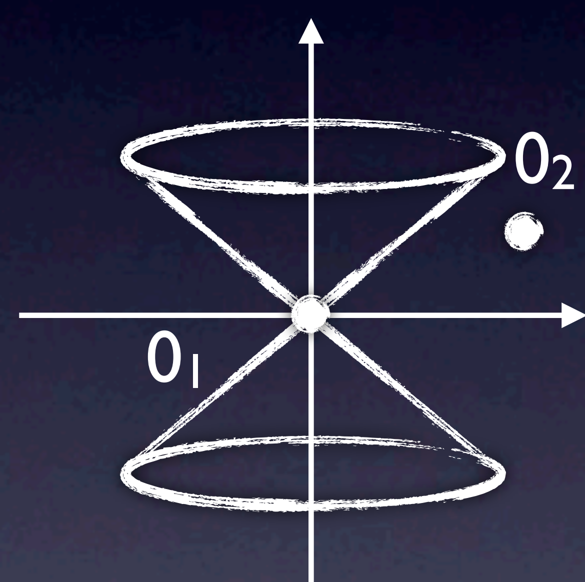
$$\begin{aligned} A(x, x', t) &= \langle x' | e^{-iHt} | x \rangle \\ &= e^{-m\sqrt{(x'-x)^2 - t^2}} \end{aligned}$$

- We see that this does not vanish outside the light cone for space-like separations,

$$(x' - x)^2 - t^2 < 0$$

Conclusions

- Particles would have to travel faster than the speed of light!



- This along with negative energy states tells us that quantising a relativistic particle is not the solution.
- We need something else, Quantum Field Theory.

Quantum Field Theory

- The quantisation of dynamical systems of fields.
- All of modern particle physics is based upon this.
- Need comes from the difficulties of trying to quantise relativistic particles (i.e. negative energy states, multiple particles, difficulties with causality.)
- We will be interested in the dynamics of fields $\varphi(\boldsymbol{x}, t)$, $\boldsymbol{x}$ is a momentum 3-vector,
  - Unlike in quantum mechanics both x and t will be labels (in QM x is a dynamical variable).

Lagrangians

- Like classical field theory QFT is described via a Lagrangian L, of one or more fields $\varphi(x)$ and their derivatives $\partial_\mu \varphi$.
- The action S is given by,

$$S = \int d^4\vec{x}\mathcal{L}(\phi, \partial_\mu \phi),$$

- The equation of motion of the field is derived using the Euler-Lagrange equation,

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0$$

The Klein-Gordon Field

- The simplest starting point is to consider the quantisation of the Klein-Gordon field,

$$\mathcal{L}_{KG} = \frac{1}{2}\left(\partial_\mu \phi\right)^2 - \frac{1}{2}m^2\phi^2$$

- With an equation of motion (using the Euler-Lagrange equation) for the field φ,

$$\left(\partial^\mu \partial_\mu + m^2\right)\phi = 0.$$

- To quantise this we will proceed in analogy with the quantisation of a simple harmonic oscillator.

The Hamiltonian

- The Hamiltonian formalism is best suited to performing this quantisation.
- The Hamiltonian, H, of a system can be defined with respect to the Lagrangian via,

$$H = \int d^3x \left[\pi(\vec{x})\dot{\phi}(\vec{x}) - \mathcal{L} \right]$$

- We also define the conjugate momentum to be,

$$\pi(\vec{x}) = \frac{\partial \mathcal{L}}{\partial \dot{\phi}(\vec{x})}$$

- $\dot{\phi}$ is the differential of the field, φ, with respect to time, t.

The Klein-Gordon field

- The Hamiltonian for the Klein-Gordon field is given by,

$$H = \int d^3x \left[\frac{1}{2}\pi^2 + \frac{1}{2}\left(\nabla\phi\right)^2 + \frac{1}{2}m^2\phi^2 \right]$$

Energy Cost to move in time

Energy Cost to Sheer in Space

Energy Cost of the field

- We no longer have a manifestly Lorentz invariant expression.

Canonical Quantisation

- In quantum mechanics quantisation of a discrete system is performed by imposing commutation relations between the position q_i and momentum p_i of one or more particles,

$$[q_i, p_j] = i\delta_{ij}, \quad [q_i, q_j] = [p_i, p_j] = 0$$

- To quantise the Klein-Gordon field we will proceed in a similar way by promoting φ, and π to operators and imposing equivalent equal time commutation relations,

$$[\phi(\vec{x}), \pi(\vec{y})] = i\delta^{(3)}(\vec{x} - \vec{y}), \quad [\phi(\vec{x}), \phi(\vec{y})] = [\pi(\vec{x}), \pi(\vec{y})] = 0$$

The simple harmonic oscillator (SHO)

- When quantising the KG field we will need to find the spectrum of the system.
- To do this we will draw an analogy with the SHO.
- The equation of motion for a SHO with frequency, $\omega_P = \sqrt{|p|^2 + m^2}$, is given by,

$$\left[\frac{\partial^2}{\partial t^2} + (|p|^2 + m^2)\right] \phi(\vec{p}, t) = 0$$

Ladder operators

- We want to promote the φ and p to operators and impose our commutation relations.
- Write φ and p in terms of "ladder" operators, a, and $a^\dagger$,

$$\phi = \frac{1}{\sqrt{2\omega}}(a + a^\dagger), \quad \vec{p} = -i\sqrt{\frac{\omega}{2}}(a - a^\dagger)$$

- If a, and $a^\dagger$ satisfy $[a, a^\dagger] = 1$ then we satisfy the commutation relation, $[\phi, p] = i$.

SHO Spectrum

- We can now investigate the spectrum of the Hamiltonian,

$$H = \frac{1}{2}p^2 + \frac{1}{2}\omega^2\phi^2 = \omega\left(a^\dagger a + \frac{1}{2}\right)$$

- Starting from the zero-point energy state $|0\rangle$, with eigenvalue $\omega/2$, (which is defined via $a|0\rangle = 0$), we can use,

$$\left[H, a^\dagger\right] = \omega a^\dagger, \;\; [H, a] = -\omega a$$

- To define the full spectrum of states, with eigenvalues $(n+1/2)\,\omega$,

$$|n\rangle = \left(a^\dagger\right)^n |0\rangle$$

Quantising the Klein-Gordon Field

- To relate the Klein-Gordon field to the SHO consider the Fourier transform of the Klein-Gordon field,

$$\phi(\vec{x},t) = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}\cdot\vec{x}} \phi(\vec{p},t)$$

- At each point x we have an SHO φ*(p,t)* with equation of motion,

$$\left[\frac{\partial^2}{\partial t^2} + (|p|^2 + m^2)\right] \phi(\vec{p},t) = 0$$

- The KG field is a continuum of SHO's.

Solutions of the Klein-Gordon field

- A solution to the KG field can be written as a plane-wave solution,

$$\phi(\vec{x},t) = ae^{-i(\omega(\vec{p})t-\vec{p}\cdot\vec{x})}$$

- The more general solution can be written as,

$$\phi(\vec{x},t) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2\omega(\vec{p})}}\left(ae^{i(\omega(\vec{p})t-\vec{p}\cdot\vec{x})} + a^* e^{-i(\omega(\vec{p})t-\vec{p}\cdot\vec{x})}\right)$$

φ and π Operators

- In analogy to the SHO we write down the field operators as,

$$\phi(\vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega(\vec{p})}} \left(a(\vec{p})e^{i\vec{p}\cdot\vec{x}} + a^\dagger(\vec{p})e^{-i\vec{p}\cdot\vec{x}}\right)$$

$$\pi(\vec{x}) = \int \frac{d^3p}{(2\pi)^3}(-i)\sqrt{\frac{\omega(\vec{p})}{2}} \left(a(\vec{p})e^{i\vec{p}\cdot\vec{x}} - a^\dagger(\vec{p})e^{-i\vec{p}\cdot\vec{x}}\right)$$

- There is a continuum of SHO states and so we label the corresponding ladder operator by the momentum p, i.e. $a(p)$ and $a^\dagger(p)$.

Commutation Relations

- There is a separate commutation relation for each *a(p)* which we write as,

$$\left[a(\vec{p}), a^\dagger(\vec{p}')\right] = (2\pi)^3\delta^{(3)}(\vec{p}-\vec{p}')$$

- These can then be used to show that the expected equal time commutation relations are satisfied,

$$[\phi(\vec{x}), \pi(\vec{y})] = i\delta^{(3)}(\vec{x}-\vec{y})$$
$$[\phi(\vec{x}), \phi(\vec{y})] = [\pi(\vec{x}), \pi(\vec{y})] = 0$$

The KG Hamiltonian

- We want to compute the spectrum of states of the system.
- The Hamiltonian can be written in terms of the ladder operators as,

$$H = \int \frac{d^3p}{(2\pi)^3}\omega(\vec{p}) \left(a^\dagger(\vec{p})a(\vec{p}) + \frac{1}{2}\left[a(\vec{p}), a^\dagger(\vec{p})\right] \right)$$

- The second term is the sum over all zero-point energies and is divergent, it is proportional to $\delta(0)$.
- In experiments we only measure relative differences to the ground state so it can be dropped.

The Energy of the system

$$H = \int \frac{d^3p}{(2\pi)^3}\omega(\vec{p})\left(a^\dagger(\vec{p})a(\vec{p}) + \frac{1}{2}\left[a(\vec{p}), a^\dagger(\vec{p})\right]\right)$$

- Acting with the Hamiltonian operator H on a state gives the energy of the state,

$$H|\phi_1\rangle = E_1|\phi_1\rangle$$

- Operators that commute with the Hamiltonian correspond to conserved quantities.

Creation & Destruction Operators

- With the rewritten form of the Hamiltonian we can then show,

$$\left[H, a^\dagger(\vec{p})\right] = \omega(\vec{p}) a^\dagger(\vec{p}) \qquad [H, a(\vec{p})] = -\omega(\vec{p}) a(\vec{p})$$

- This means that

$$a^\dagger(p)|0\rangle = \omega(p)|p\rangle \qquad \langle 0|a(p) = \langle p|\omega(p)$$

- Which we can interpret to mean that the $a^\dagger(p)$ operators are creating particles of energy ω while the $a(p)$ destroy them, with energy

$$\omega(p) = \sqrt{p^2 + m^2}$$

Constructing States

- The ground state, with energy zero, is defined to be

$$a(p)|0\rangle = 0$$

- We build up the set of all states by acting on this ground state with creation, $a^\dagger(p)$, operators.

- Multi particle states can be built up by applying multiple creation operators,

$$a^\dagger(p_1)a^\dagger(p_2)\ldots a^\dagger(p_n)|0\rangle$$
$$= \omega(p_1) + \omega(p_2) + \ldots + \omega(p_n)|p_1, p_2, \ldots, p_n\rangle$$

The Fock Space

- These states obey Bose-Einstein statistics as

$$a(p)^\dagger a(q)^\dagger |0\rangle \equiv a(q)^\dagger a(p)^\dagger |0\rangle$$

- The spectrum of all particles is the Hilbert space of our theory.
- By acting with all possible combinations of we span the entire space.
- This type of space is known as a ***Fock*** space.

The Number Operator

- We can count the number of states in a particular state using the number operator,

$$N = \int \frac{d^3p}{(2\pi)^3} a^\dagger(\vec{p}) a(\vec{p})$$

- So for example

$$N|p_1, \ldots, p_n\rangle = n|p_1, \ldots, p_n\rangle$$

- As this commutes with the Hamiltonian then particle number is conserved. This will change when we consider interacting theories.

Normalisation

- The vacuum state is normalised so that

$$\langle 0|0\rangle = 1$$

- We normalise the states in a Lorentz invariant way,

$$\langle \vec{p}|\vec{q}\rangle 2\omega(\vec{p})(2\pi)^3\delta^{(3)}(\vec{p}-\vec{q})$$

- Also useful will be the Lorentz invariant integral measure, $\displaystyle\int \frac{d^3p}{(2\pi)^3 2\omega(\vec{p})}$

Creating States

- To create a state of the field φ we act on the ground state with the operator φ*(x)*,

$$\phi(\vec{x})|0\rangle = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega(\vec{p})} e^{-i\vec{p}\cdot\vec{x}} |\vec{p}\rangle$$

- This creates a linear superposition of well-defined single-momentum states.
- This is not a "particle" which is localised in space because φ*(x)* is not a good operator on the Fock space of states, its normalisation is proportional to a delta function,

$$\langle 0|\phi(\vec{x})\phi(\vec{x})|0\rangle = \langle \vec{x}|\vec{x}\rangle = \delta^{(3)}(0)$$

Dual Interpretation

- Dual interpretation of a quantum field,
- Wave: The field is a linear combination of solutions of the KG equation.
- Particle: The ladder operators a and $a^\dagger$ are Hilbert space operators which annihilate and create particles.

Summary

- We now know why we need quantum field theory.
- We understand the basics of a quantum field theory through the study of the simplest example, the Klein-Gordon Field.
- Quantised this field by using the simple harmonic oscillator and ladder operators.

Lecture 2

- The time evolution of states and fields
  - The Schrodinger Picture.
  - The Heisenberg Picture.
- Causality.
- Propagators.
- Spinors.

Schrodinger Picture

- All the operators and states we have discussed so far have been in the Schrodinger Picture.
- The operators are independent of time and we have applied equal time commutation relations.
- The states evolve in time, via the Schrodinger equation,

$$i\frac{d|\vec{p}(t)\rangle}{dt} = H|\vec{p}(t)\rangle \Rightarrow |\vec{p}(t)\rangle = e^{-i\omega(\vec{p})t}|\vec{p}\rangle$$

Heisenberg Picture

- We want a more Lorentz Covariant framework.
- Let us now make our operators depend upon time by switching to the Heisenberg picture,

$$\phi(x) = e^{iHt}\phi(\vec{x})e^{-iHt}$$

- The evolution of an operator in time is given by,

$$\frac{d\mathcal{O}_H}{dt} = -\left[H, \mathcal{O}_H\right]$$

- The states no longer evolve in time.

Ladder operators & the Heisenberg Picture

- We note that the Hamiltonian H acting on the ladder operator a gives,

$$Ha(\vec{p}) = a(\vec{p})(H - \omega(\vec{p}))$$

- So that for n applications of H we have

$$H^n a(\vec{p}) = a(\vec{p})(H - \omega(\vec{p}))^n$$

- So we can commute the $\exp(iHt)$ through the ladder operator,

$$e^{iHt}a(\vec{x})e^{-iHt} = a(\vec{p})e^{-i\omega(\vec{p})t}, \;\; e^{iHt}a^\dagger(\vec{x})e^{-iHt} = a^\dagger(\vec{p})e^{i\omega(\vec{p})t}$$

The Field operator in the Heisenberg Picture

- We can now shift from the Schrodinger picture for the field,

$$\phi(\vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega(\vec{p})}} \left(a(\vec{p})e^{i\vec{p}\cdot\vec{x}} + a^\dagger(\vec{p})e^{-i\vec{p}\cdot\vec{x}}\right)$$

- To the Heisenberg picture,

$$\phi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega(\vec{p})}} \left(a(\vec{p})e^{-ip\cdot x} + a^\dagger(\vec{p})e^{ip\cdot x}\right)$$

- Where the 4-vector $p = (\omega(\vec{p}), \vec{p})$.

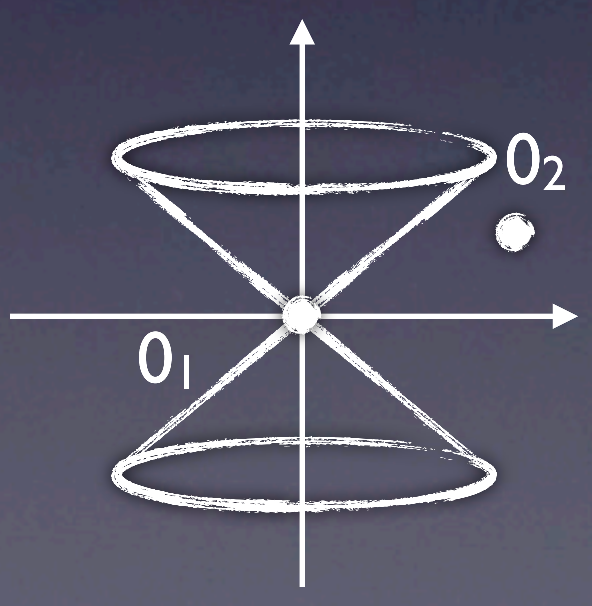
Causality

- One question we can ask about our quantised theory is if a two space-like separated measurements can effect each other.
- Restated we are asking if space-like operators commute outside the light cone.
- So far we have only ensured that the commutation relations hold for equal time

$$[\phi(x), \phi(y)] = 0 \;\; x^0 = y^0$$

- What we want to show

$$[\phi(x), \phi(y)] = 0 \;\; \forall \;\; (x-y)^2 < 0$$

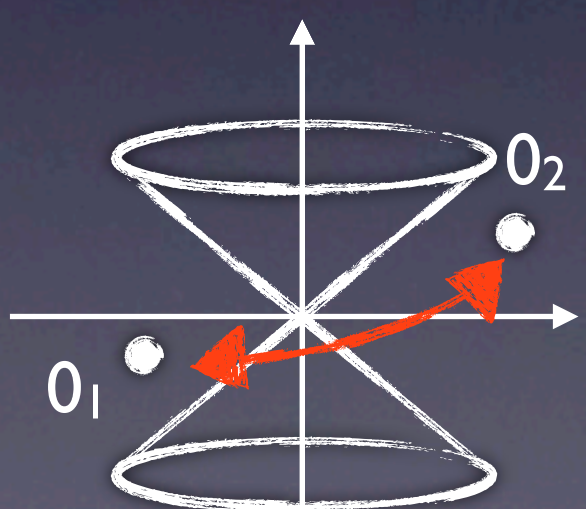


Commutators

- Let us compute this commutator,

$$[\phi(x), \phi(y)] = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega(\vec{p})} \left(e^{-ip\cdot(x-y)} - e^{ip\cdot(x-y)} \right)$$

- We have two Lorentz invariant terms.
- Furthermore we can rotate $p \to -p$ in the second term when $(x-y)^2 < 0$.
- The two terms then cancel.



Can perform a Lorentz transform

Interpretation

- The vanishing commutator tells us that causality is preserved.
- We can interpret the two separate exponentials as a *particle propagating* from x to y and an *anti-particle propagating* from y to x. The particle and the anti-particle cancel each other.

$$[\phi(x), \phi(y)] = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega(\vec{p})} \left(e^{-ip\cdot(x-y)} - e^{ip\cdot(x-y)} \right)$$

- Fortunately this cancellation cannot occur when $(x-y)^2 > 0$.

Complex Fields

- This becomes clearer when we look at a complex scalar field, the field operators now look like,

$$\phi(\vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega(\vec{p})}} \left(a(\vec{p})e^{i\vec{p}\cdot\vec{x}} + b^\dagger(\vec{p})e^{-i\vec{p}\cdot\vec{x}}\right)$$

$$\phi^\dagger(\vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega(\vec{p})}} \left(b(\vec{p})e^{i\vec{p}\cdot\vec{x}} + a^\dagger(\vec{p})e^{-i\vec{p}\cdot\vec{x}}\right)$$

- There are two types of ladder operators, we can interpret one as creating a "particle" of the field and the other as creating the "anti-particle".

Propagators

- We have seen that the commutator of the fields tells us about how the fields propagate through space-time,

$$[\phi(x), \phi(y)] = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega(\vec{p})} \left(e^{-ip\cdot(x-y)} - e^{ip\cdot(x-y)} \right)$$

- Each exponential term represents the propagation of the particle from x to y, we label this $D(x\text{-}y)$,

$$D(x-y) = \langle 0|\phi(x)\phi(y)|0\rangle = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega(\vec{p})} e^{-ip\cdot(x-y)}$$

Propagators Properties

- Write the commutator as,

$$\Delta(x) = -i[D(x) - D(-x)]$$

- This object and hence the propagator has the following properties,
  - i) A solution of the KG eq. $(\partial_\mu \partial^\mu + m^2)\Delta(x) = 0$
  - ii) Is Lorentz Invariant.
  - iii) Preserves causality, $\Delta(x) = 0$ if $x^2 < 0$.

Propagators

- We would like to write the propagator in a more obviously Lorentz invariant form.
- This is given by $$\int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m^2} e^{-ip\cdot(x-y)}$$
- To connect these two expressions let us consider the integral over p_0 as a contour integral,

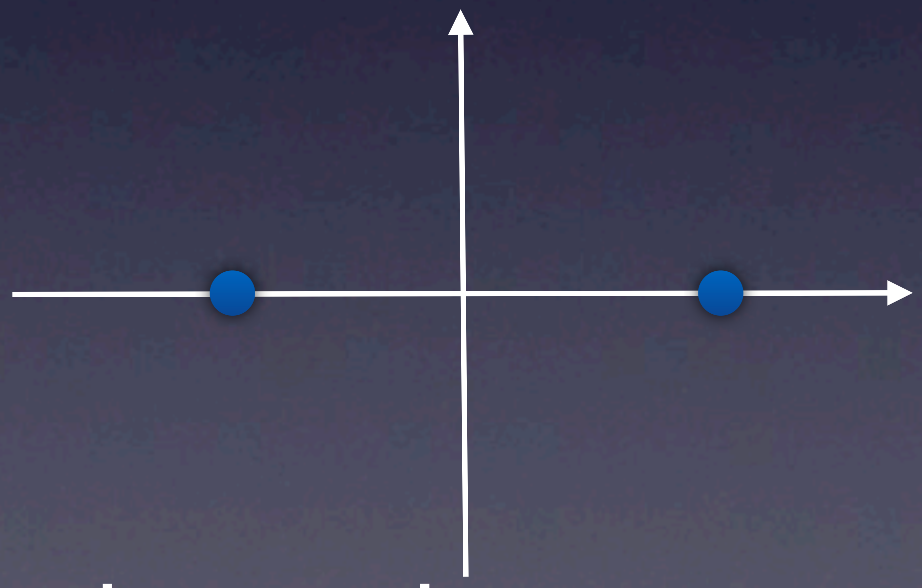
$$\int \frac{dp_0}{2\pi} \frac{1}{p^2 - m^2} e^{-ip\cdot(x-y)} = \int \frac{dp_0}{2\pi} \frac{1}{p_0^2 - \omega(\vec{p})^2} e^{-ip\cdot(x-y)}$$

$$= \int \frac{dp_0}{2\pi} \frac{1}{(p_0 - \omega(\vec{p}))(p_0 + \omega(\vec{p}))} e^{-ip_0\cdot(x^0-y^0)+i\vec{p}\cdot(\vec{x}-\vec{y})}$$

Choosing a Contour

$$\int \frac{dp_0}{2\pi} \frac{1}{(p_0 - \omega(\vec{p}))(p_0 + \omega(\vec{p}))} e^{-ip_0 \cdot (x^0 - y^0) + i\vec{p} \cdot (\vec{x} - \vec{y})}$$

- There are two poles $p_0 = \pm\omega(\vec{p})$ and so the complex plane for the two particle correlation function looks like

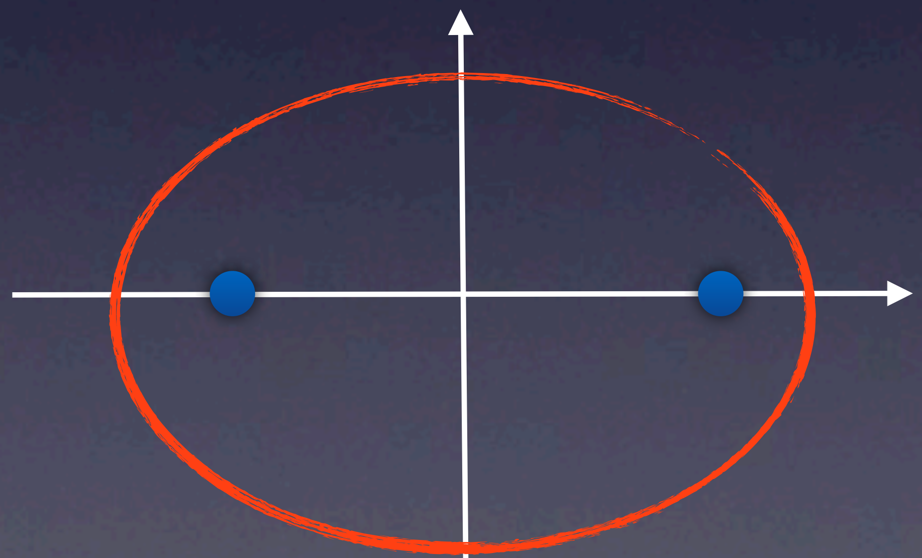


- How do we choose the contour to integrate over?

Choosing a Contour

$$\int \frac{dp_0}{2\pi} \frac{1}{(p_0 - \omega(\vec{p}))(p_0 + \omega(\vec{p}))} e^{-ip_0 \cdot (x^0 - y^0) + i\vec{p} \cdot (\vec{x} - \vec{y})}$$

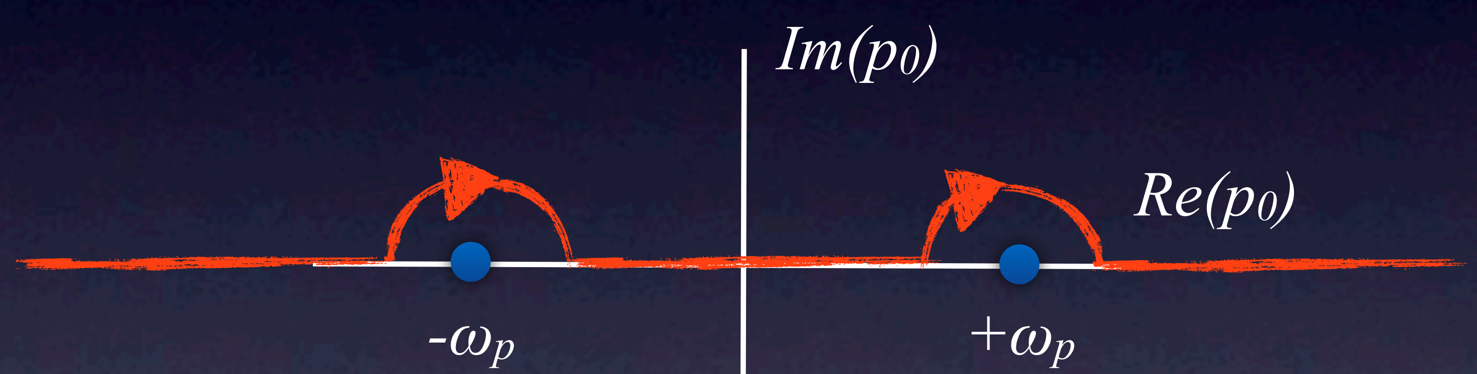
- There are two poles $p_0 = \pm\omega(\vec{p})$ and so the complex plane for the two particle correlation function looks like



- How do we choose the contour to integrate over?

Retarded Propagator

- When $x_0 < y_0$ then we will want to close the contour at -∞ and we get



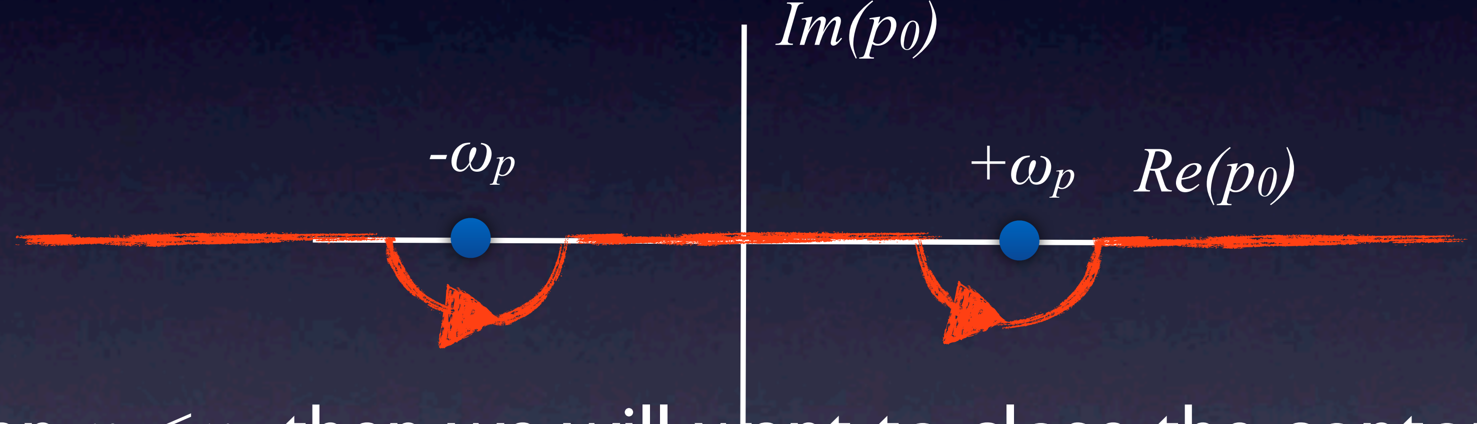
- When $x_0 < y_0$ then we will want to close the contour at +∞ and we miss both poles and get zero.
- This gives the expression for the retarded propagator,

$$\Delta_{\mathrm{ret}}(x) = \Theta(x_0)\Delta(x)$$

- This uses the state in the infinite past as its initial condition.

Advanced Propagator

- When $x_0 > y_0$ then we will want to close the contour at $+\infty$ and we get



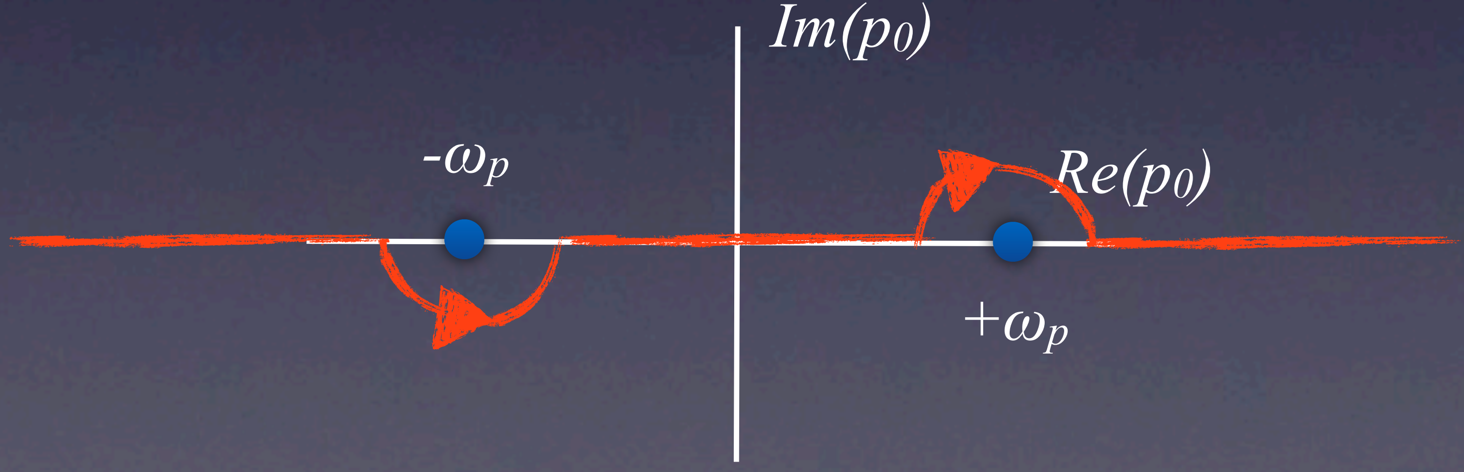
- When $x_0 < y_0$ then we will want to close the contour at $-\infty$ and we miss both poles and get zero.
- This gives the expression for the advanced propagator,

$$\Delta_{\text{adv}}(x) = \Theta(-x_0)\Delta(x)$$

- This uses the state in the infinite future as its initial condition.

Feynman Contour

- The contour we will want to use the most is the Feynman propagator.
  - When $x_0 > y_0$ we close the contour above and enclose the $-\omega_p$ pole.
  - When $x_0 < y_0$ we close the contour below and enclose the $+\omega_p$ pole.



Feynman Propagator

- This choice of contour then means we get contributions for both time orderings of the field,

$$D_F(x-y) = \Theta(x_0 - y_0)D(x-y) + \Theta(y_0 - x_0)D(y-x)$$

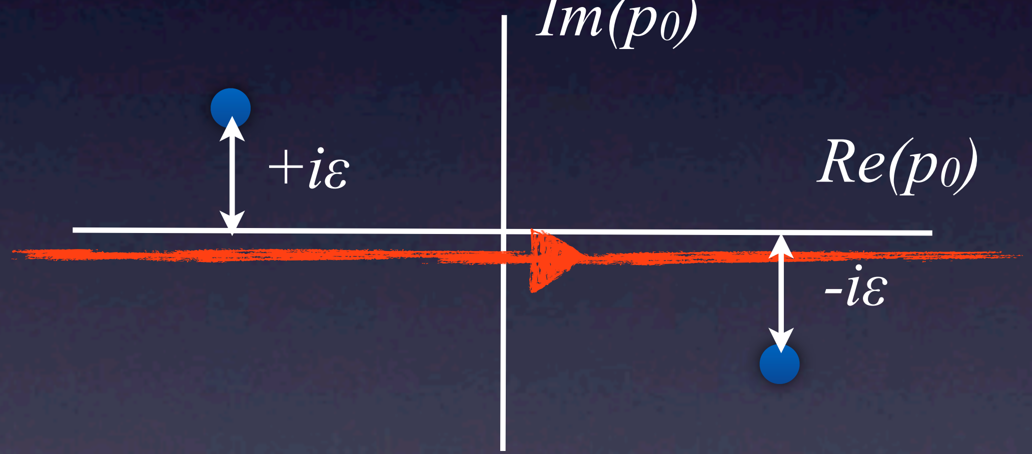
$$D_F(x-y) = \Theta(x_0 - y_0)\langle 0|\phi(x)\phi(y)|0\rangle + \Theta(y_0 - x_0)\langle 0|\phi(y)\phi(x)|0\rangle$$

- This can be written in a more compact notation using the Time Ordering operation,

$$D_F(x-y) = \langle 0|T\{\phi(x)\phi(y)\}|0\rangle$$

Feynman Propagator

- Alternatively we can generate the same effect by shifting the poles $\omega(\vec{p}) \to \omega(\vec{p}) + i\epsilon$



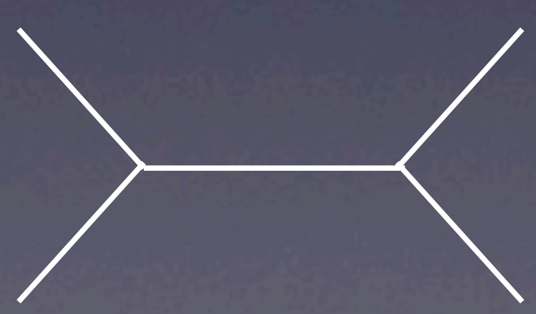
- The final form for the Feynman propagator in a non-interacting real scalar field is

$$D_F(x-y) = \langle 0|T\{\psi(x)\psi(y)\}|0\rangle = \int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2 - m^2 + i\epsilon}e^{-ip\cdot(x-y)}$$

Feynman Propagator

$$D_F(x-y) = \langle 0|T\{\phi(x)\phi(y)\}|0\rangle = \int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2-m^2+i\epsilon}e^{-ip\cdot(x-y)}$$

- This form of the propagator will be the foundation upon which we set up our computational framework.
- As preview of what is to come we will see that this will become the internal lines in a diagrammatic representation of the field computations.



Summary

- We have seen the time evolution of states and fields
  - The Schrodinger Picture.
  - The Heisenberg Picture.
- Have Shown that our quantised real scalar field theory preserves causality.
- Propagators: Retarded, Advanced and the Feynman propagator.