# *Phenomenology of hadronic collisions*

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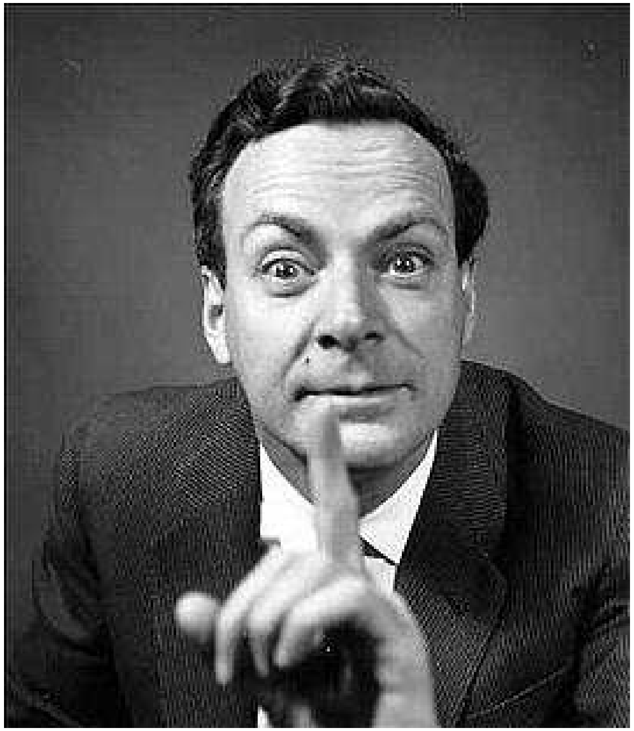
# Plan

You are now experts in computing Feynman diagrams

You (hopefully) want to know how to compute things at hadronic colliders

(the LHC in particular)



# *Disclaimer*

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- *ATLAS TDR (Detector and Physics Performance)*: 1852 pages
- *CMS TDR (2 volumes)*: 1317 pages

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- *QCD and Collider Physics*, R. K. Ellis, W. J. Stirling and B. R. Webber (447 pages)

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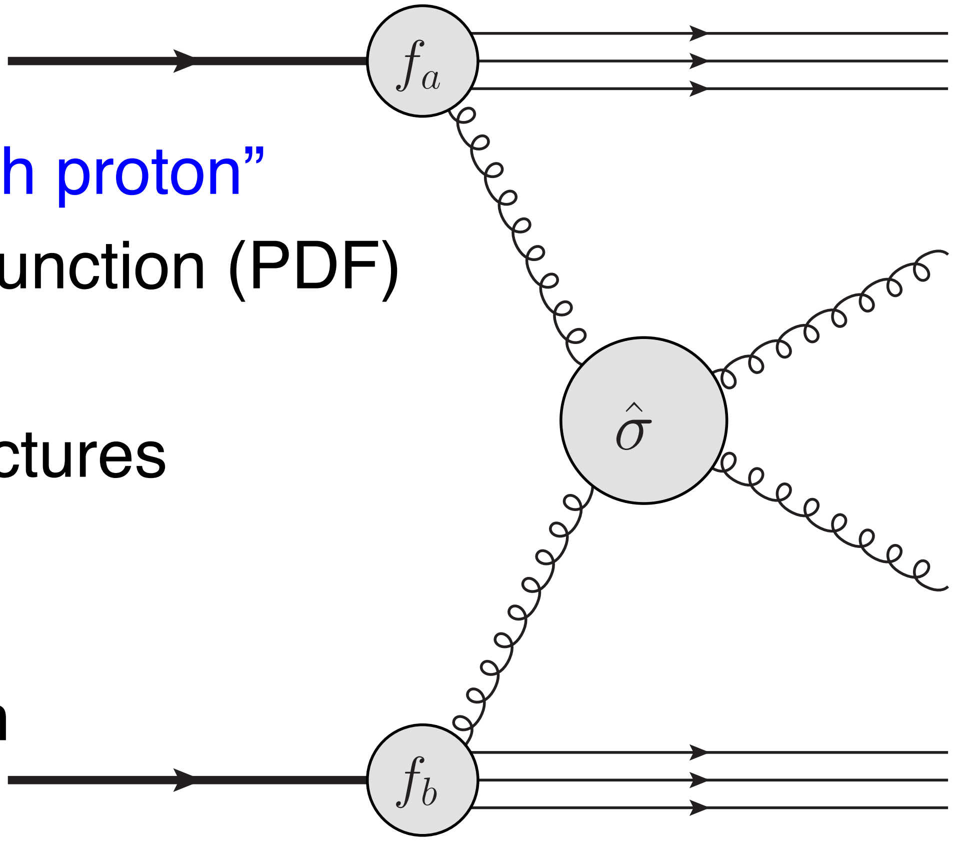
I won’t be able to cover all that in 6+2 hours!

# Plan #2

How to describe a collision between 2 hadrons?

# The very fundamental collision

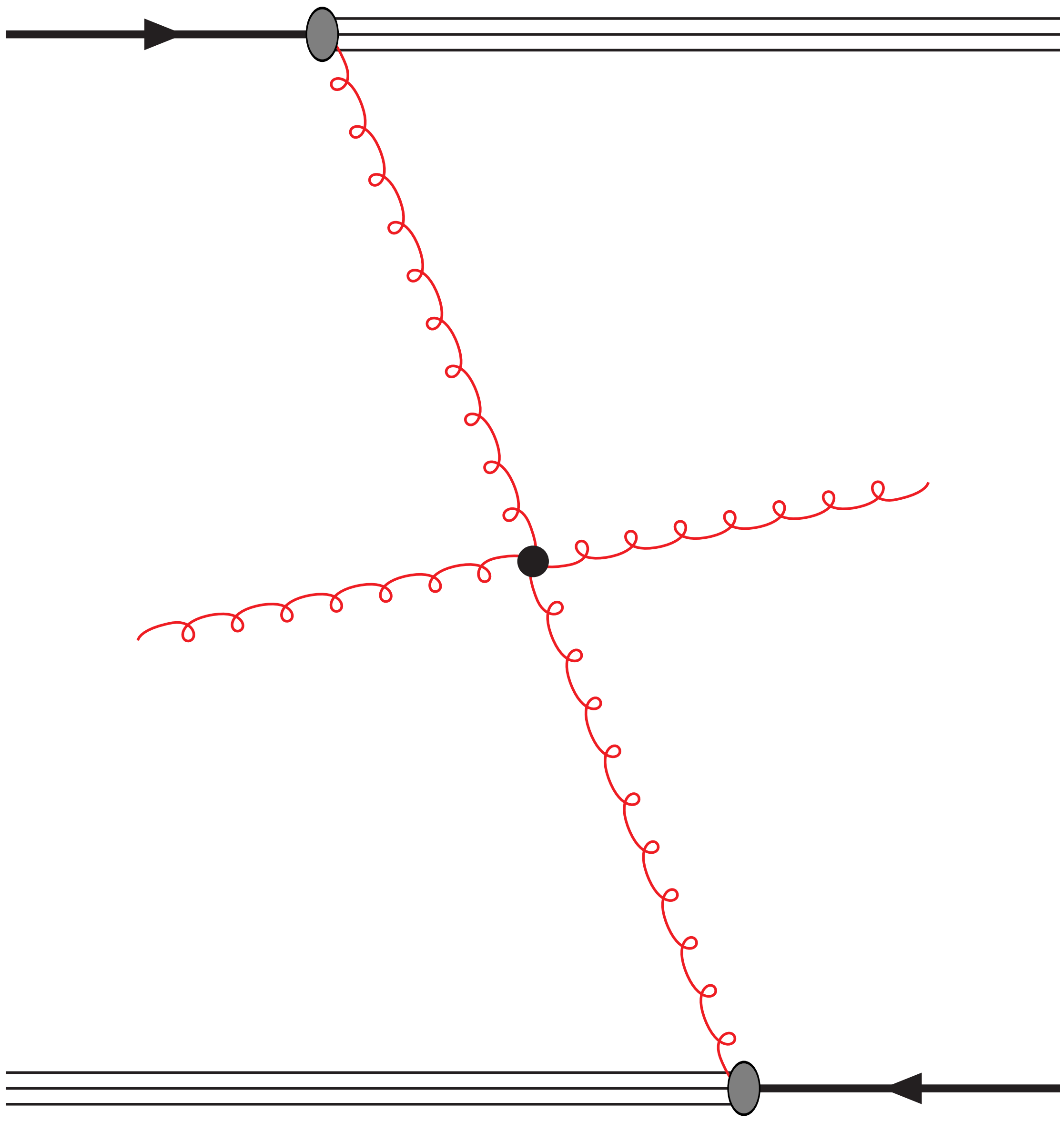
$$\sigma = f_a \otimes f_b \otimes \hat{\sigma}$$




- "take a parton out of each proton"
  $f_a \equiv$ parton distribution function (PDF)
  for quark and gluons
  a big chapter of these lectures

- hard matrix element
  perturbative computation
  Forde-Feynman rules

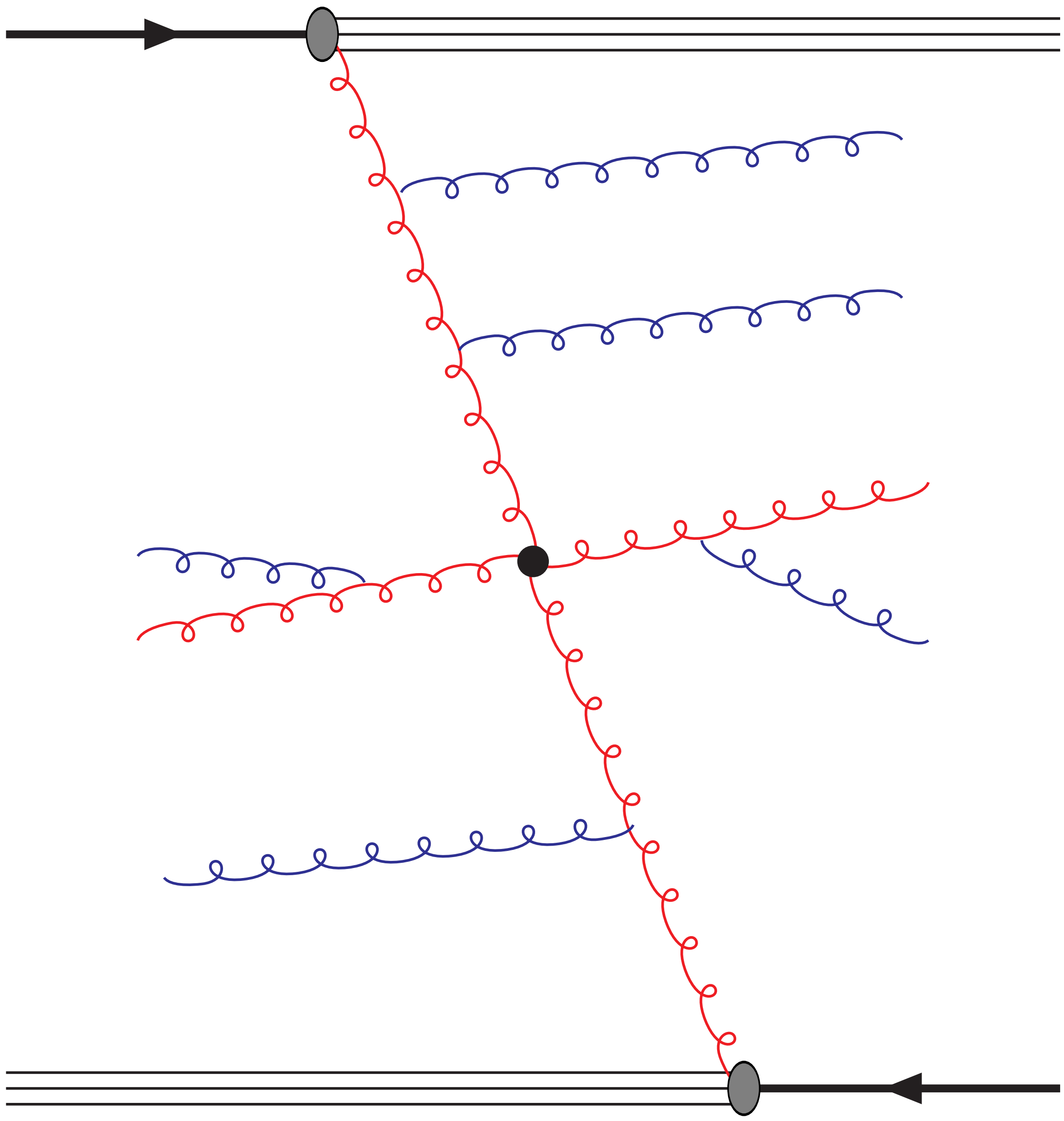
# The more realistic version



- Hard ME

  perturbative

# The more realistic version



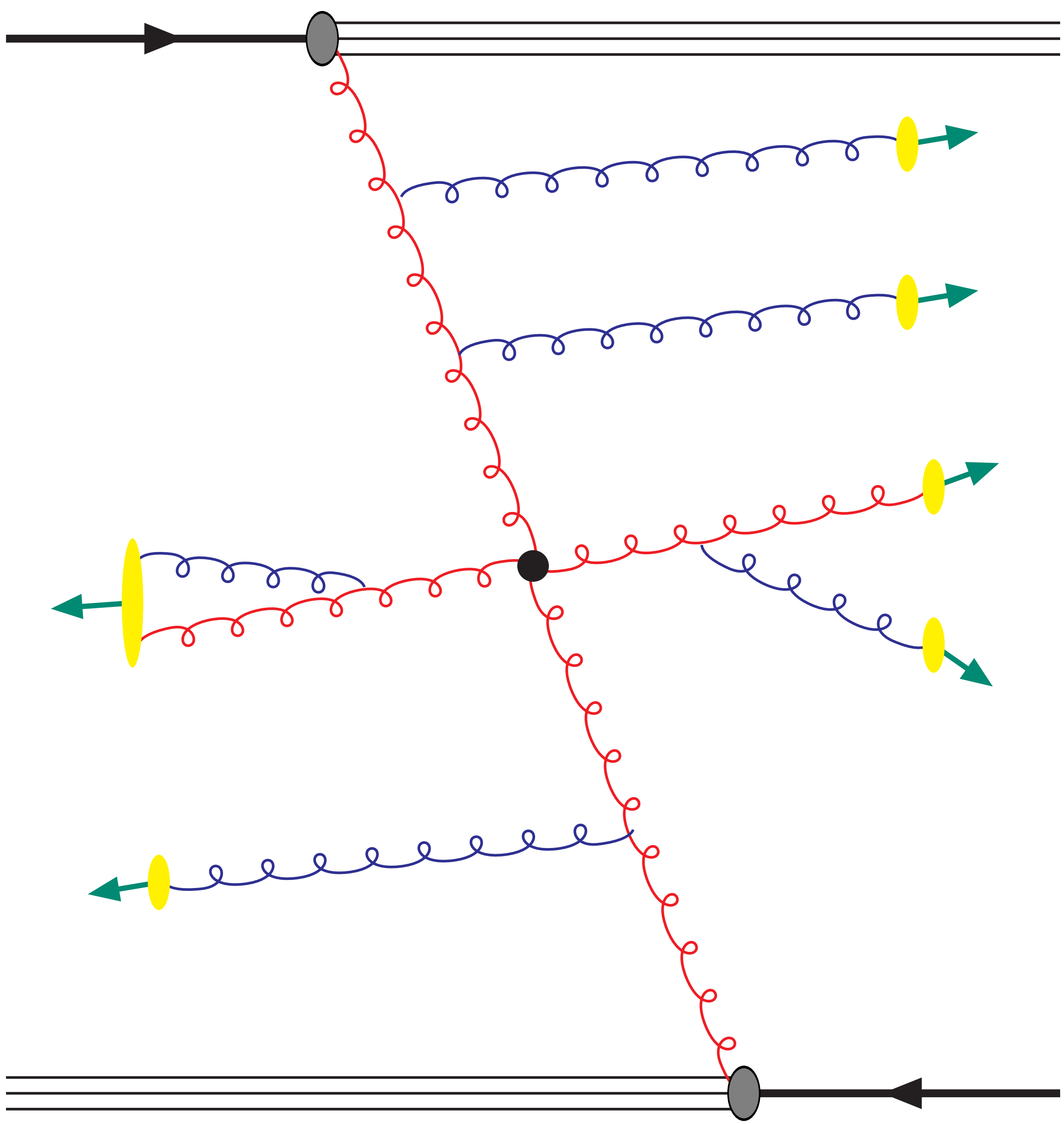
- Hard ME

  perturbative

- Parton branching

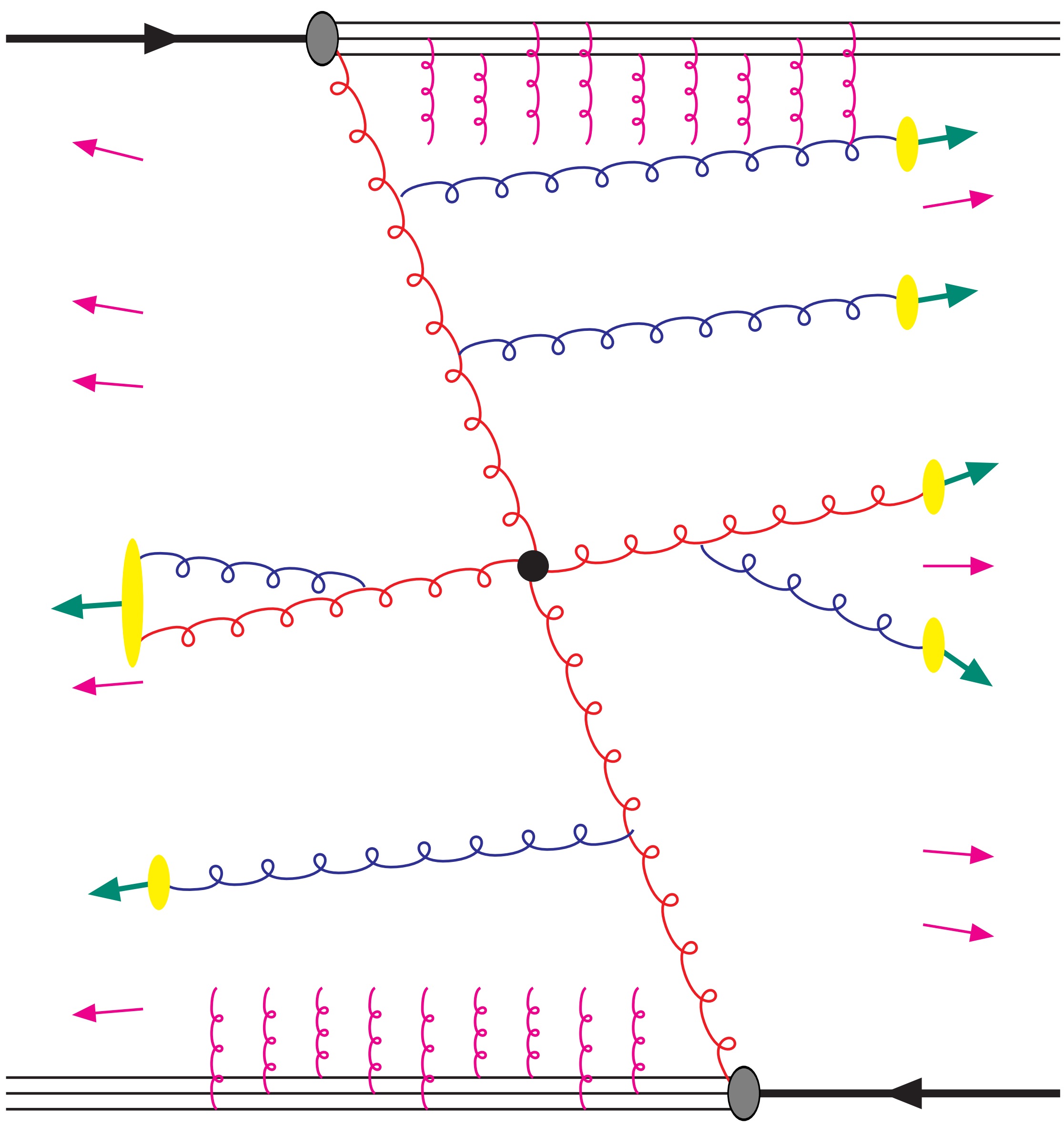
  initial+final state radiation

# The more realistic version



- Hard ME
  perturbative
- Parton branching
  initial+final state radiation
- Hadronisation
  $q, g \to$ hadrons

# The more realistic version



- Hard ME
  perturbative
- Parton branching
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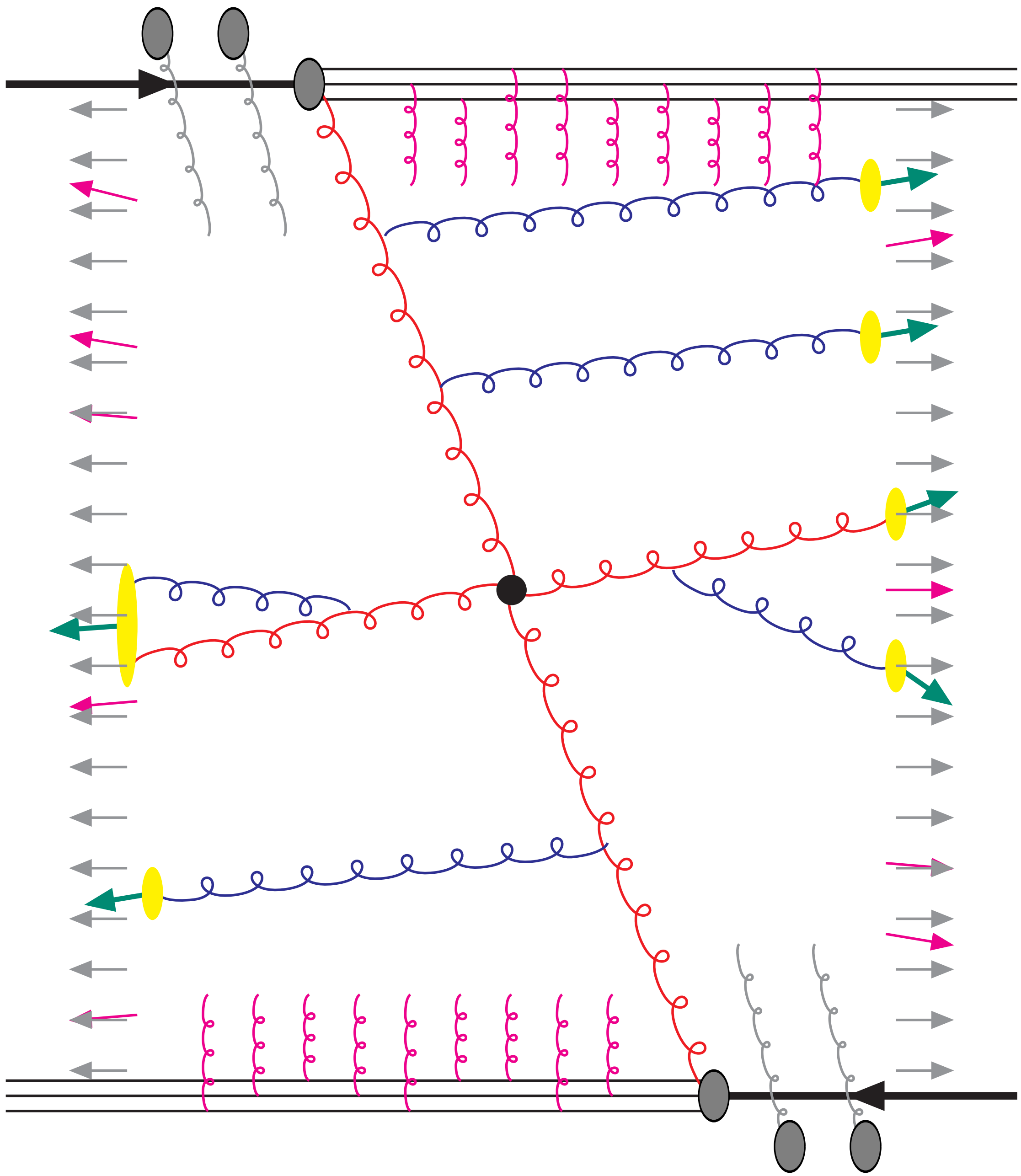
- Hadronisation
  $q, g \to$ hadrons
- Multiple interactions
  Underlying event (UE)

# The more realistic version



- Hard ME
  perturbative
- Parton branching
  initial+final state radiation
- Hadronisation
  $q, g \to$ hadrons
- Multiple interactions
  Underlying event (UE)
- Pile-up
  $\lesssim 25\ pp$ at the LHC

# Step by step...

We shall investigate those effects one by one:

- $e^+e^-$ collisions for QCD final state (and hadronisation)
- $ep$ collisions *aka* Deep Inelastic scattering (DIS) for the Parton Distribution Functions
- $pp$ collisions: put everything together
  - kinematics
  - Monte-Carlo
  - jets + various processes ($W/Z$, Higgs, top, ...)

# *Step by step...*

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# Tutorial

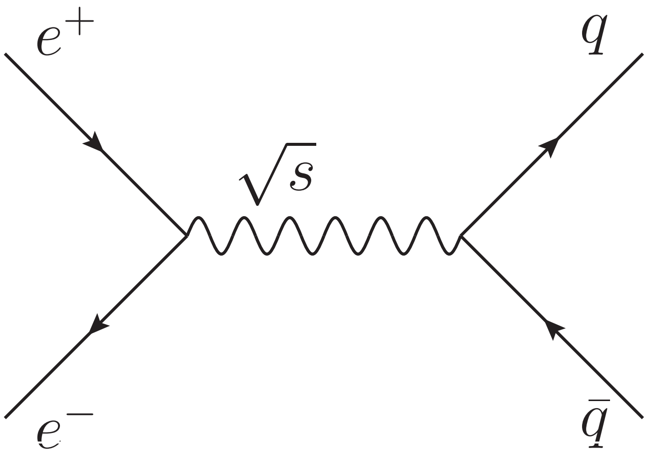
The plan is to play with Pythia 8 (the C++ version) and FastJet.
You can get them (and a few sample codes) from the link at

`http://soyez.fastjet.fr`

## $e^+e^-$ ***collisions***

# QCD final state

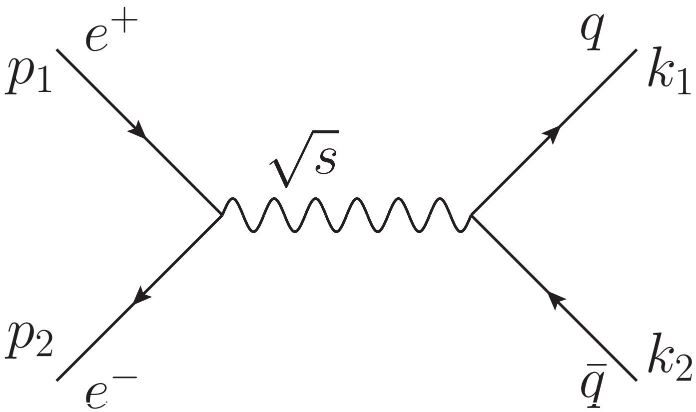
$e^+e^-$ collisions give QCD final state without initial-state/beam contamination




Useful for many QCD studies

Intermediate state can be $\gamma$ or $Z$, we only consider $\gamma$ for simplicity

# QCD final state: basic QCD



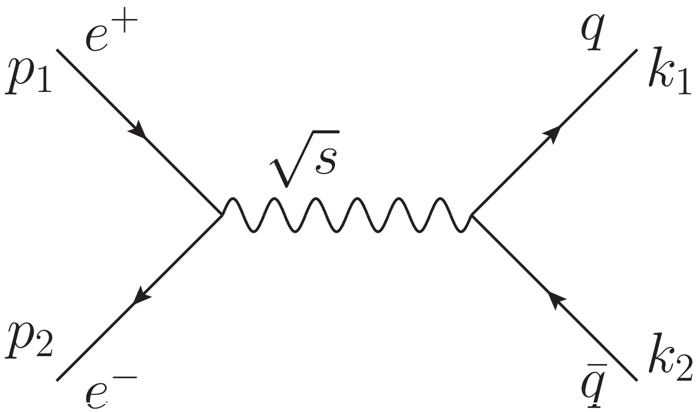

$$p_1 \equiv \frac{\sqrt{s}}{2}(0, 0, 1, 1)$$

$$p_2 \equiv \frac{\sqrt{s}}{2}(0, 0, -1, 1)$$

$$k_1 \equiv \frac{\sqrt{s}}{2}(\sin(\theta), 0, \cos(\theta), 1)$$

$$k_2 \equiv \frac{\sqrt{s}}{2}(-\sin(\theta), 0, -\cos(\theta), 1)$$

# QCD final state: basic QCD




$$p_1 \equiv \frac{\sqrt{s}}{2}(0,0,1,1)$$

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$$k_1 \equiv \frac{\sqrt{s}}{2}(\sin(\theta),0,\cos(\theta),1)$$

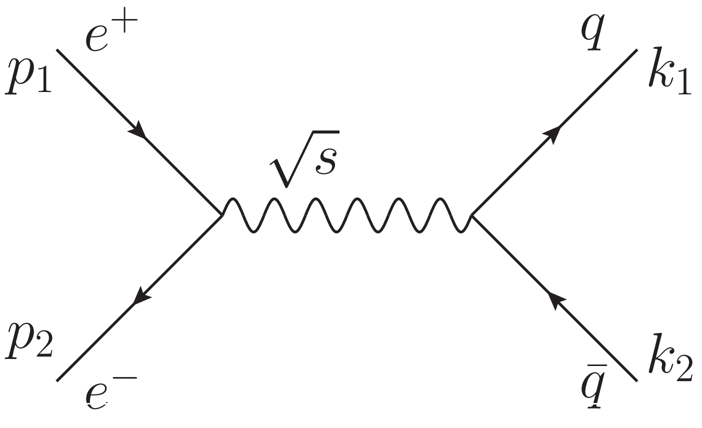
$$k_2 \equiv \frac{\sqrt{s}}{2}(-\sin(\theta),0,-\cos(\theta),1)$$

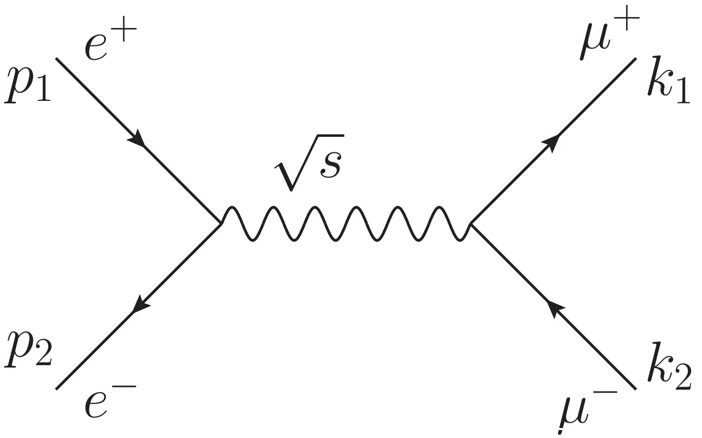
$$\frac{d\sigma}{d\cos(\theta)} = e_q^2 N_c \frac{\pi\alpha_e^2}{2s}[1+\cos^2(\theta)]$$

$$\sigma(e^+e^- \to q\bar{q}) = N_c \left(\sum_q e_q^2\right) \sigma_0$$

$$\sigma_0 = \frac{4\pi\alpha_e^2}{3s}$$

# QCD final state: basic QCD

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$$\sigma(e^+e^- \to q\bar{q}) = N_c \left(\sum_q e_q^2\right)\sigma_0 \qquad \sigma(e^+e^- \to \mu^+\mu^-) = \sigma_0$$

$$\sigma_0 = \frac{4\pi\alpha_e^2}{3s}$$

# QCD final state: basic QCD

$$R = \frac{\sigma(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \approx N_c \left( \sum_q e_q^2 \right)$$

- $u, d, s$: $R = 3\left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9}\right) = 2$
- $u, d, s, c$: $R = 3\left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9}\right) = \frac{10}{3}$
- $u, d, s, c, b$: $R = 3\left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9} + \frac{4}{9}\right) = \frac{14}{3}$

Test of

- The 3 colours in QCD ($N_c = 3$)
- The number of quark flavours

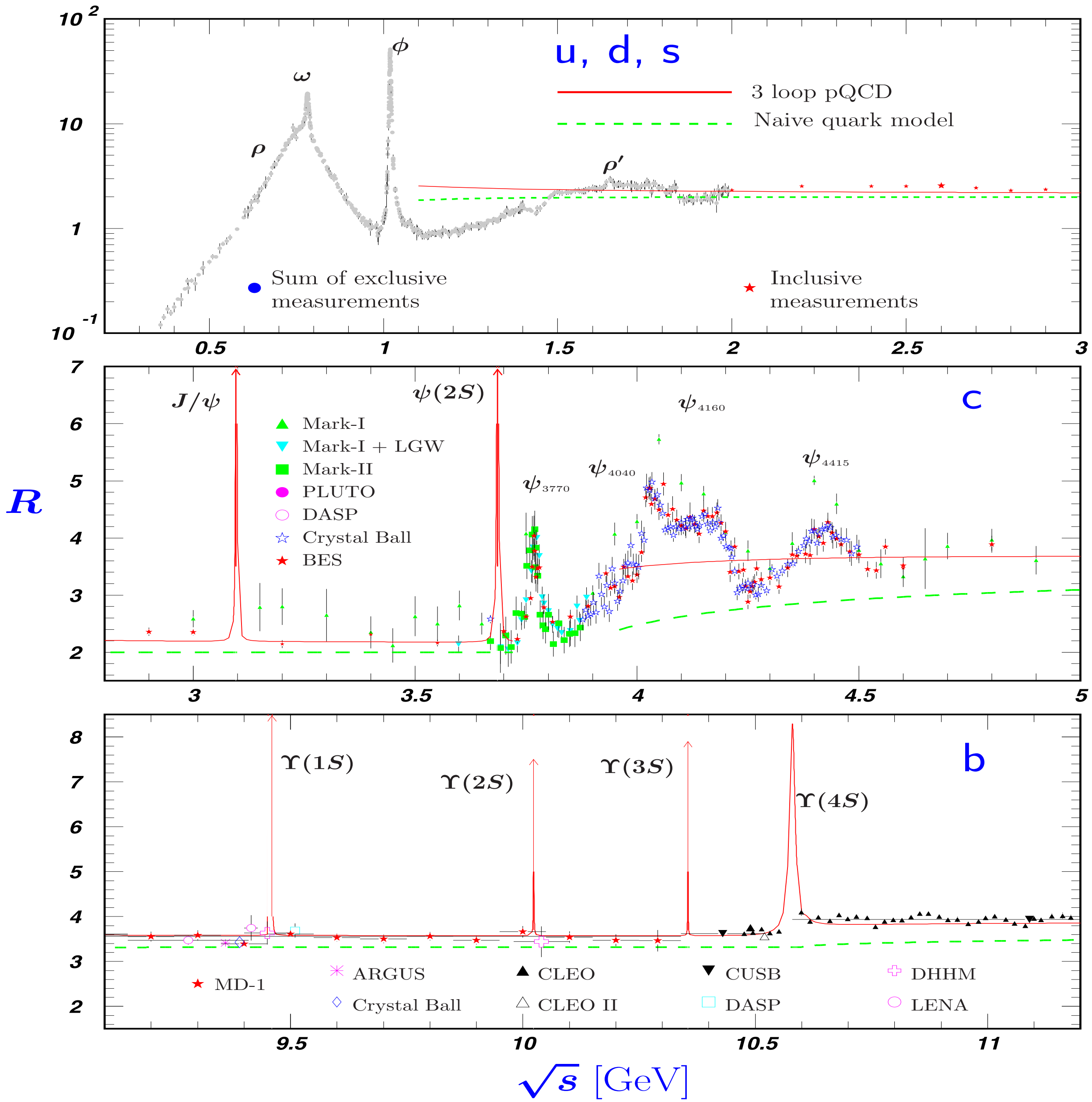
# QCD final state: basic QCD

$R$

- $u, d, s$:
- $u, d, s,$
- $u, d, s,$

Test of

- The 3
- The nu



# QCD final state: basic QCD

$R$

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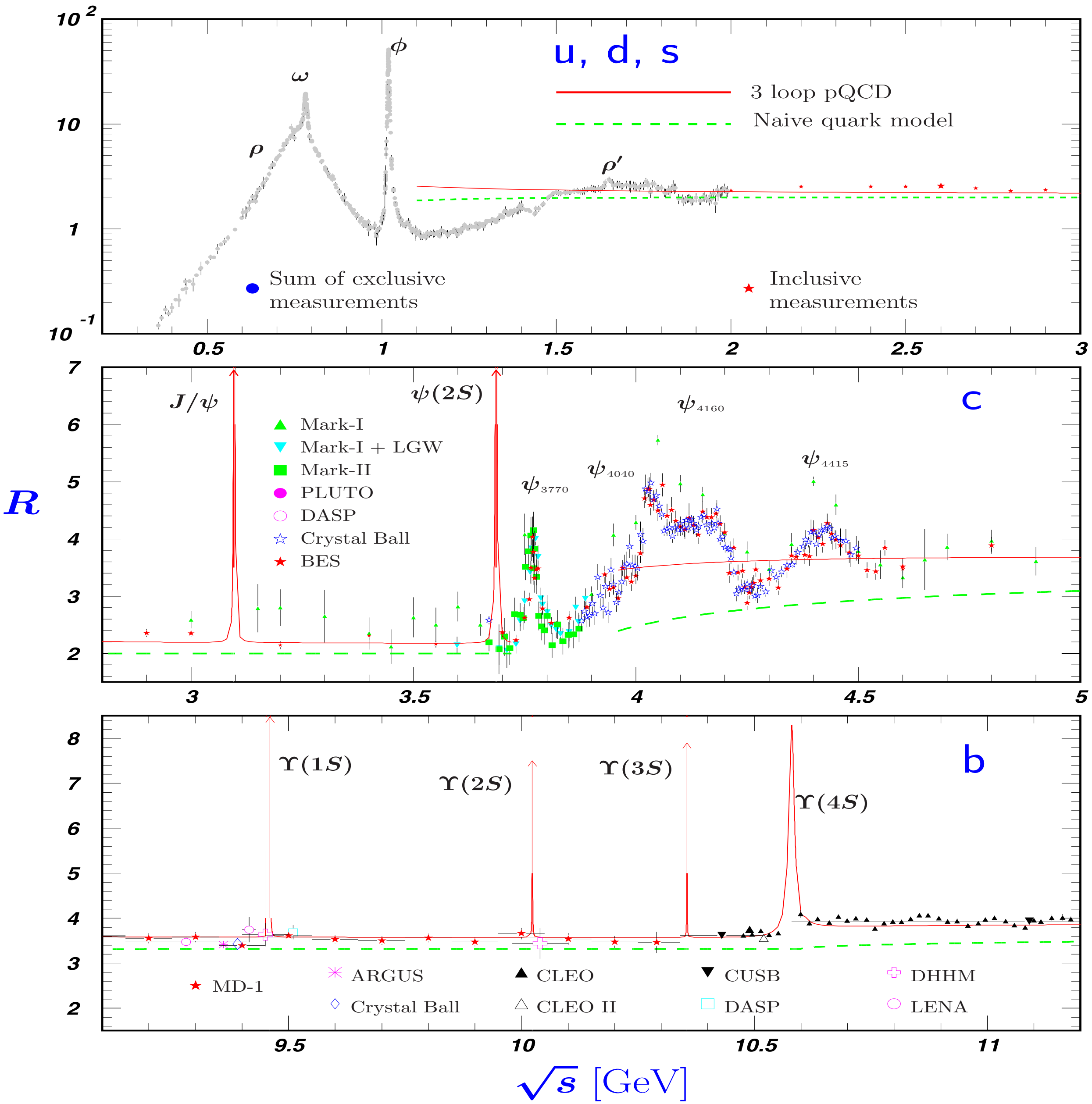
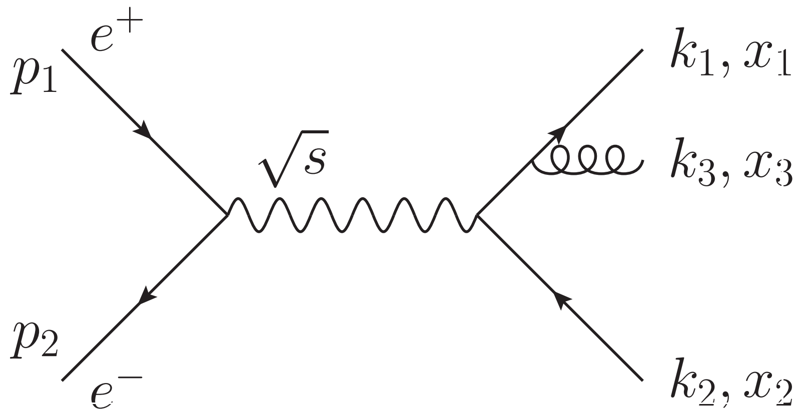



Test of

- The 3
- The nu

Q: why $\sigma(e^+e^- \to \mu^+\mu^-)$ and not $\sigma(e^+e^- \to e^+e^-)$?

# QCD final state: QCD dynamics




$3 \times (4-1) - 4 = 5$ d.o.f.

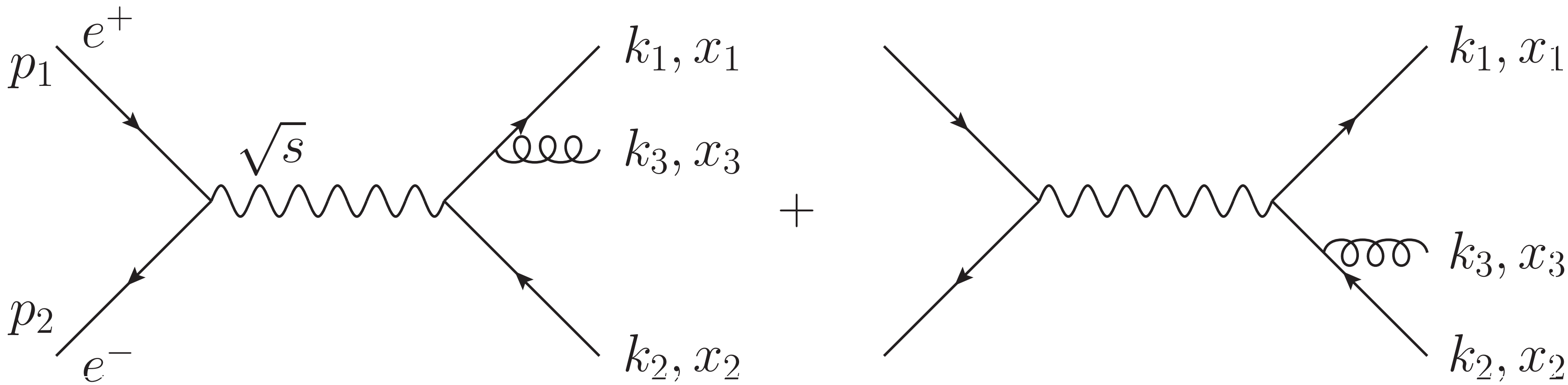
- 3 Euler angles
- $x_i = 2E_i/\sqrt{s}$, $x_1 + x_2 + x_3 = 2$
- or $\theta_{13}$, $\theta_{23}$

$$\int d\Phi_3 = \prod_{i=1}^{3} \frac{d^3 k_i}{(2\pi)^3 2E_i} (2\pi)^4 \, \delta^{(4)}(p_1 + p_2 - k_1 - k_2 - k_3)$$

$$= \frac{s}{32(2\pi)^5} \int d\alpha \, d\cos\beta \, d\gamma \, dx_1 \, dx_2$$

$$\cos(\theta_{13}) = -\frac{x_1^2 + x_3^2 - x_2^2}{2x_1 x_3} \qquad \cos(\theta_{23}) = -\frac{x_2^2 + x_3^2 - x_1^2}{2x_2 x_3}$$

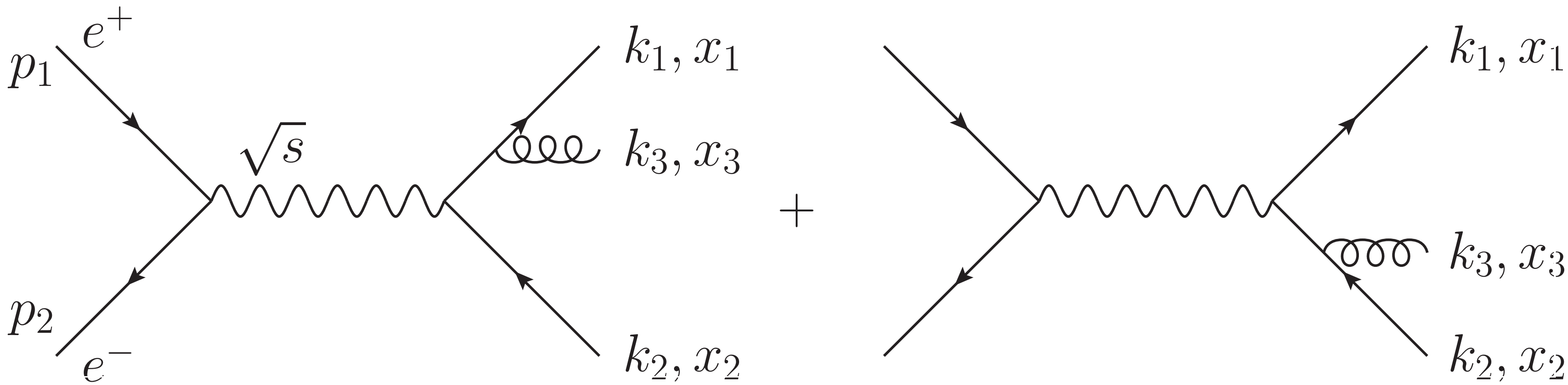
# QCD final state: QCD dynamics




$$\bar{\sum} |\mathcal{M}|^2 = 4(4\pi)^3 \alpha_e^2 \alpha_s C_F N_c \frac{(p_1.k_1)^2 + (p_1.k_2)^2 + (p_2.k_1)^2 + (p_2.k_2)^2}{s(k_1.k_3)(k_2.k_3)}$$

$$\frac{d^2\sigma}{dx_1\, dx_2} = e_q^2\, N_c\, \sigma_0\, \frac{\alpha_s C_F}{2\pi}\, \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

# QCD final state: QCD dynamics




QCD coupling

QCD colour

$e^+e^- \to q\bar{q}$

QCD dynamics

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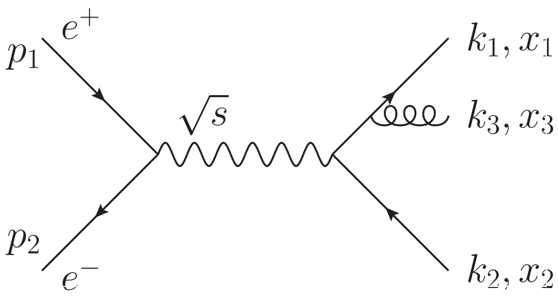


$$\bar{\sum} |\mathcal{M}|^2 \propto \frac{(p_1.k_1)^2 + (p_1.k_2)^2 + (p_2.k_1)^2 + (p_2.k_2)^2}{s(k_1.k_3)(k_2.k_3)}$$

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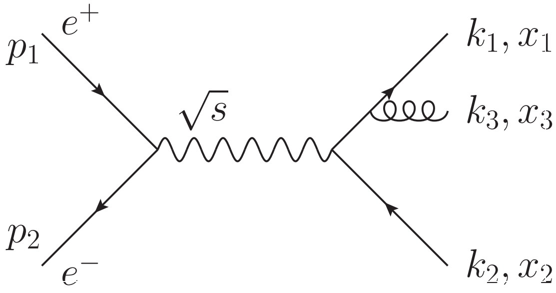
# QCD final state: QCD dynamics

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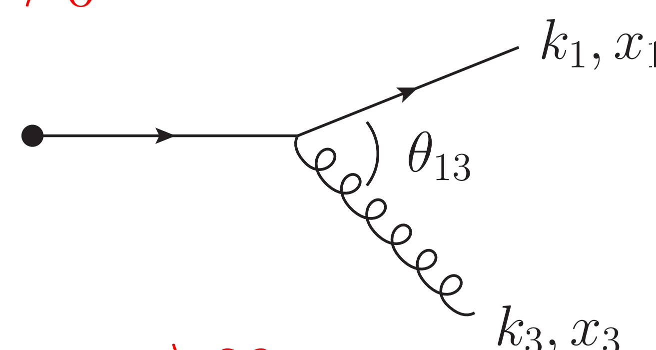
Divergent when $k_1.k_3 \to 0$ or $k_2.k_3 \to 0$




$k_1.k_3 \to 0 \Rightarrow (k_1+k_3)^2 \to 0$ *i.e.*

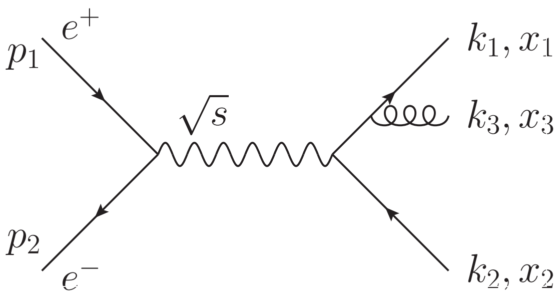
parent quark propag $= \dfrac{1}{(k_1+k_3)^2} \to \infty$

Physical origin of the divergence!

They are **infrared divergences** ($(k_1+k_3)^2 \to 0$, not $\infty$)

(one power cancelled by phase-space $\Rightarrow$ log divergence)

# QCD final state: QCD dynamics

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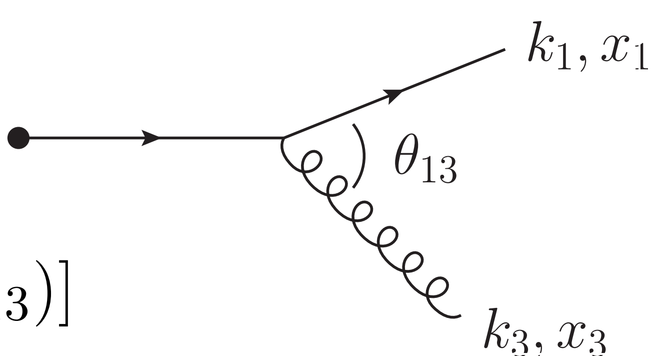
Divergent when $x_1$ (or $x_2$) $\to 1$




$$1 - x_2 = \frac{1}{2} x_1 x_3 \left[1 - \cos(\theta_{13})\right]$$

- $\theta_{13} \to 0$ (or $\theta_{23}$): collinear divergence divergence
- $x_3 \to 0$ (*i.e.* $E_g \to 0$): soft divergence

# QCD final state: coll and soft divergences

Collinear and soft divergences

- fundamental/omnipresent in QCD! (also in QED)
  we will meet them often through these lectures

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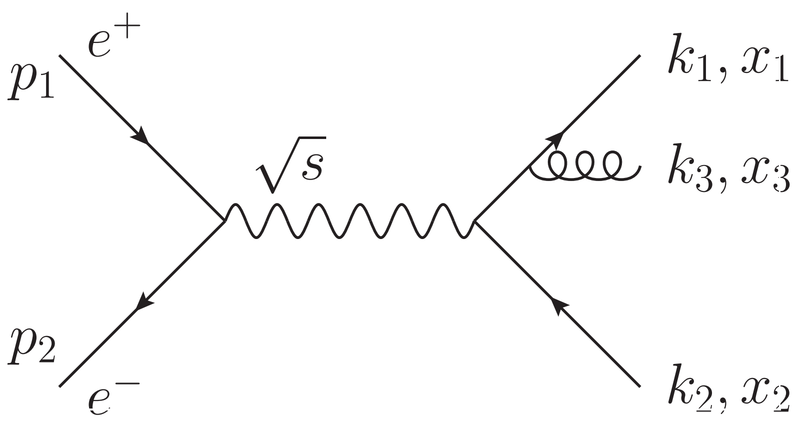
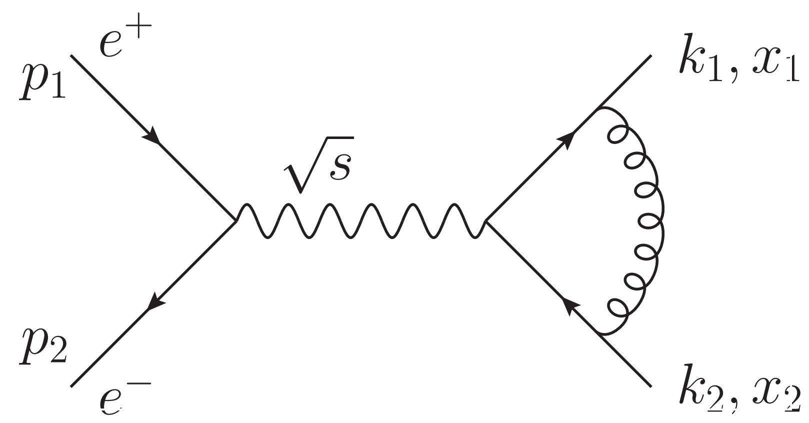
- also present for $g \to gg$ ($\neq$ QED; $C_F \to C + A$)
- cancelled by virtual corrections




Real




Virtual

# QCD final state: coll and soft divergences

Collinear and soft divergences

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- also present for $g \to gg$ ($\neq$ QED; $C_F \to C + A$)
- cancelled by virtual corrections
  Dimensional regularisation $d = 4 - 2\varepsilon$:

$$\sigma_{\mathrm{real}}^{(q\bar{q}g)} = e_q^2 N_c \sigma_0 \frac{\alpha_s C_F}{2\pi} T(\epsilon) \left[ \frac{2}{\varepsilon^2} + \frac{3}{\varepsilon} + \frac{19}{2} + \mathcal{O}(\varepsilon) \right]$$

$$\sigma_{\mathrm{virt}}^{(q\bar{q}g)} = e_q^2 N_c \sigma_0 \frac{\alpha_s C_F}{2\pi} T(\epsilon) \left[ \frac{-2}{\varepsilon^2} - \frac{3}{\varepsilon} - 8 + \mathcal{O}(\varepsilon) \right]$$

$$\sigma_{\mathcal{O}(\alpha_s)}^{(q\bar{q}g)} = e_q^2 N_c \sigma_0 \frac{3\alpha_s C_F}{4\pi} = e_q^2 N_c \sigma_0 \frac{\alpha_s}{\pi}$$

# QCD final state: coll and soft divergences

Collinear and soft divergences

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- also present for $g \to gg$ ($\neq$ QED; $C_F \to C + A$)
- cancelled by virtual corrections
- cancellation order-by-order in perturbation theory

Block-Nordsieck, Kinoshita-Lee-Nauenberg theorems

# *QCD final state: coll and soft divergences*

Collinear and soft divergences

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- also present for $g \to gg$ ($\neq$ QED; $C_F \to C + A$)
- cancelled by virtual corrections
- cancellation order-by-order in perturbation theory

Block-Nordsieck, Kinoshita-Lee-Nauenberg theorems

- Terminology issue: 'soft' divergence sometimes called 'infrared' divergence (though both soft and coll are infrared)

# QCD final state: IRC safety

Cancellation of divergence not true for any observable

Example: "number of partons in the final state", $dP/dn$

- LO ($\mathcal{O}(\alpha_s^0)$): $dP/dn = \delta(n-2)$
- NLO ($\mathcal{O}(\alpha_s^1)$):

  (i) real emission: $n = 3$

  (ii) virtual correction: $n = 2$

  $\Rightarrow$ $dP/dn = [1 - \infty\alpha_s]\delta(n-2) + \infty\alpha_s\delta(n-3)$

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  $\Rightarrow$ $dP/dn = [1 - \infty\alpha_s]\delta(n-2) + \infty\alpha_s\delta(n-3)$

Observables for which cancellation happens
are called **INFRARED-AND-COLLINEAR SAFE**
Necessary for perturbative QCD computation
to make sense!!

# QCD final state: IRC safety

Observable $\mathcal{O}$:

$$\mathcal{O} = \sum_{n=0}^{\infty} \int \underbrace{d\Psi_n(k_1, \ldots, k_n)}_{\text{phasespace}} \underbrace{\frac{d\sigma}{d\Psi_n}(k_1, \ldots, k_n)}_{\text{matrix element}} \underbrace{\mathcal{O}_n(k_1, \ldots, k_n)}_{\text{observable}}$$

- IR safety: "adding a soft particle does not change $\mathcal{O}$"

$$\mathcal{O}_{n+1}(k_1, \ldots, k_n, k_{n+1}) \stackrel{k_{n+1} \to 0}{=} \mathcal{O}_n(k_1, \ldots, k_n)$$

- Collinear safety: "a collinear splitting does not change $\mathcal{O}$"

$$\mathcal{O}_{n+1}(k_1, \ldots, \lambda k_n, (1-\lambda)k_n) = \mathcal{O}_n(k_1, \ldots, k_n)$$

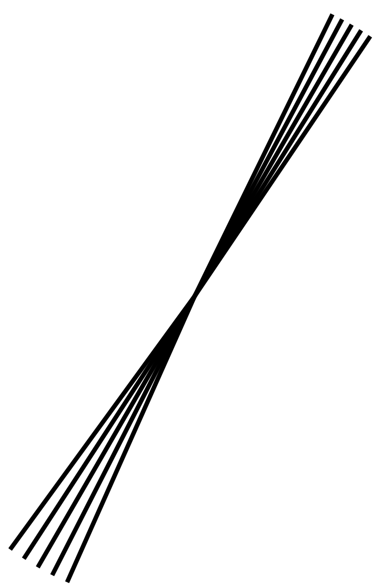
for $0 < \lambda < 1$

# QCD final state: IRC safety

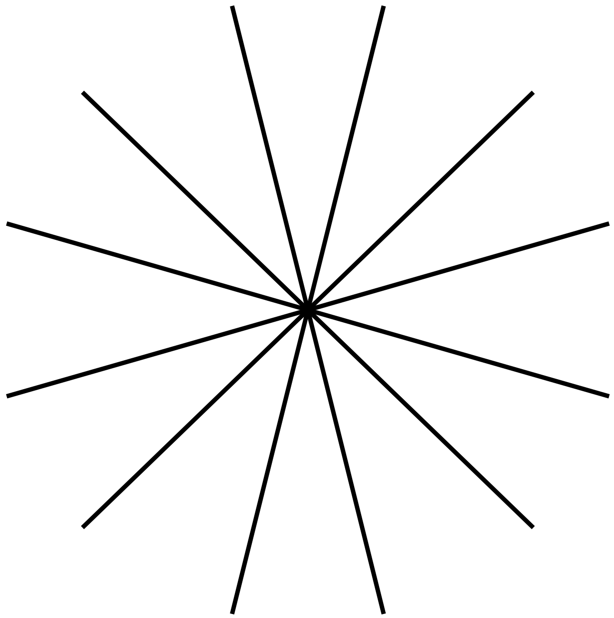
## Example #1: event-shapes in $e^+e^-$

*thrust, sphericity, thrust major, thrust minor, ...*

Thrust: $$T_n = \max_{|\vec{u}|=1} \frac{\sum_{i=0}^{n} |\vec{k}_i.\vec{u}|}{\sum_{i=0}^{n} |\vec{k}_i|}$$



pencil-like: $T \lesssim 1$



spherical: $T \gtrsim 1/2$

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- the thrust is infrared safe: for $k_{n+1} \to 0$

$$T_{n+1} = \max_{|\vec{u}|=1} \frac{\sum_{i=0}^{n+1} |\vec{k}_i.\vec{u}|}{\sum_{i=0}^{n+1} |\vec{k}_i|} = \max_{|\vec{u}|=1} \frac{\sum_{i=0}^{n} |\vec{k}_i.\vec{u}|}{\sum_{i=0}^{n} |\vec{k}_i|} = T_n$$

- the thrust is collinear safe

$$0 < \lambda < 1 \Rightarrow \begin{cases} |\vec{u}.(\lambda\vec{k} + (1-\lambda)\vec{k})| = |\vec{u}.\vec{k}| \\ |\lambda\vec{k} + (1-\lambda)\vec{k}| = |\vec{k}| \end{cases}$$

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## Example #1: event-shapes in $e^+e^-$

*thrust, sphericity, thrust major, thrust minor, ...*

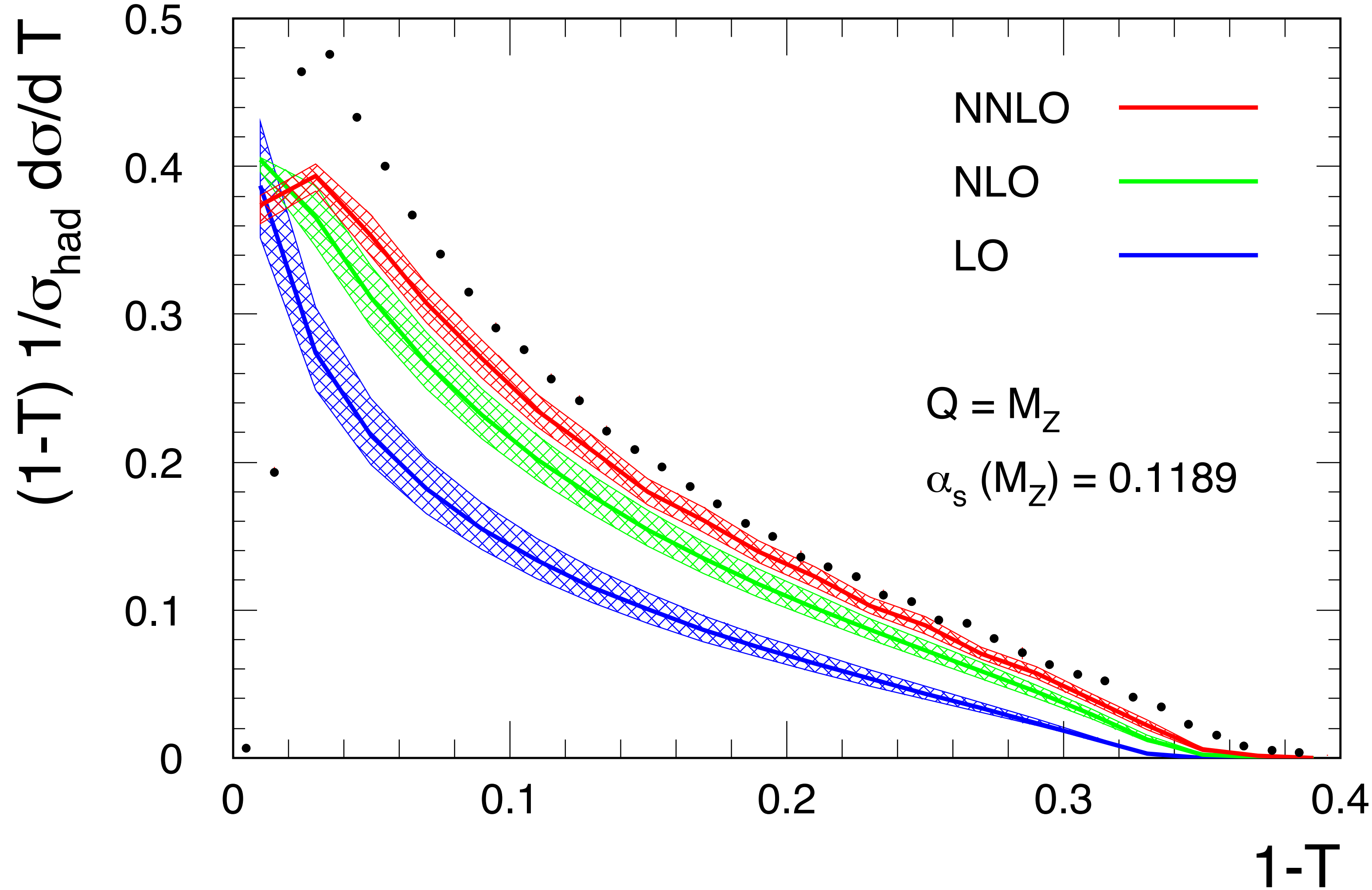
Thrust: $$T_n = \max_{|\vec{u}|=1} \frac{\sum_{i=0}^{n} |\vec{k}_i . \vec{u}|}{\sum_{i=0}^{n} |\vec{k}_i|}$$

Computation in perturbative QCD (from the matrix element given earlier)

$$\frac{1}{\sigma}\frac{d\sigma}{dT} = \frac{\alpha_s C_F}{2\pi}\left[\frac{2(2-3T+3T^2)}{T(1-T)}\log\left(\frac{2T-1}{1-T}\right) - \frac{3(2-T)(3T-2)}{1-T}\right]$$

- Allows for test of QCD (*e.g.* at LEP)
- “log” is a reminiscence from the soft and collinear divergence

# *Thrust*



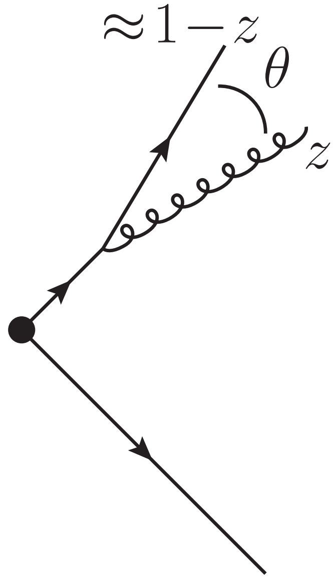

comparison with LEP data: peaked at $T = 1$

# $e^+e^-$: QCD divergences

Typical behaviour of divergences:

- Collinear limit:

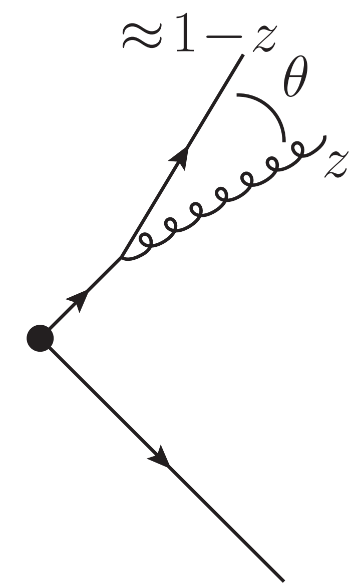
$$\frac{1}{\sigma_0} d\sigma \approx \underbrace{\frac{\alpha_s}{2\pi} \frac{1+(1-z)^2}{z}}_{\text{splitting proba}} \underbrace{\frac{d\theta^2}{\theta^2}}_{\text{coll.div}}$$




For different situations (different parton types), the branching probability changes but the $d\theta/\theta$ is generic!

# $e^+e^-$: QCD divergences

Typical behaviour of divergences:

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- Soft limit:

$$d\sigma_{q\bar{q}g} = d\sigma_{q\bar{q}} \frac{\alpha_s C_F}{\pi^2} \frac{(k_1.k_2)}{(k_1.k_3)(k_2.k_3)} d^4k_3\, \delta(k^2) \propto \frac{dE_3}{E_3} \propto \frac{dz}{z}$$

Antenna formula — soft-gluon emission

# $e^+e^-$: QCD divergences

Frequent appearance in computations:

Both soft and collinear divergences are logarithmic
$\Rightarrow$ the emission of a gluon comes with a factor $\alpha_s \log$

Example:

soft emissions for the thrust : $\alpha_s \log(1-T)$

At some point, $\alpha_s \log \sim 1$ *i.e.* NLO$\sim$LO in the perturbative series
$\Rightarrow$ At order $n$, we will have $\alpha_s^n \log^n$ all of the same order
$\Rightarrow$ ALL have to be considered: **resummation**

# *Other interests in* $e^+e^-$ *collisions*

- Fragmentation functions

  "parton $\to$ hadron transition", $D_{p/\pi}(z, p_t)$

- Hadronisation

  *e.g.* Lund strings

- Jets

  Collinear divergence $\longrightarrow$ a parton develops into a bunch of collimated particles

We will postpone (part of) this to the "hadronic collisions" chapter

# $e^+e^-$: Summary

- $e^+e^-$ collisions: good framework to test QCD (final state)
- emission of a gluon has 2 divergences: soft and collinear
  - cancel between “real” and “virtual” daigrams
  - ... provided the observable is IRC safe
  - give rise to “logarithms” in perturbative computations
  - ... resummed to all orders when $\alpha_s \log \sim 1$
  - ... done analytically or by parton cascade MC
- collinear divergence+parton branching $\rightarrow$ jets

***Time for questions!***

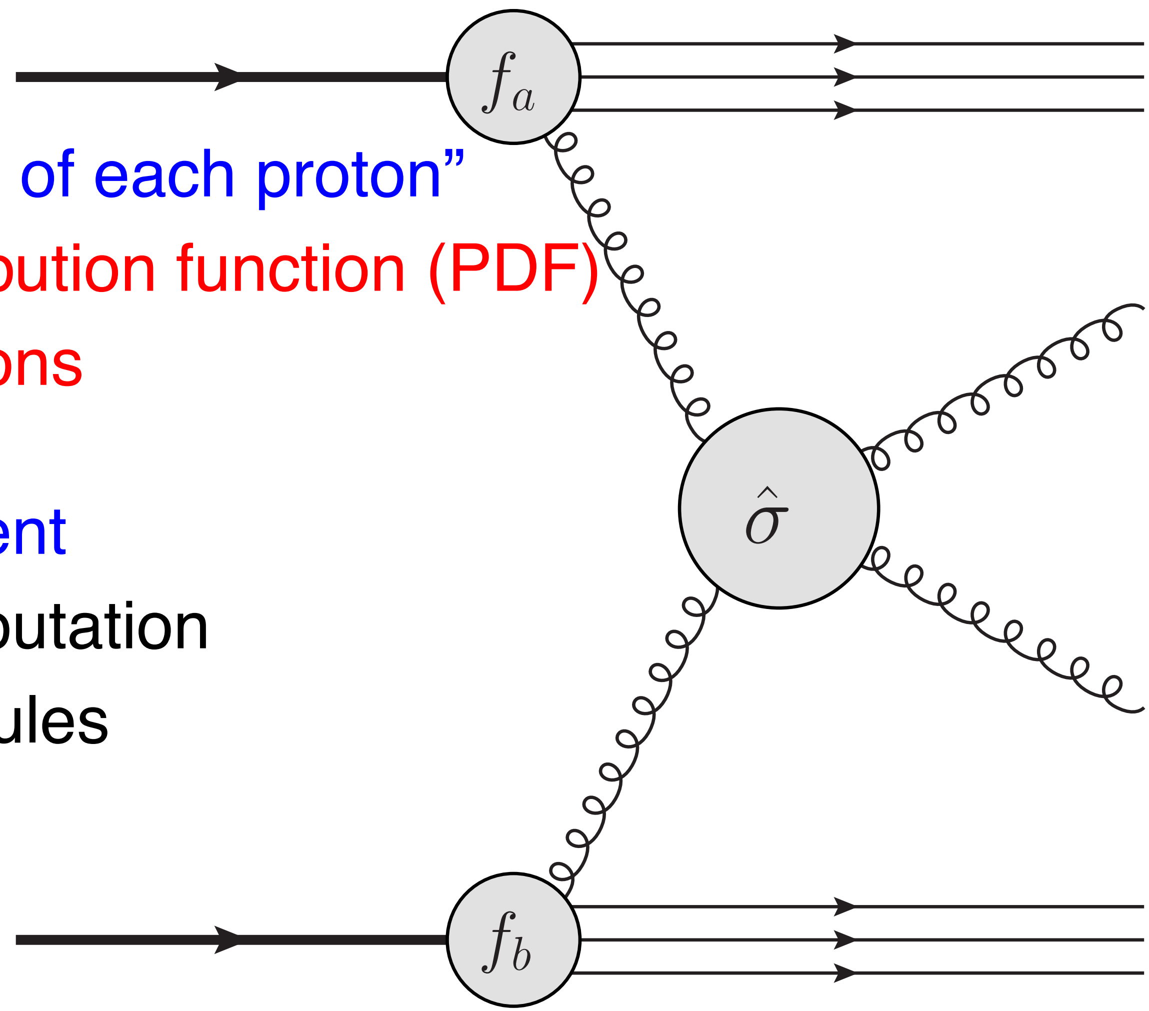
***<interlude hadronic collisions>***

***kinematics***

***jets***

# The very fundamental collision

$$\sigma = f_a \otimes f_b \otimes \hat{\sigma}$$




- "take a parton out of each proton"
  $f_a \equiv$ parton distribution function (PDF)
  for quark and gluons

- hard matrix element
  perturbative computation
  Forde-Feynman rules

# Kinematics

## Incoming partons:

$$
\begin{aligned}
p_1 &\equiv x_1\frac{\sqrt{s}}{2}(0,0,\ \ \ 1,1) \\
p_2 &\equiv x_2\frac{\sqrt{s}}{2}(0,0,-1,1)
\end{aligned}
$$

- carry a fraction of the beam's (longitudinal) momentum
- Energy$^2$ in the hard collision: $(p_1+p_2)^2 = x_1 x_2 s \leq s$
- the partonic centre-of-mass is shifted/boosted compared to the lab/$pp$ centre-of-mass
  $\Rightarrow$ need variables (longitudinally) boost-invariant

# Kinematics

## Final-state particles: commonly-used variables

$$k \equiv (k_x, k_y, k_z, E) \equiv E(\sin(\theta)\cos(\phi), \sin(\theta)\sin(\phi), \cos(\theta), 1)$$

- $E$ and $\theta$ are not suited!

# *Kinematics*

## Final-state particles: commonly-used variables

- Transverse plane
  - azimuthal angle $\phi$
  - transverse momentum $p_t = \sqrt{p_x^2 + p_y^2}$

# *Kinematics*

## Final-state particles: commonly-used variables

- Transverse plane
  - azimuthal angle $\phi$
  - transverse momentum $p_t = \sqrt{p_x^2 + p_y^2}$
- Longitudinal variable
  - Rapidity: $y = \frac{1}{2} \log \left( \frac{E+p_z}{E-p_z} \right)$

Boost: $$y \quad \to \quad \frac{1}{2} \log \left( \frac{\gamma(E - \beta p_z) + \gamma(p_z - \beta E)}{\gamma(E - \beta p_z) - \gamma(p_z - \beta E)} \right)$$
$$= \quad \frac{1}{2} \log \left( \frac{\gamma(1-\beta)(E+p_z)}{\gamma(1+\beta)(E-p_z)} \right) = y + \frac{1}{2} \log \left( \frac{(1-\beta)}{(1+\beta)} \right)$$

not boost-invariant itself but $\Delta y = y_2 - y_1$ is ($\Delta\theta$ is not)

# Kinematics

## Final-state particles: commonly-used variables

- Transverse plane
  - azimuthal angle $\phi$
  - transverse momentum $p_t = \sqrt{p_x^2 + p_y^2}$
- Longitudinal variable
  - Rapidity: $y = \frac{1}{2}\log\left(\frac{E+p_z}{E-p_z}\right)$

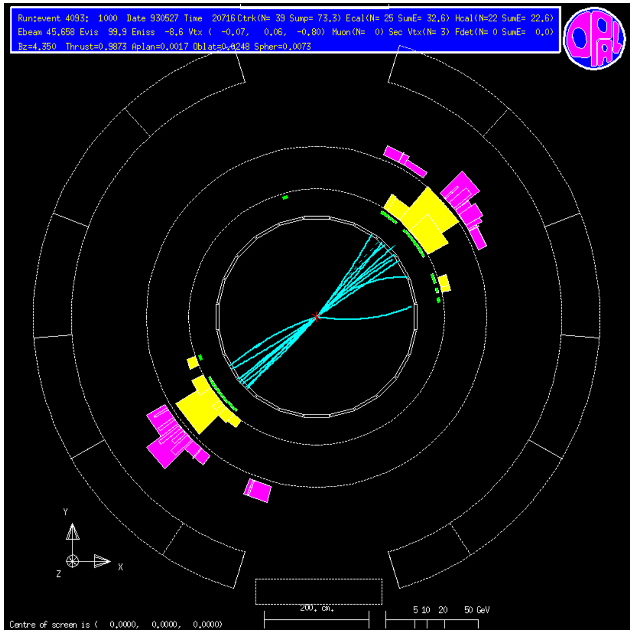
    $$k \equiv (k_t \cos(\phi), k_t \sin(\phi), m_t \sinh(y), m_t \cosh(y))$$

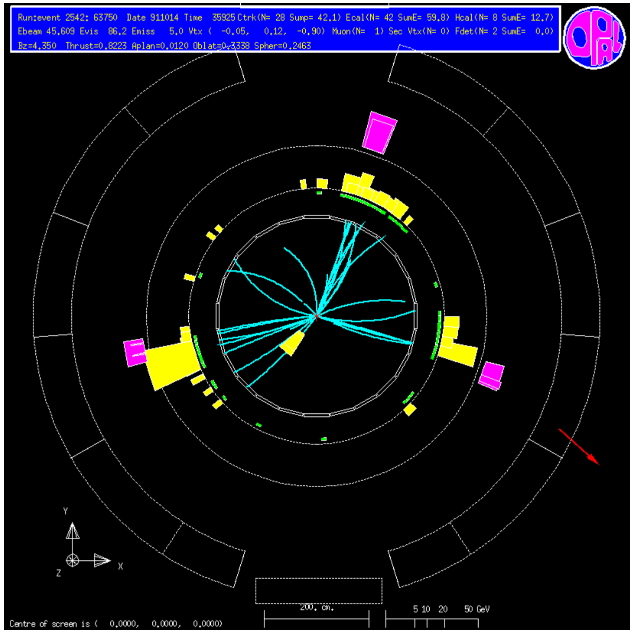
    Transverse mass: $m_t^2 = k_t^2 + m^2$
  - Pseudo-rapidity: $\eta = \frac{1}{2}\log\left(\tan(\theta/2)\right)$

    $\Delta\eta$ boost-invariant if massless
  - For massless particles: $y = \eta$

# Jets

- We have seen in the $e^+e^-$ studies (thrust) that the final state is pencil-like

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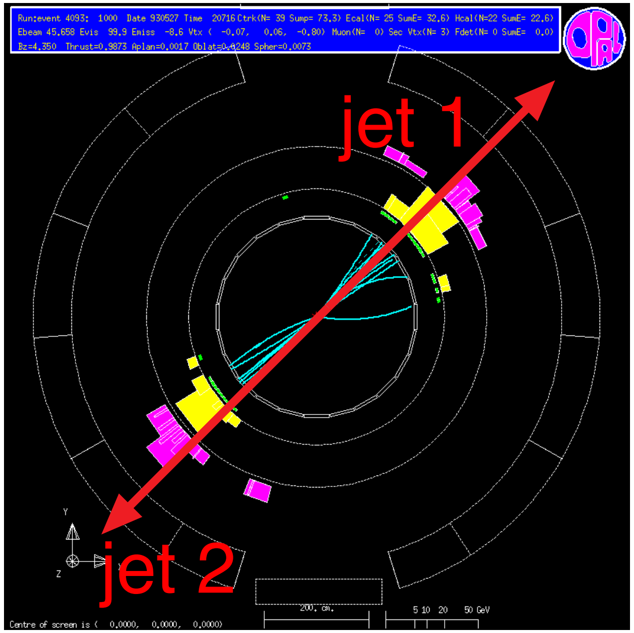

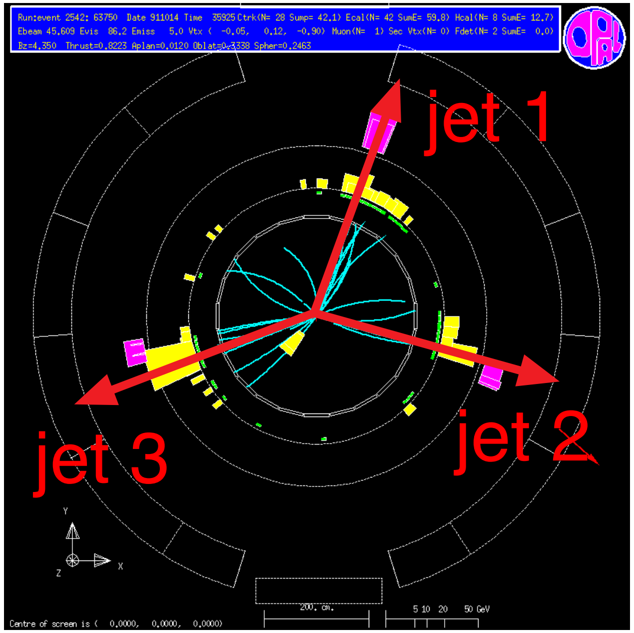


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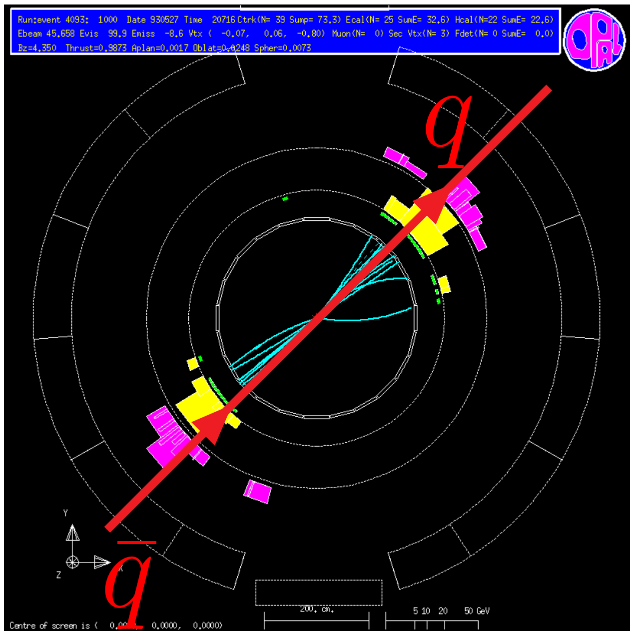


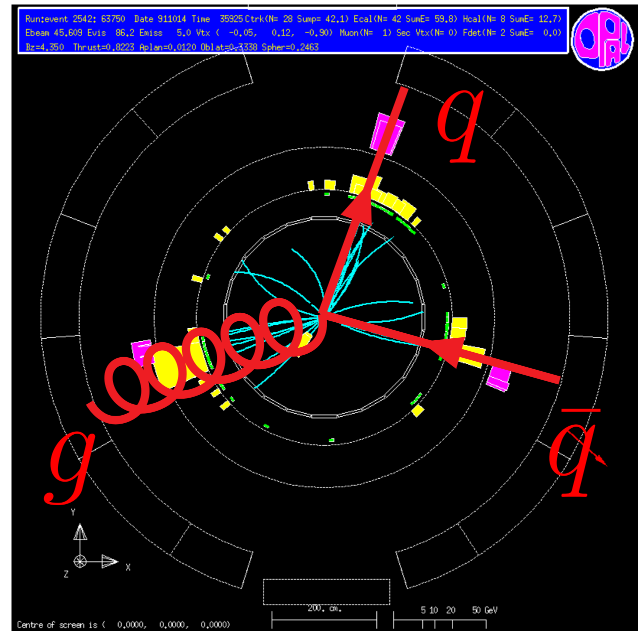


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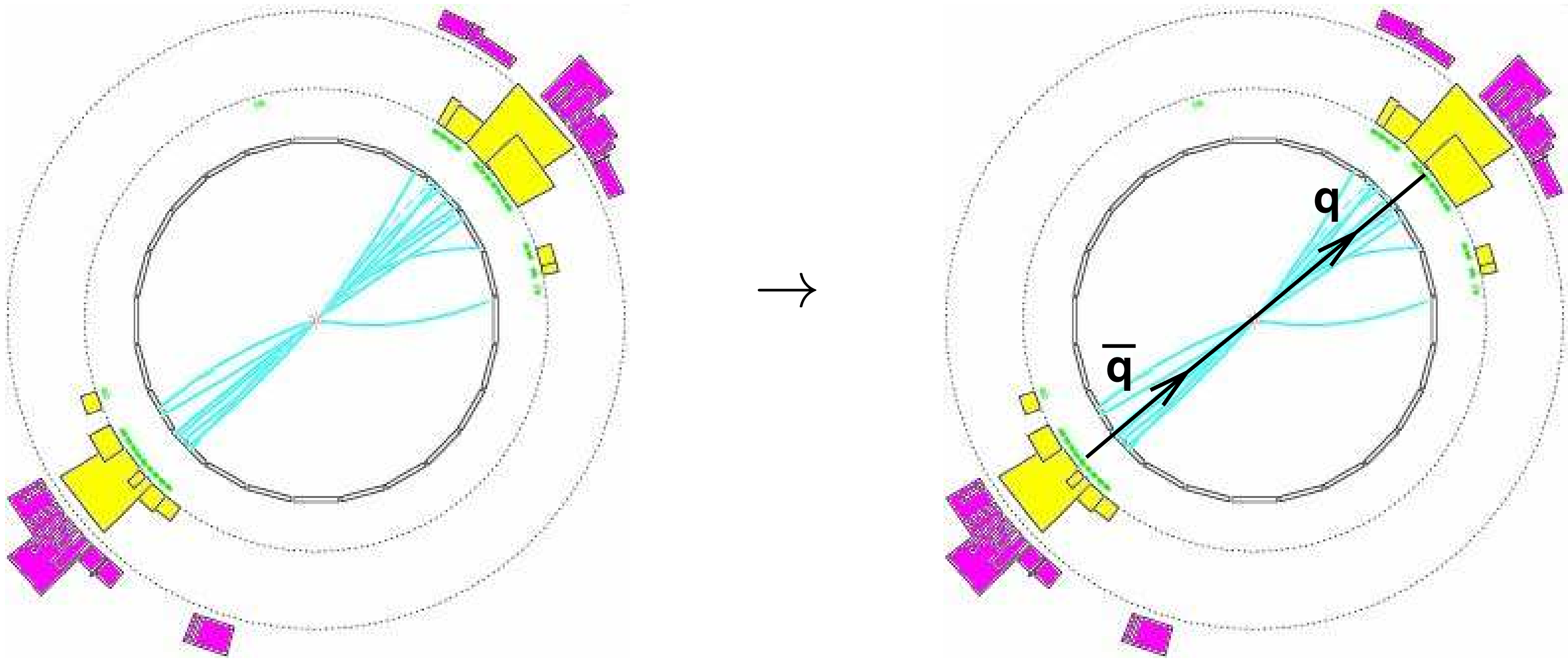
# Jets

“Jets” $\equiv$ bunch of collimated particles $\cong$ hard partons

obviously 2 jets



# Jets

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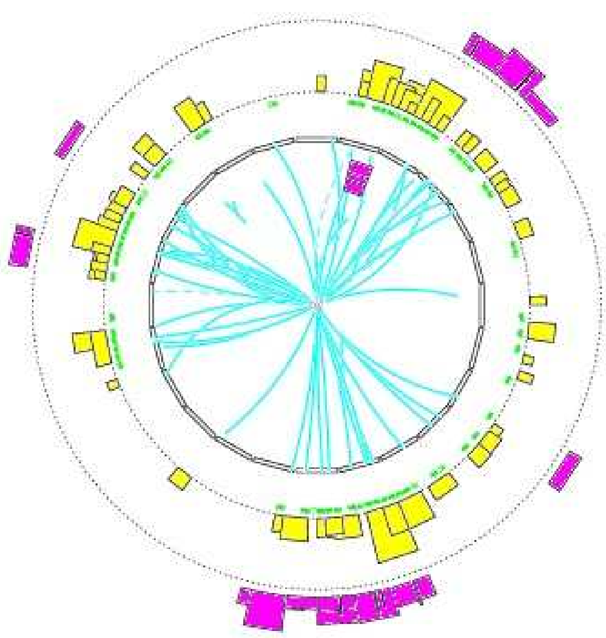
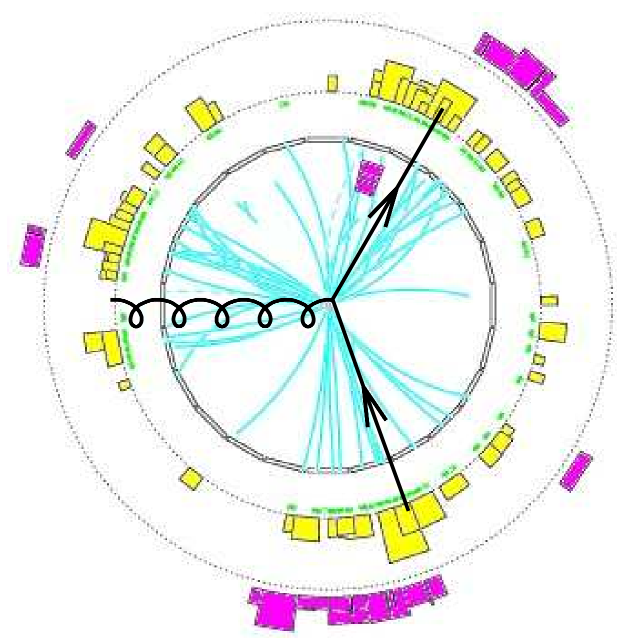
3 jets



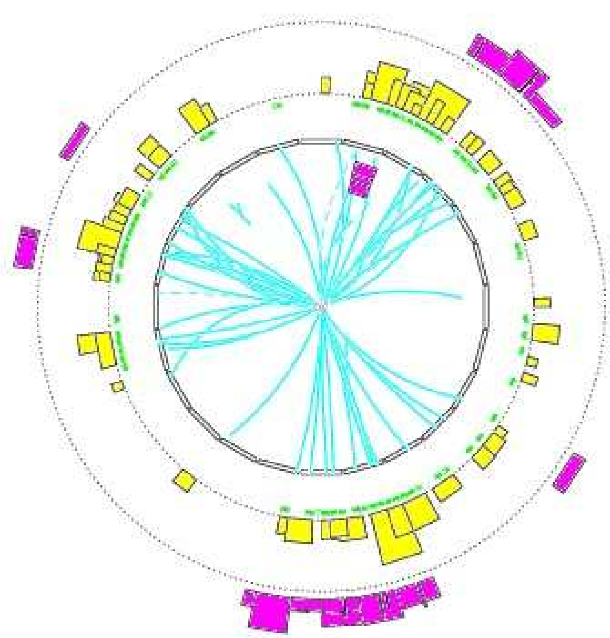
$\rightarrow$



# *Jets*

"Jets" $\equiv$ bunch of collimated particles $\cong$ hard partons

3 jets... or 4?



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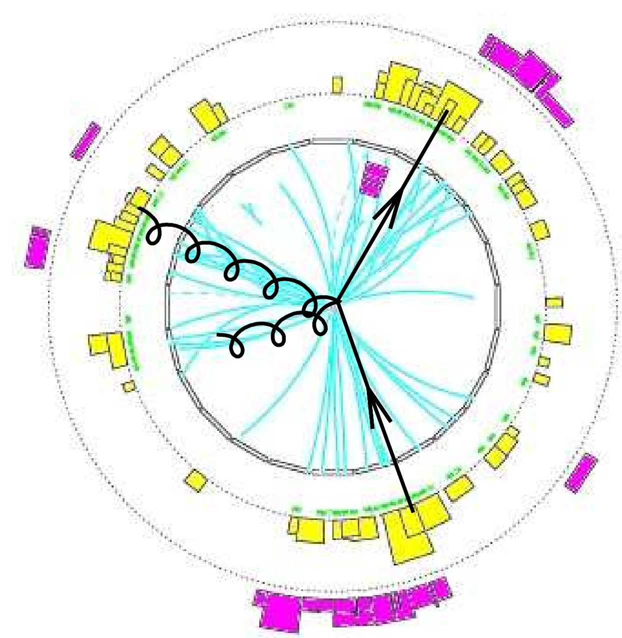

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# *Jets*

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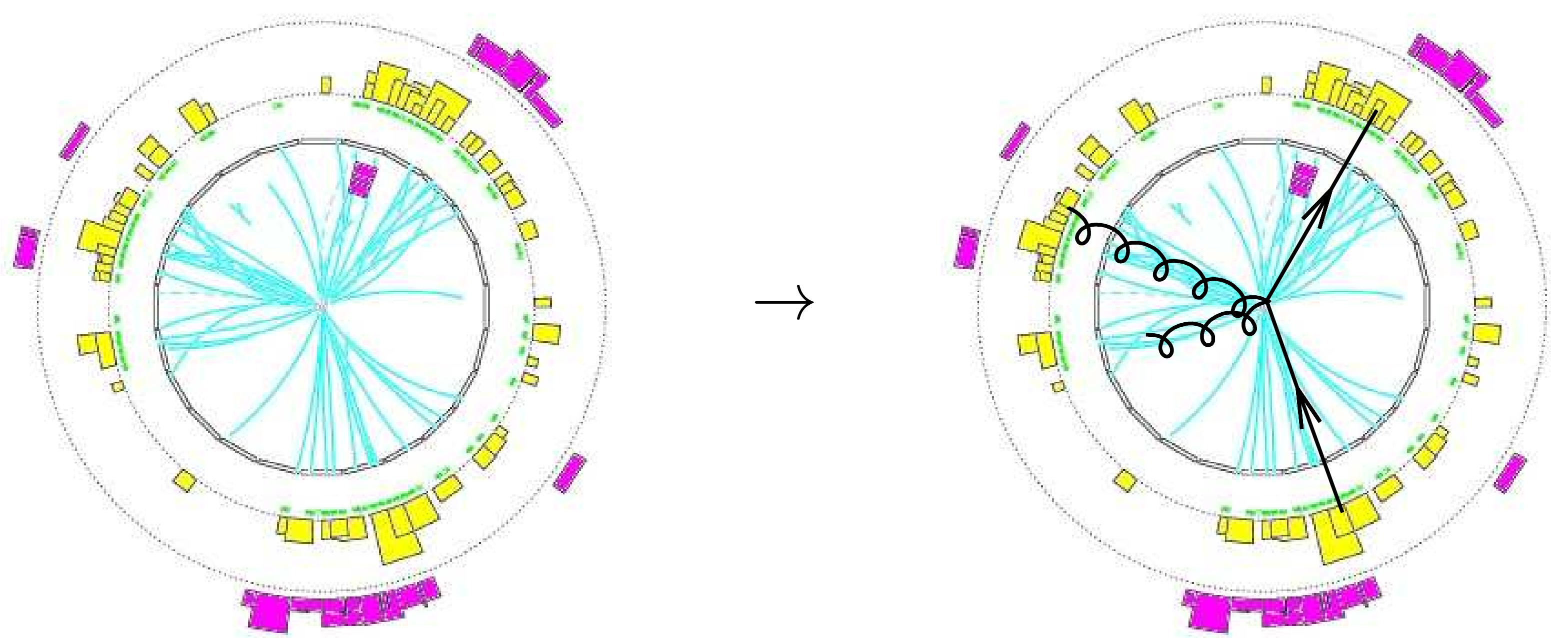
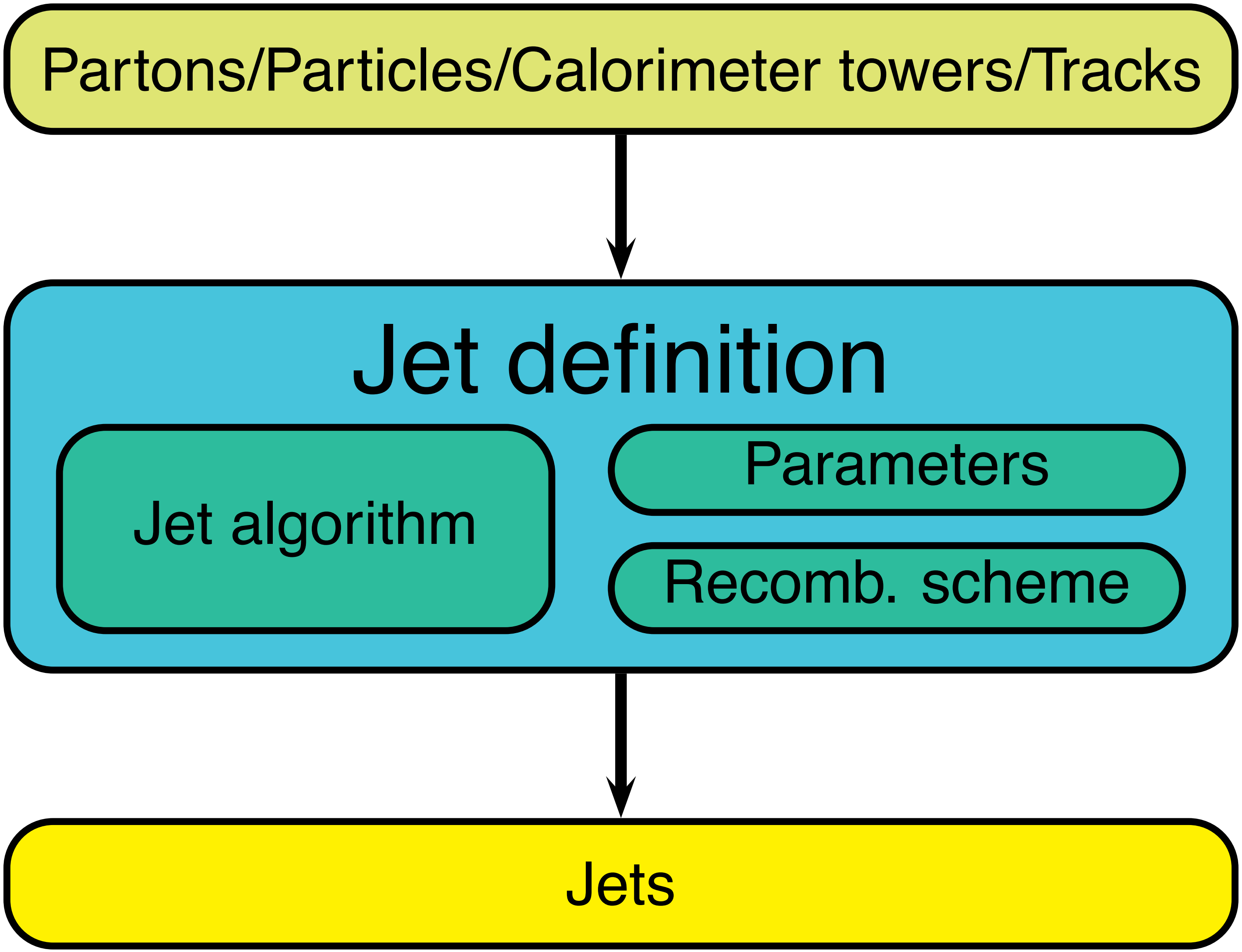


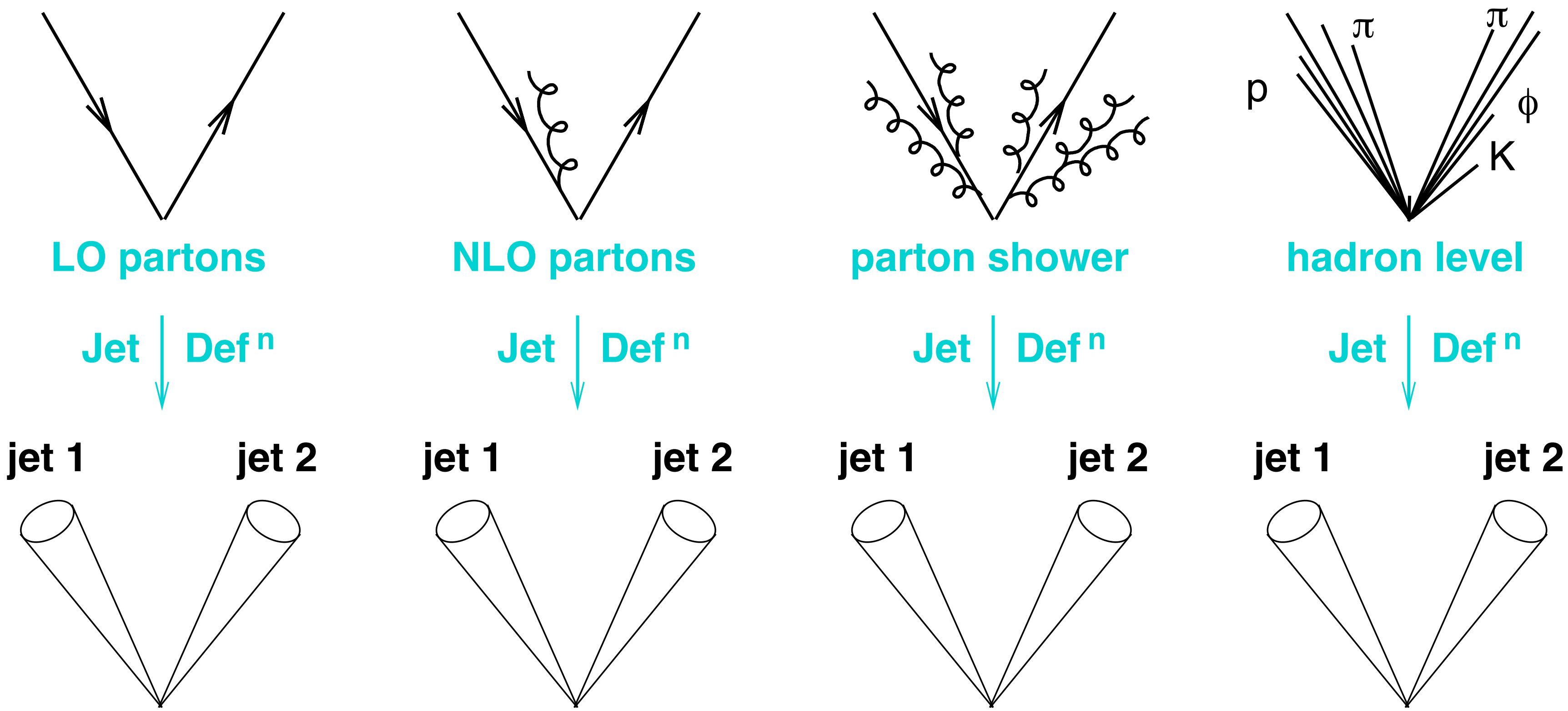
- “collinear” is arbitrary
- “parton” concept strictly valid only at LO

# Jets


Partons/Particles/Calorimeter towers/Tracks
Jet definition
Jet algorithm
Parameters
Recomb. scheme
Jets

# Jets

A jet definiton is supposed to be (as) consistent (as possible) across different view of an event



# Jet definitions: constraints

SNOWMASS accords (FermiLab, 1990)

Several important properties that should be met by a jet definition are [3]:

1. Simple to implement in an experimental analysis;
2. Simple to implement in the theoretical calculation;
3. Defined at any order of perturbation theory;
4. Yields finite cross section at any order of perturbation theory;
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30 years later, these are only recently satisfied!!!

# Jet definitions: cone

## Cone algorithm

- Concept of *stable cone* as a direction of energy flow
  - *"cone"*: circle of fixed radius $R$ in the $(y, \phi)$ plane
  - *"stable"*: sum of the particles (4-mom.) inside the cone points in the direction of its centre

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  - start from an initial direction (seed) for the cone centre
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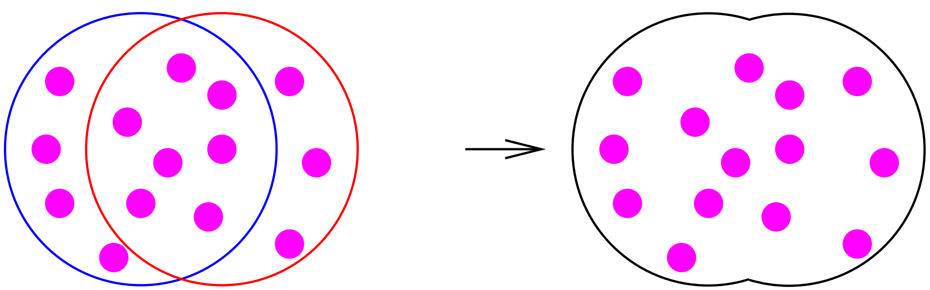
  - iterate until stable
- Stable cones $\equiv$ jets ... up to overlaps!

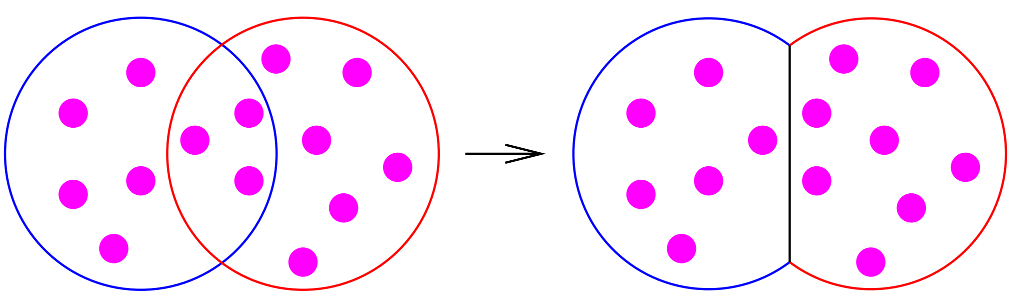
# Jet definitions: cone with SM

## Cone algorithm: (1) cone with split–merge

- Step 1: find the stable cones with the seeds
  1. input particles (over a seed threshold)
  2. midpoints of the stable cones found above
- Step 2: split–merge (with threshold $f$)



$$p_{t,\text{common}} > f\, p_{t,\text{hard}}$$



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<u>Examples</u>: main algorithm at the Tevatron

- CDF JetClu (1)
- CDF MidPoint (1+2)
- D0 Run II Cone (1+2)
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Examples: main algorithm at the Tevatron

- CDF JetClu (1) IR unsafe (2 hard+1 soft)
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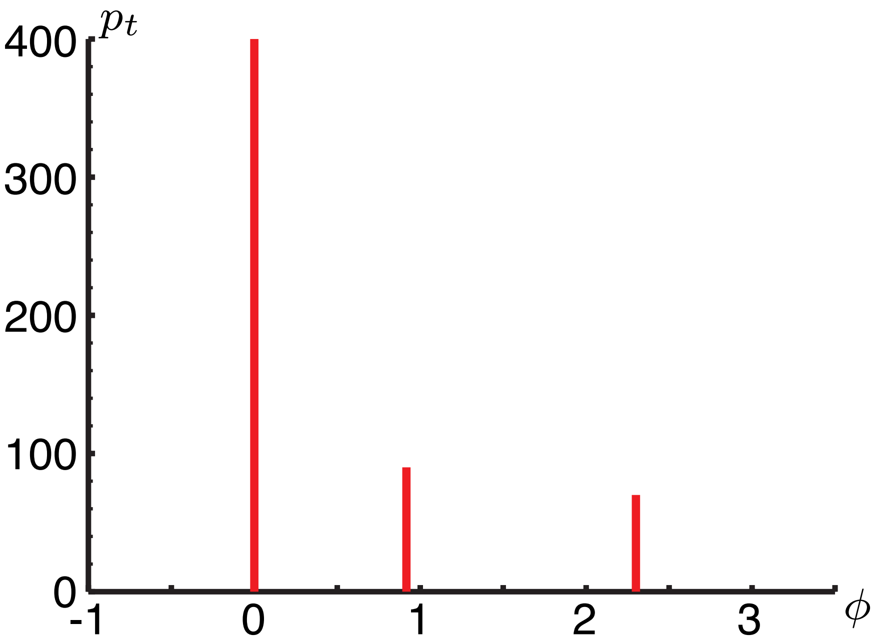
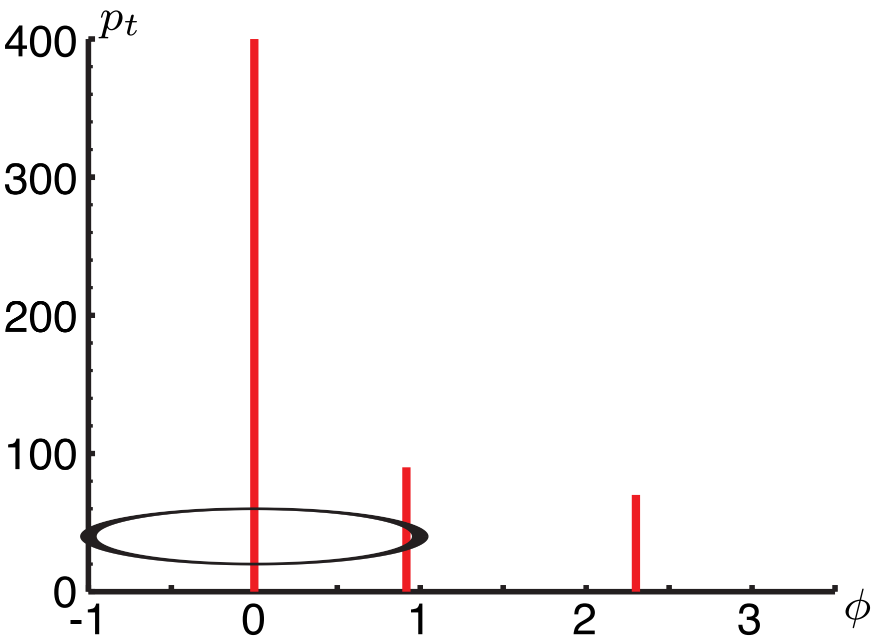
# IR unsafety of the Midpoint alg



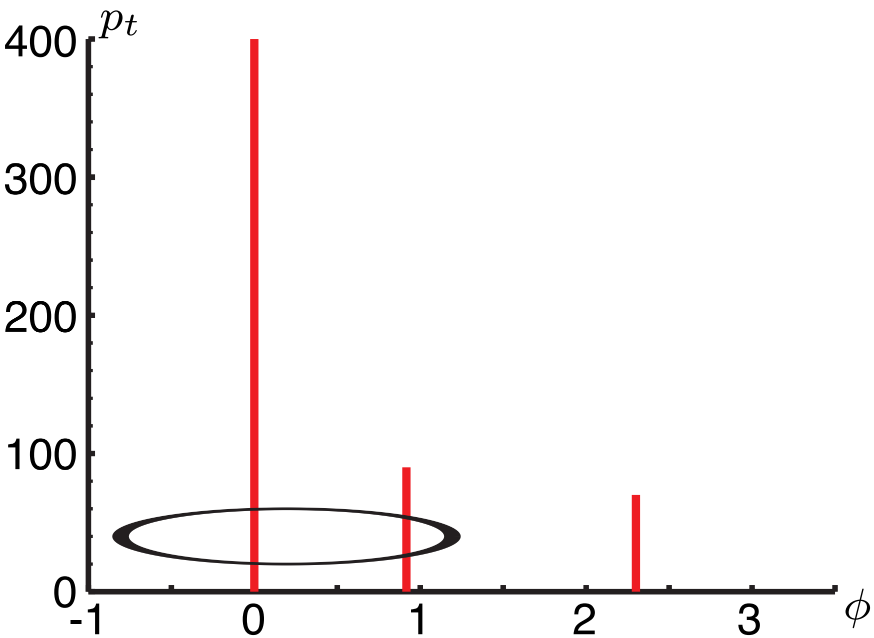

3-particle event — MidPoint clustering

# *IR unsafety of the Midpoint alg*



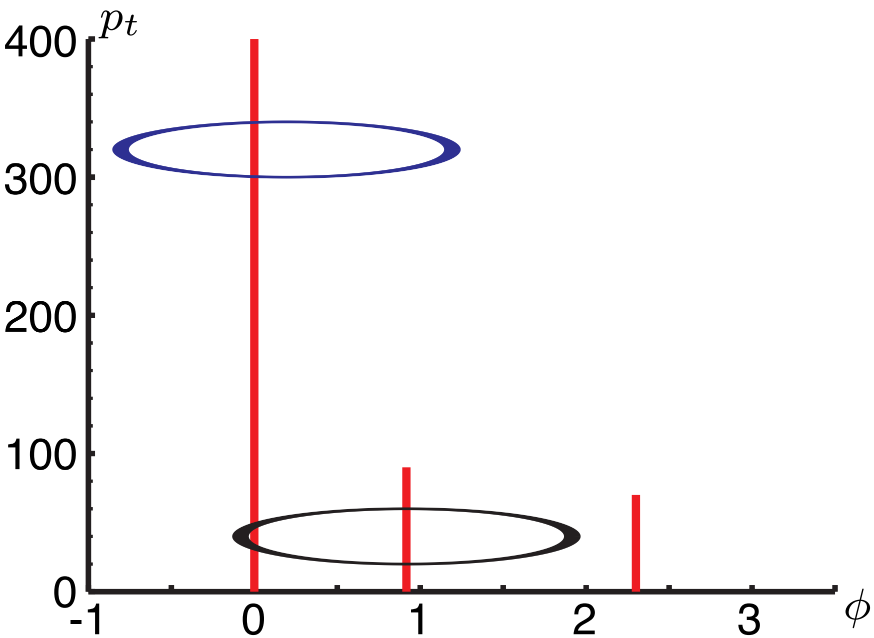

1st seed

# IR unsafety of the Midpoint alg




iterate

# IR unsafety of the Midpoint alg




stable; 2nd seed

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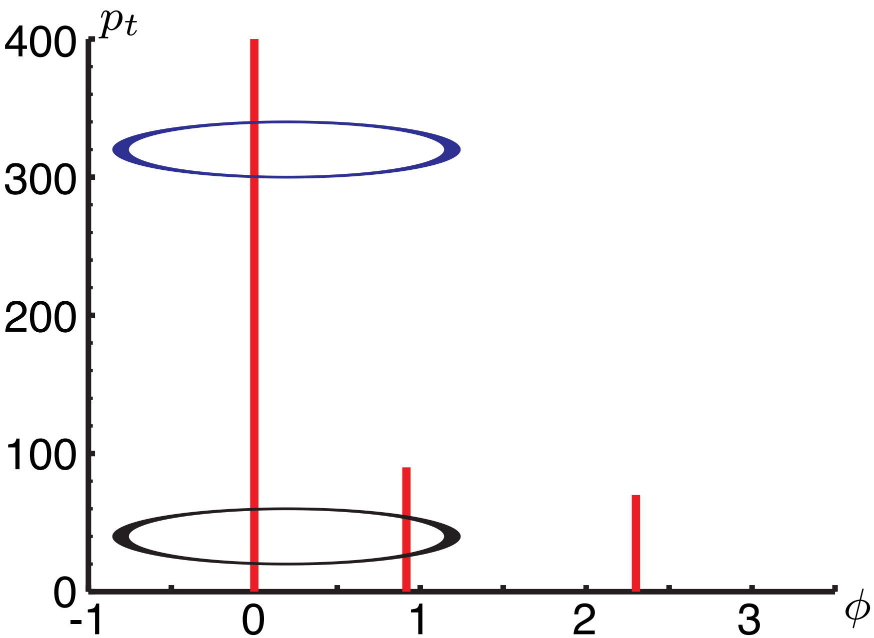


iterate

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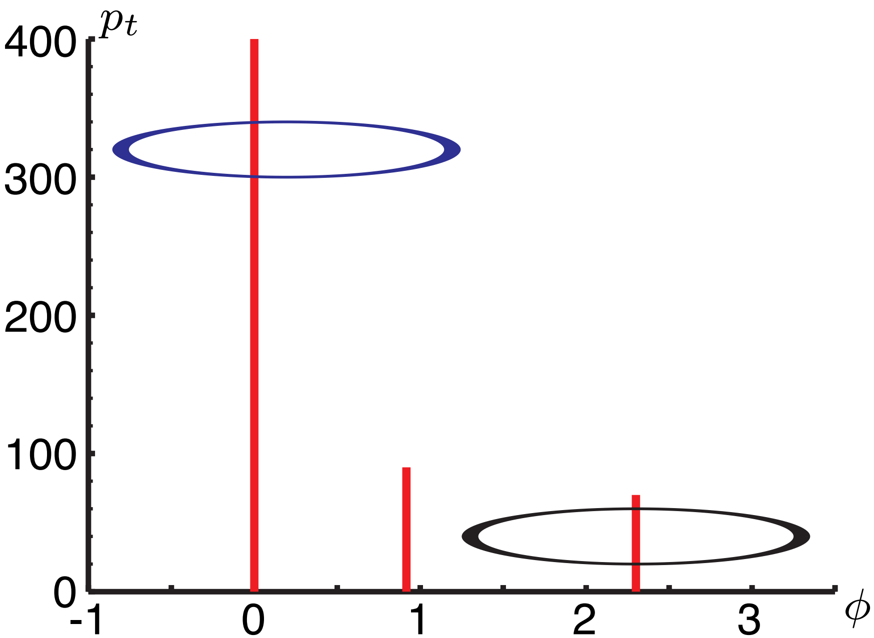
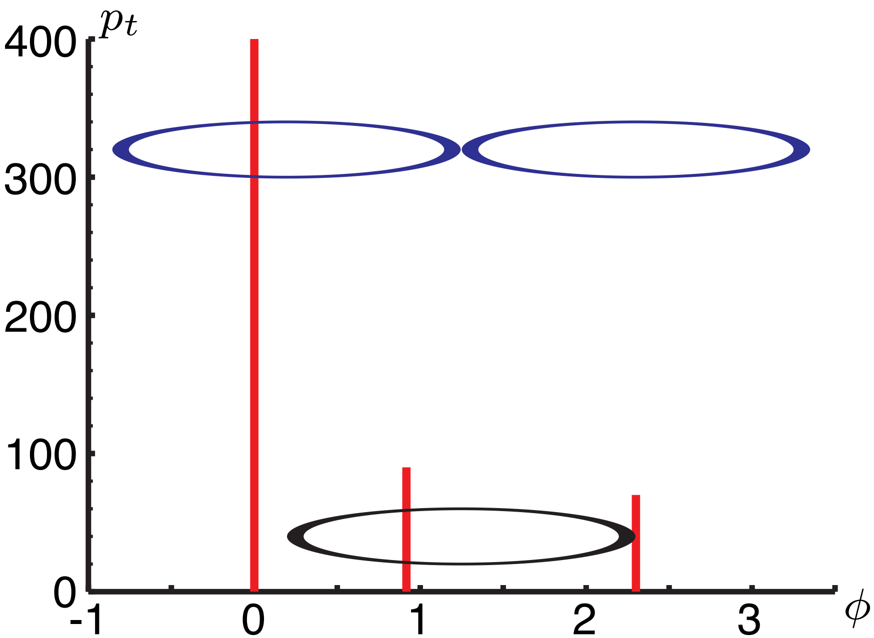



stable; 3rd seed

# IR unsafety of the Midpoint alg




stable; midpoint seed

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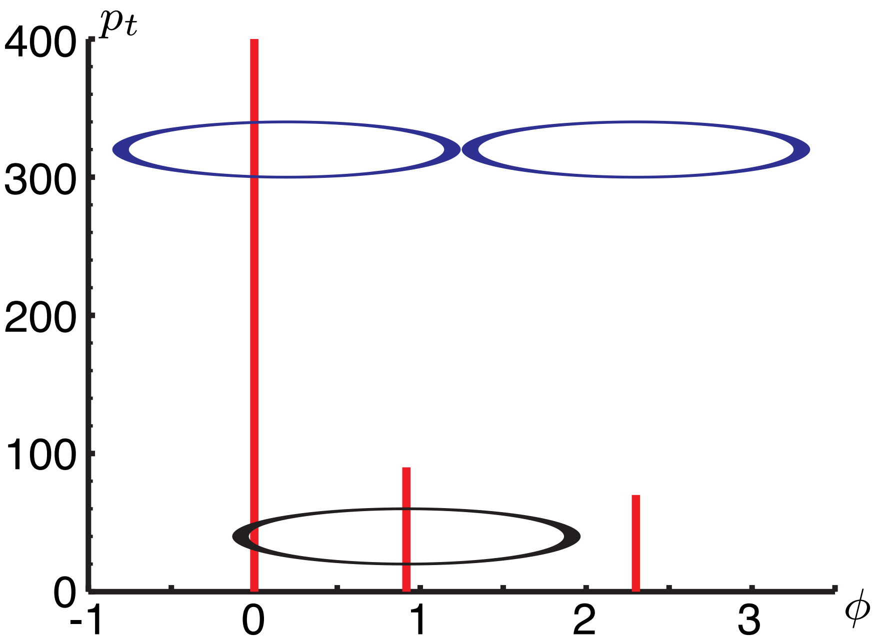


iterate

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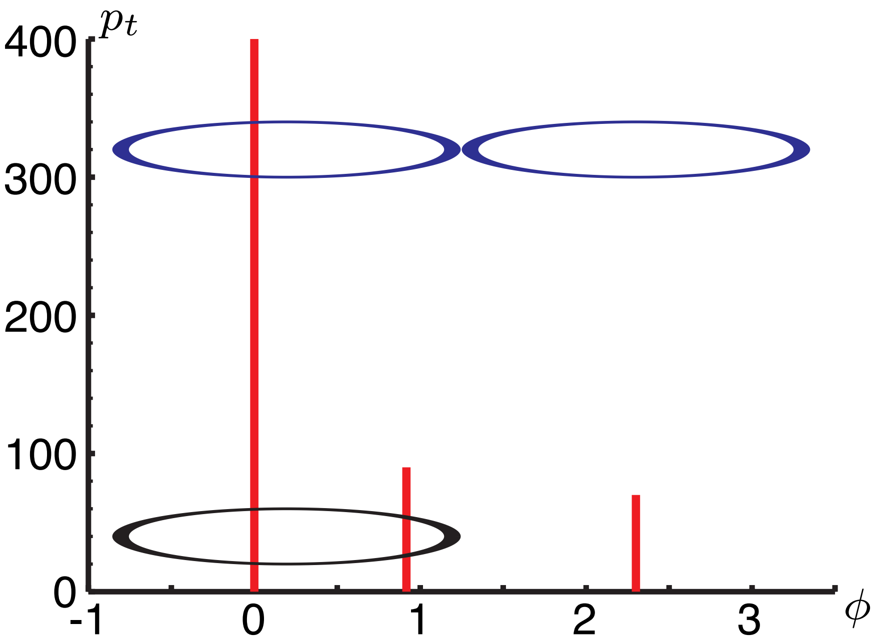


iterate

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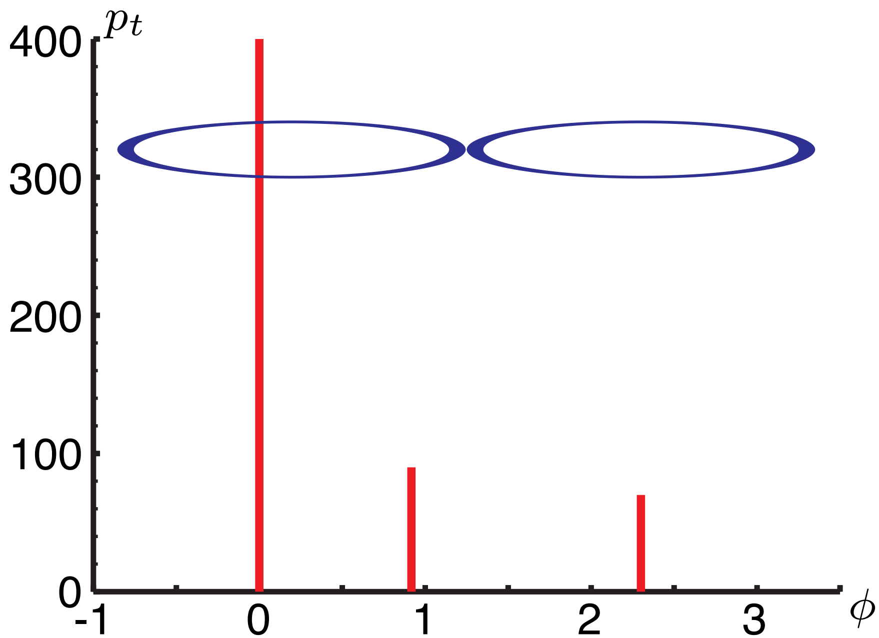
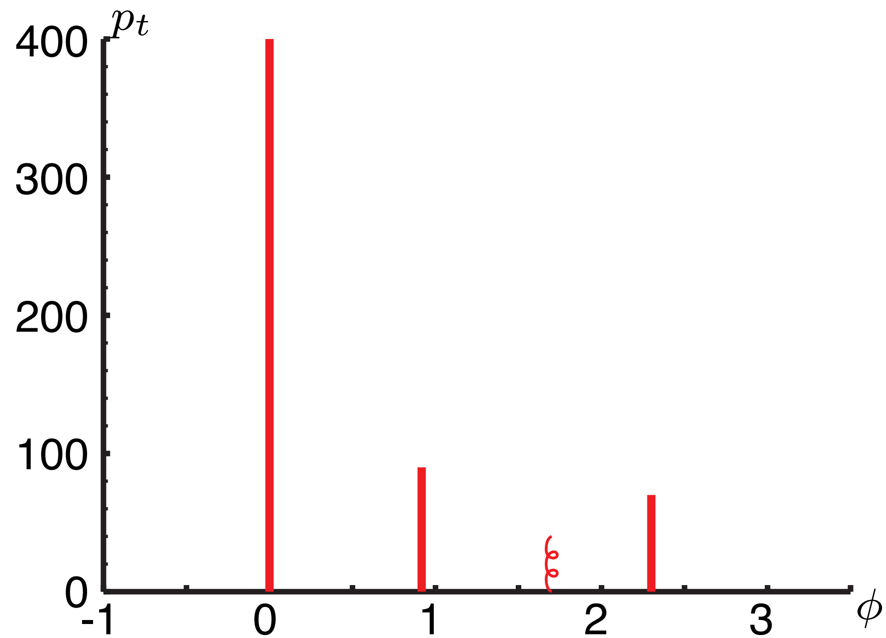
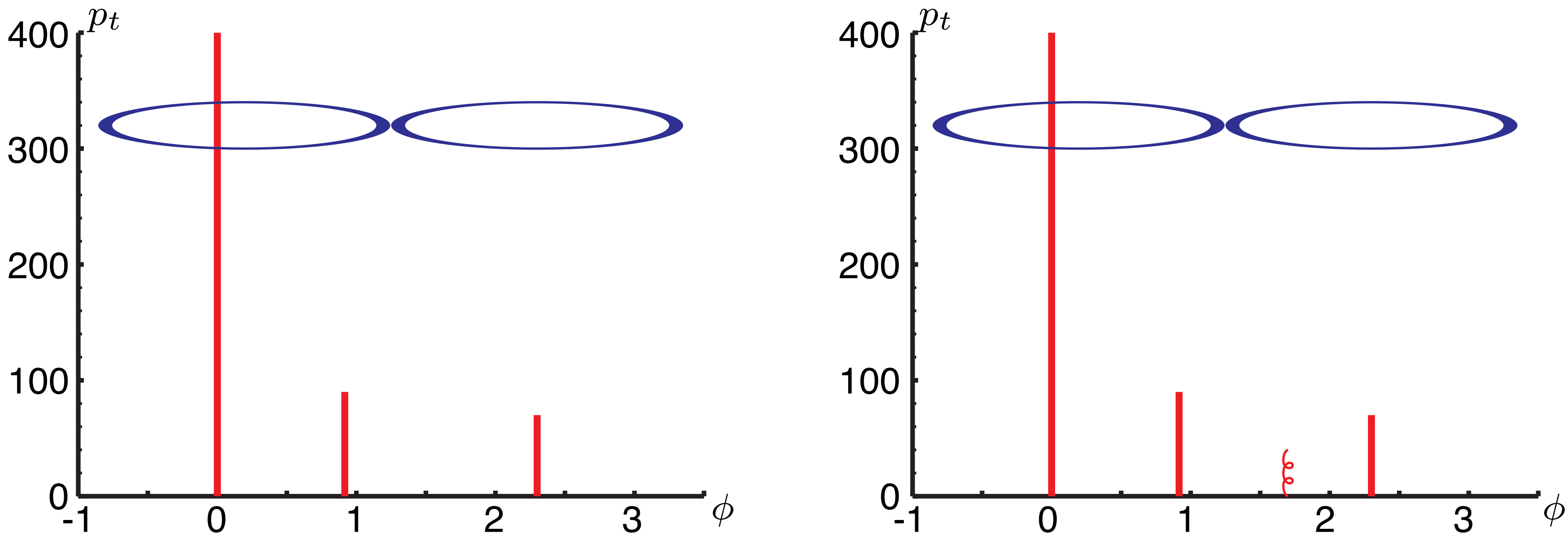


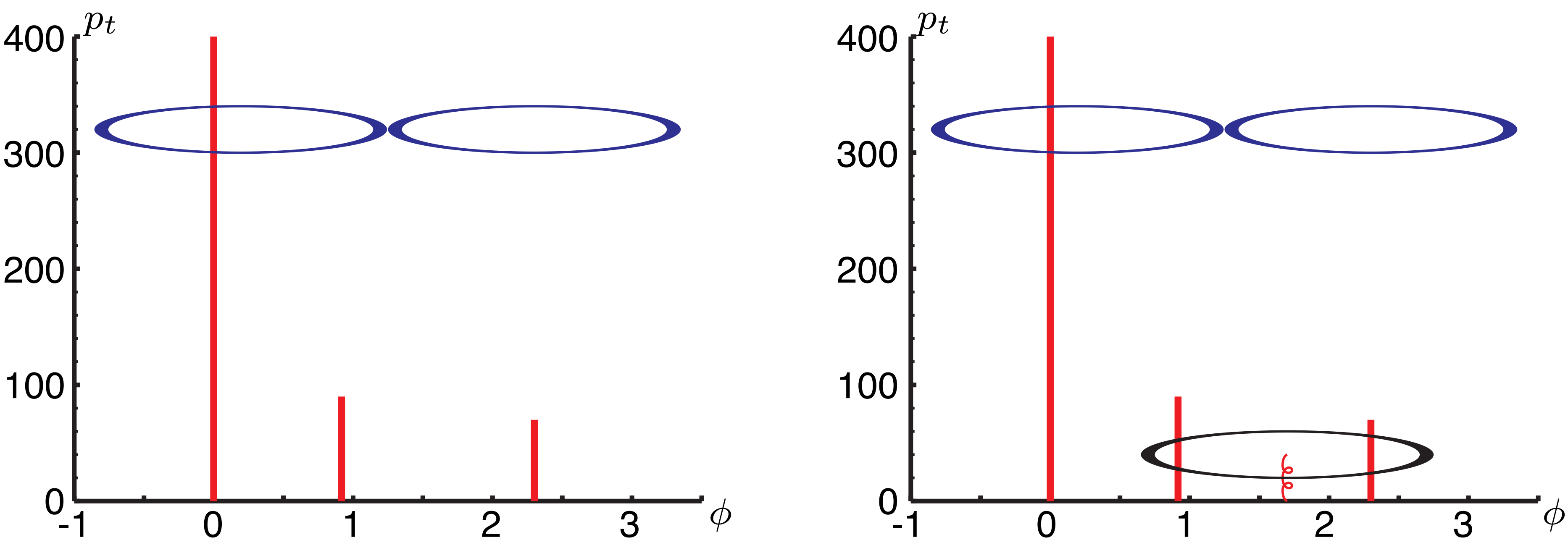

add an infinitely soft particle

# IR unsafety of the Midpoint alg



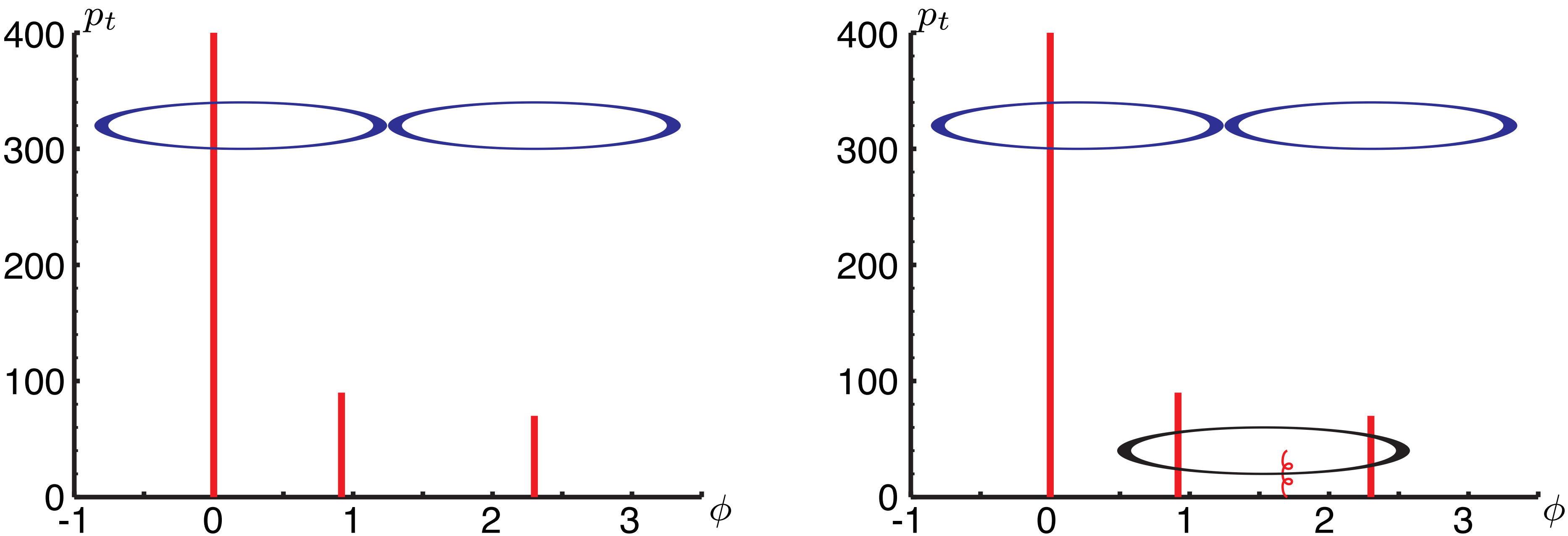

3 hard seeds + midpoint seed $\rightarrow$ 2 stable cones

# IR unsafety of the Midpoint alg




new seed!

# IR unsafety of the Midpoint alg




iterate

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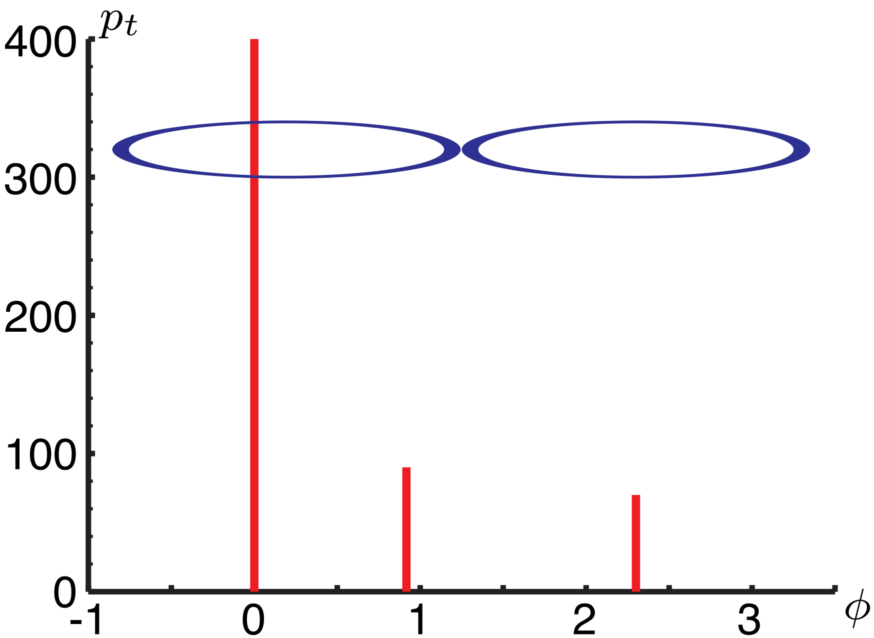
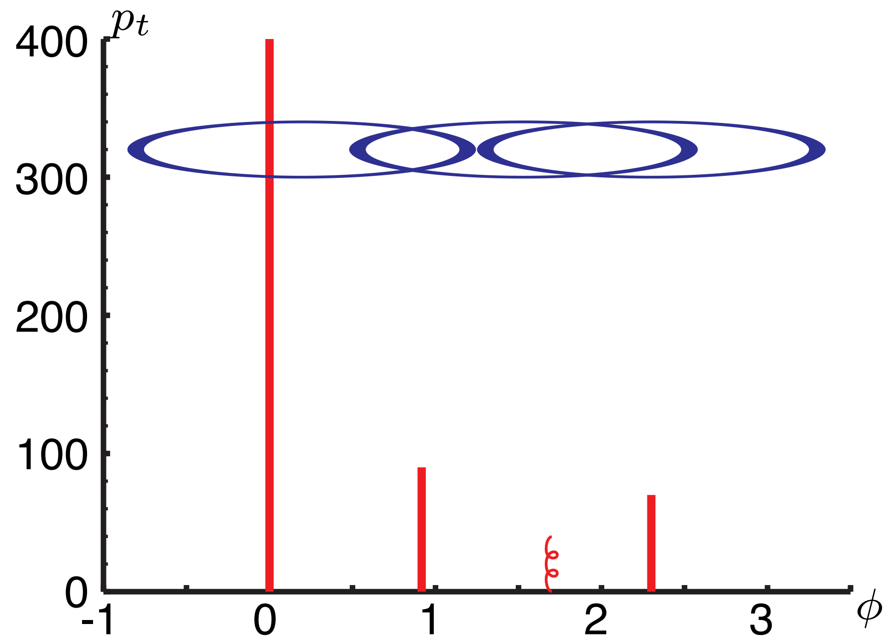
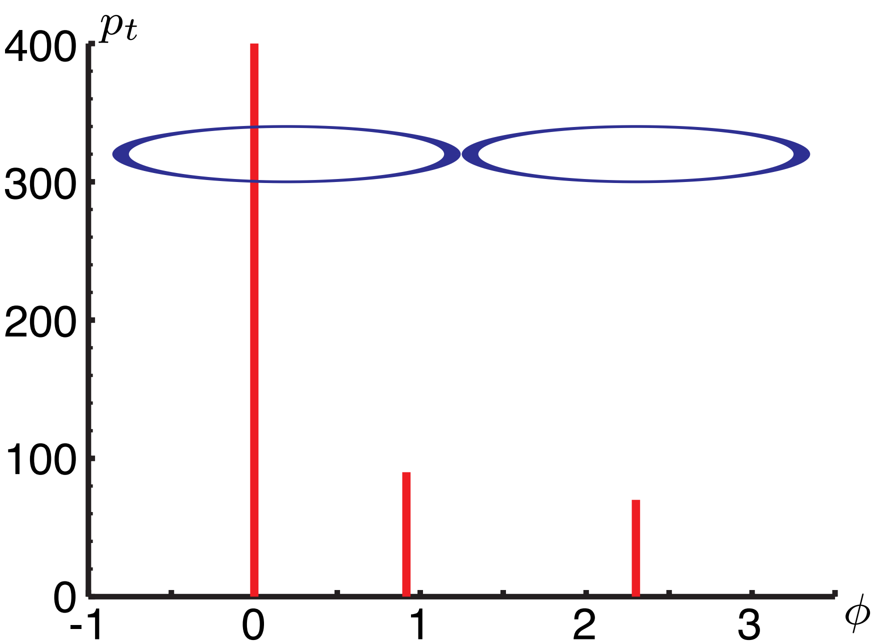


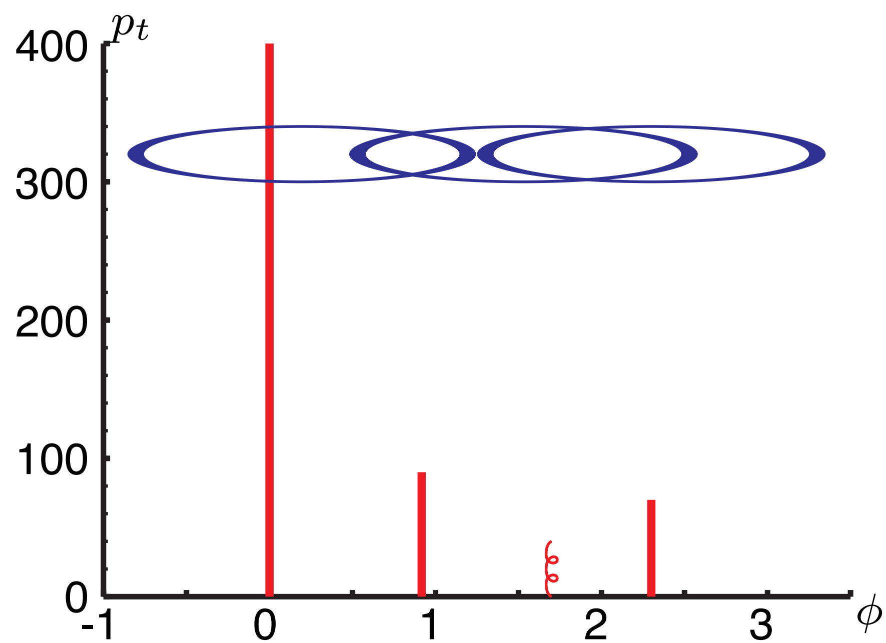




Stable cones:

| | | |
|---|---|---|
| Midpoint: | {1,2} & {3} | {1,2} & {3} & {2,3} |

# IR unsafety of the Midpoint alg




Stable cones:

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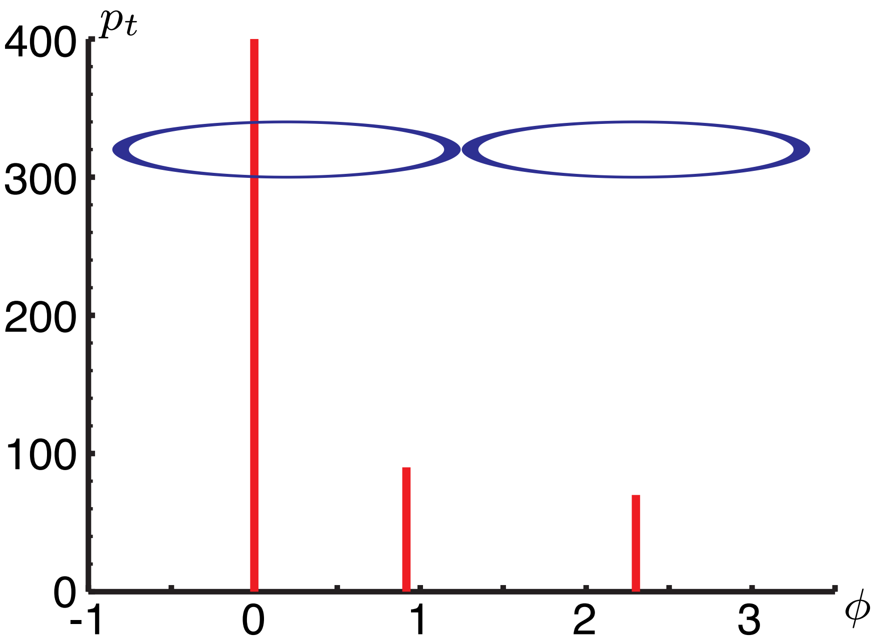
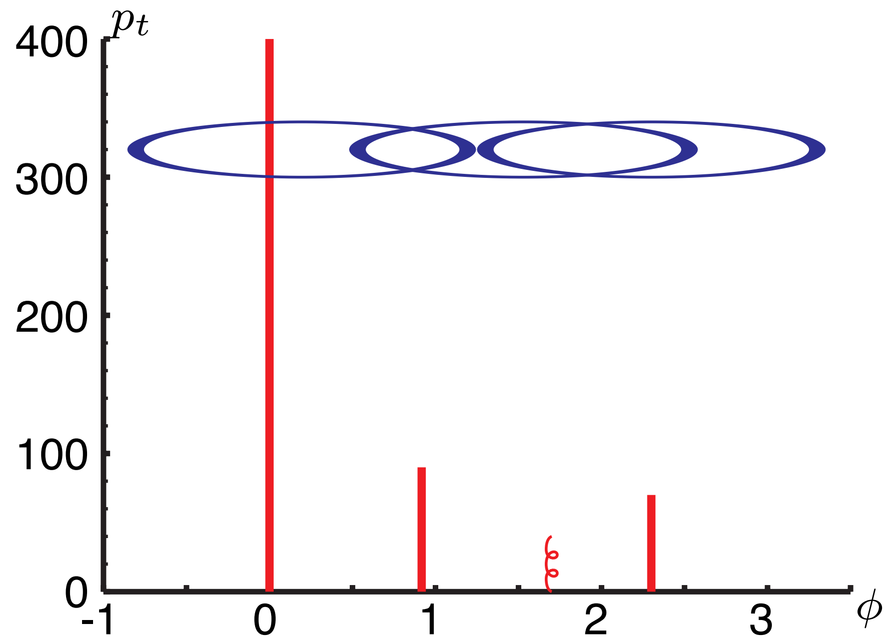
Jets: ($f = 0.5$)

| | | |
|---|---|---|
| Midpoint: | {1,2} & {3} | {1,2,3} |

# *IR unsafety of the Midpoint alg*



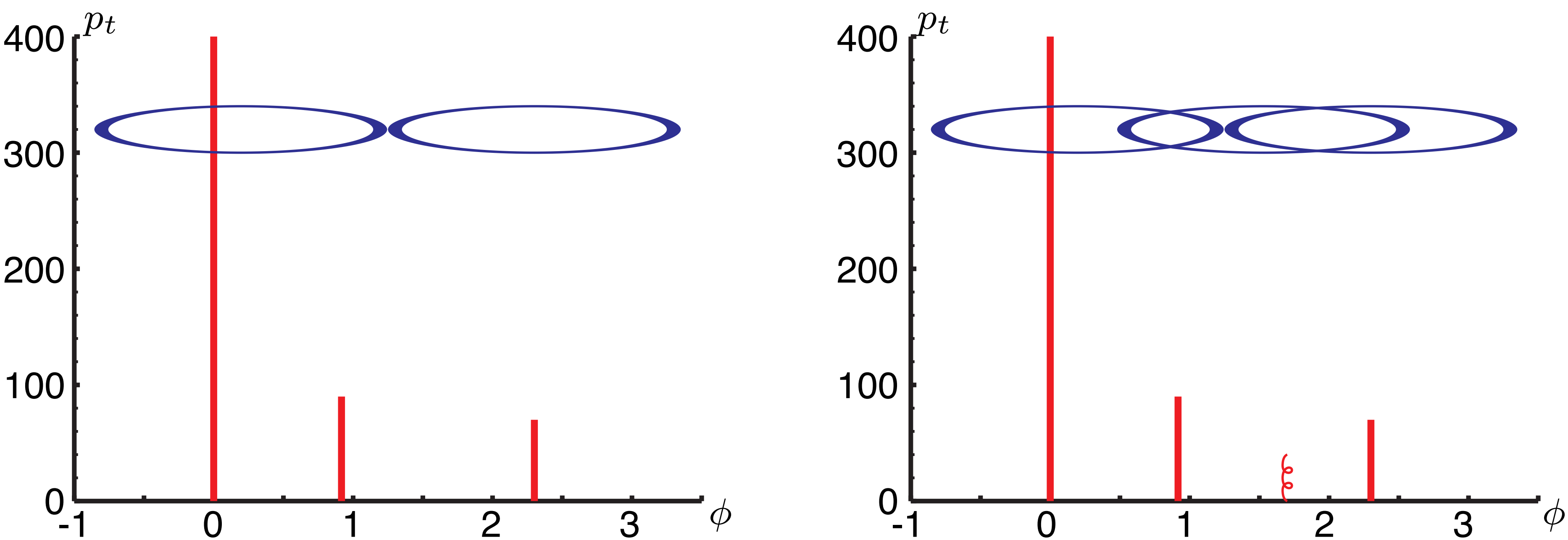

Stable cones:

| | | |
|---|---|---|
| Midpoint: | {1,2} & {3} | {1,2} & {3} & {2,3} |
| Seedless: | {1,2} & {3} & {2,3} | {1,2} & {3} & {2,3} |

Jets: ($f = 0.5$)

| | | |
|---|---|---|
| Midpoint: | {1,2} & {3} | {1,2,3} |
| Seedless: | {1,2,3} | {1,2,3} |

# IR unsafety of the Midpoint alg




Stable cones:

| | | |
|---|---|---|
| Midpoint: | {1,2} & {3} | {1,2} & {3} & {2,3} |
| Seedless: | {1,2} & {3} & {2,3} | {1,2} & {3} & {2,3} |

Jets: ($f = 0.5$)

| | | |
|---|---|---|
| Midpoint: | {1,2} & {3} | {1,2,3} |
| Seedless: | {1,2,3} | {1,2,3} |

**Stable cone missed $\longrightarrow$ MidPoint is IR unsafe**

# *Jet definitions*

Cone algorithm: (1) cone with split–merge

- Step 1: find ALL stable cones in a reasonable time
  - MidPoint: time $\propto N^3$
  - All-Naive: time $\propto 2^N$
  - SISCone: time $\propto N^2 \log(N)$
- Step 2: split–merge (with threshold $f$)

Example: SISCone Seedless Infrared-Safe Cone

2007!!!

# Jet definition: cone with PR

Cone algorithm: (2) cone with progressive removal

- Recipe:
  - start with the hardest particle as a seed
  - iterate to find a stable cone
  - stable cone $\rightarrow$ $1^{\text{st}}$ jet
  - remove its constituents
  - continue with the next hardest particle left

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- Benchmark: circular/soft-resilient hard jets

# Jet definition: cone with PR

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- Example: CMS Iterative Cone

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  - stable cone $\rightarrow$ 1$^{\mathrm{st}}$ jet
  - remove its constituents
  - continue with the next hardest particle left
- Benchmark: circular/soft-resilient hard jets
- Example: CMS Iterative Cone
  BUT Collinear unsafe (3 hard+1 coll.splitting) !!

# Jet definition: successive recombinations

Idea: Undo the QCD cascade

- Define an inter-particle distance $d_{ij}$
  and a beam distance $d_{iB}$
- Successively
  - Find the minimum of all $d_{ij}$, $d_{iB}$
  - If $d_{ij}$, recombine $i + j \to k$ (remove $i$, $j$; add $k$)
  - If $d_{iB}$, call $i$ a jet (remove $i$)
- Until all particles have been clustered

# *Jet definition: successive recombinations*

Typical choice of distances:

$$
\begin{aligned}
d_{ij}^2 &= \min(k_{t,i}^{2p}, k_{t,j}^{2p})(\Delta y_{ij}^2 + \Delta\phi_{ij}^2) \\
d_{iB}^2 &= k_{t,i}^{2p} R^2
\end{aligned}
$$

- $p = 1$: $k_t$ algorithm (1993)

- $p = 0$: Cambridge-Aachen algorithm (1997)

- $p = -1$: anti-$k_t$ algorithm (2008)

- parameter $R$ (jet separation)
- trivially IRC-safe

# Jet definition: successive recombinations

Typical choice of distances:

$$
\begin{aligned}
d_{ij}^2 &= \min(k_{t,i}^{2p}, k_{t,j}^{2p})(\Delta y_{ij}^2 + \Delta\phi_{ij}^2) \\
d_{iB}^2 &= k_{t,i}^{2p} R^2
\end{aligned}
$$

- $p = 1$: $k_t$ algorithm (1993)
  (as close as possible to pQCD)
- $p = 0$: Cambridge-Aachen algorithm (1997)
  (close to pQCD; useful for substructure)
- $p = -1$: anti-$k_t$ algorithm (2008)
  (circular/soft-resilient jets; replaces it. cone)

Variants for $e^+e^-$ collisions (+JADE)

# *Jet definitions: IRC safety matters*

As said in $e^+e^-$: **IRC safety matters** if you want to compare to QCD computations

| Process | Last OK order IR$_{2+1}$ | IR/Coll$_{3+1}$ | safe | today's pQCD |
|---|---|---|---|---|
| Incl. jet x-sect | LO | NLO | any | NLO |
| W/Z/H+1 jet | LO | NLO | any | NLO |
| 3-jet x-sect | none | LO | any | NLO |
| W/Z/H+2 jet | none | LO | any | NLO |
| jet mass in 3-jet | none | none | any | LO |

# *Jet definitions: IRC safety matters*

As said in $e^+e^-$: **IRC safety matters** if you want to compare to QCD computations

| Process | Last OK order $\mathrm{IR}_{2+1}$ | $\mathrm{IR/Coll}_{3+1}$ | safe | today's pQCD |
|---|---|---|---|---|
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| 3-jet x-sect | none | LO | any | NLO |
| W/Z/H+2 jet | none | LO | any | NLO |
| jet mass in 3-jet | none | none | any | LO |

$\Rightarrow$ Use an IRC-safe algorithm like $k_t$, C/A, anti-$k_t$ or SISCone

# *Jet definitions: comparison*

## Quick comparison of the algorithms

| | $k_t$ | C/A | anti-$k_t$ | SISCone |
|---|---|---|---|---|
| pQCD | ✓✓✓ | ✓✓✓ | ✓✓ | ✓✓ |
| soft (UE) | ✗ | $\sim$ OK | ✓✓ | ✓✓✓ |
| speed | ✓✓✓ | ✓✓✓ | ✓✓✓ | ✓ |
| substruct | ✓✓ | ✓✓✓ | ✗ | ✗ |
| calibr. | ✓ | ✓ | ✓✓✓ | ✓✓ |

# *Jet clustering: usage/access*

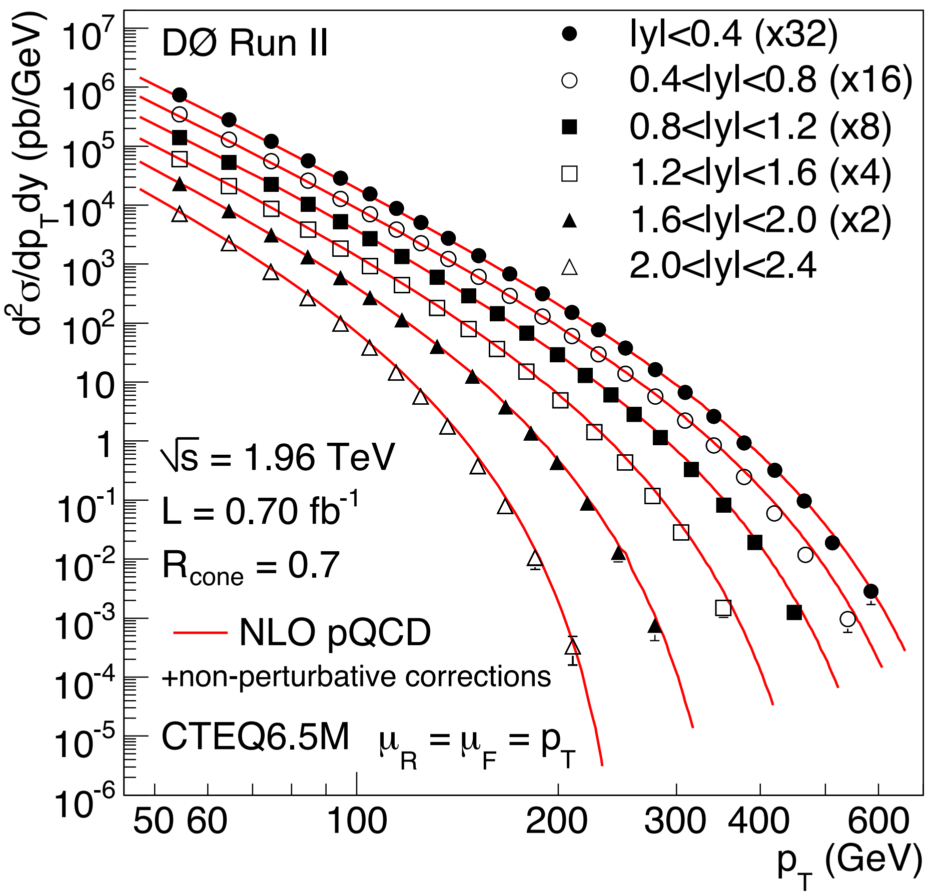
## FastJet

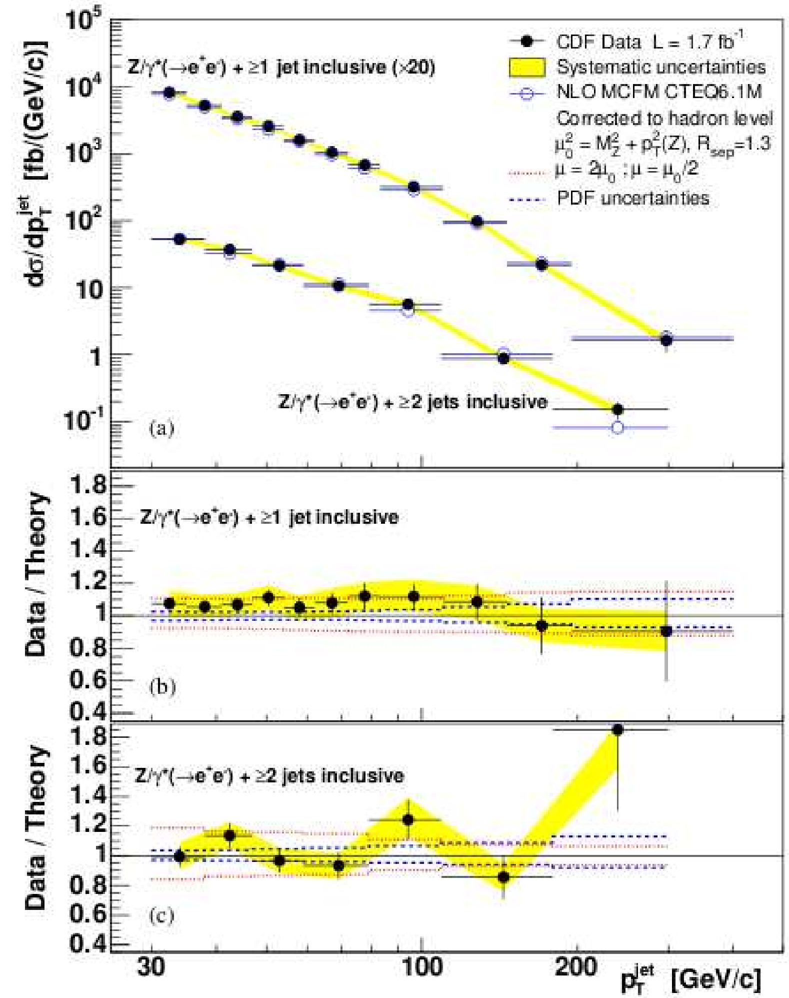
[M.Cacciari, G.Salam, GS]

- Fast implementation of recomb. algs ($N\ \log(N)$)
- Plugins for all common algs
  (SISCone; CDF, D0, ATLAS, CMS algs; $e^+e^-$ algs)
- Other tools (like jet areas)
- More in the tutorial part!

# Jets: experimentally

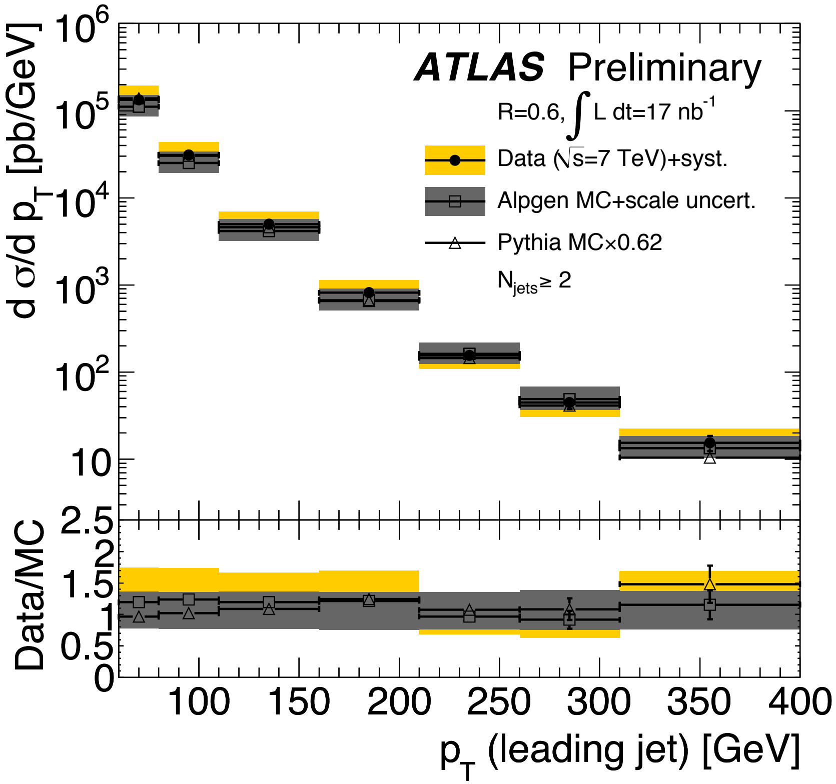
- Tevatron
  Use of IR-unsafe JetClu or MidPoint and sometimes $k_t$

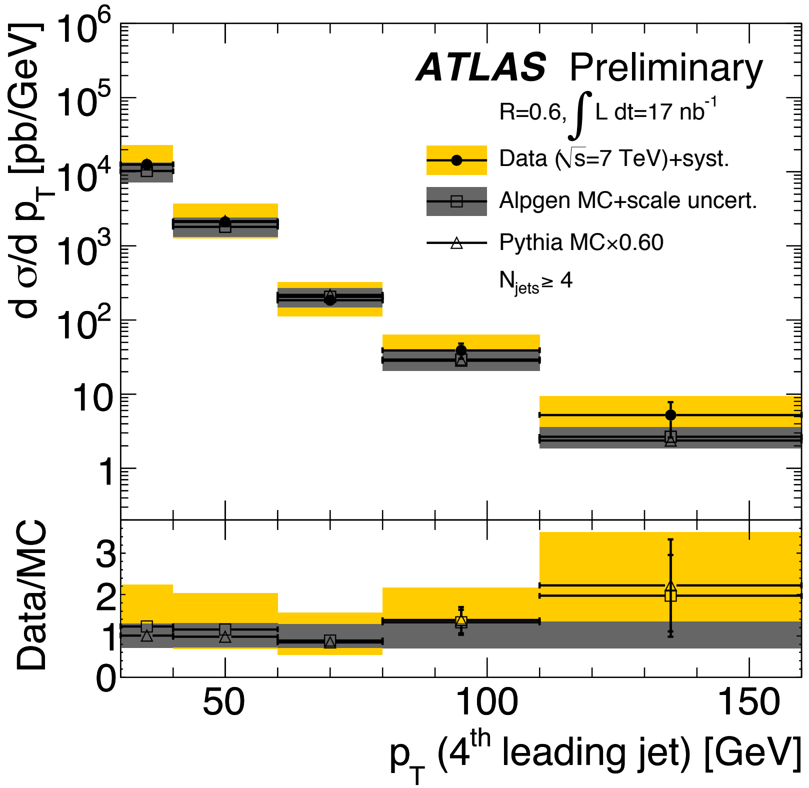


# Jets: experimentally

- Tevatron
  Use of IR-unsafe JetClu or MidPoint and sometimes $k_t$
- LHC: anti-$k_t$ by default



# Jets: hadronic colliders

At hadronic colliders, many "contaminations" to a jet:

- radiation from partons in the initial state
- Underlying event/Multiple interactions
  - shift: UE $\approx$ uniform soft background *i.e.* contamination $\propto$ jet area $\propto R^2$
  - smearing: due to UE fluctuations
  - typical scale: a few GeV
- Pile-up: many $pp$ interactions in 1 bunch-crossing:

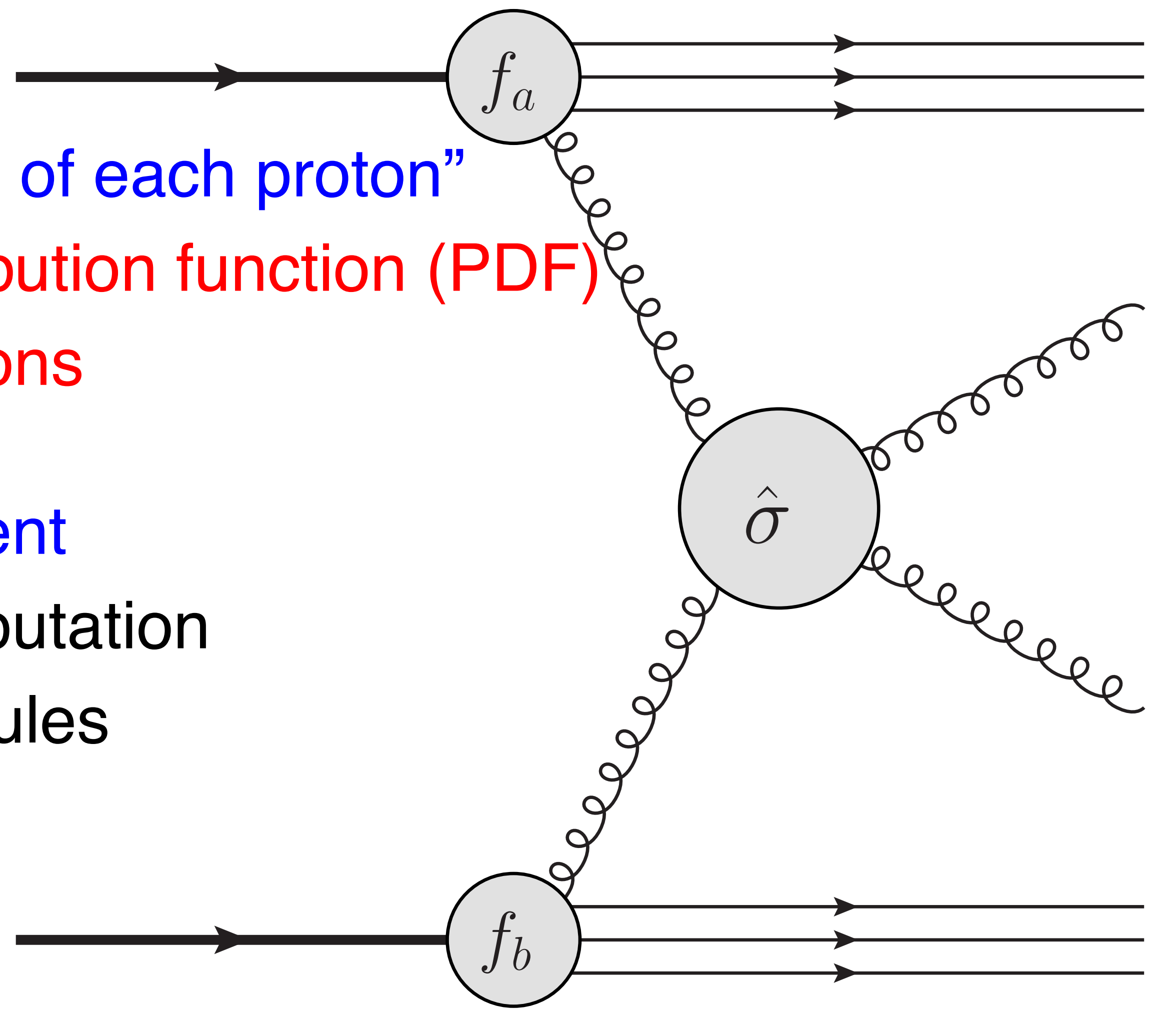
$$n \approx \mathcal{L}\,\Delta t_{\text{bunch}}\,\sigma_{pp} \approx 10^{34}\,25.10^{-9}\,100.10^{-27} \approx 25$$

Again: shift + smearing
Typical scale: 20-30 GeV
Need for subtraction techniques

</interlude>

# The very fundamental collision

$$\sigma = f_a \otimes f_b \otimes \hat{\sigma}$$



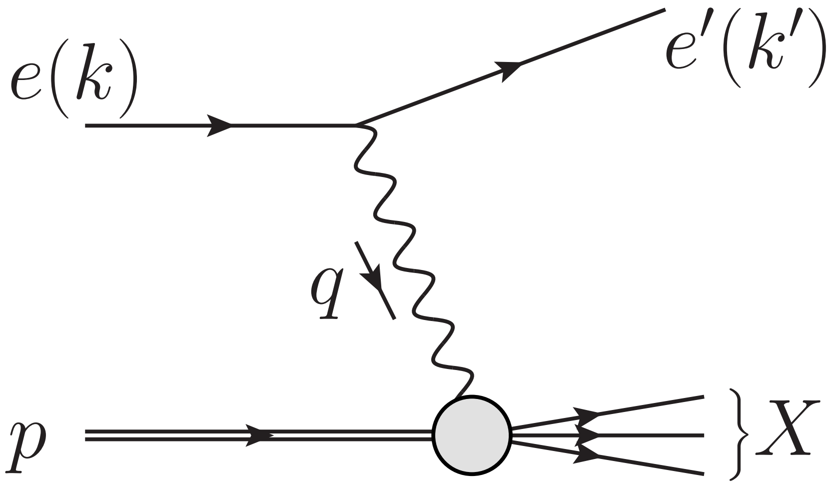

- "take a parton out of each proton"
  $f_a \equiv$ parton distribution function (PDF)
  for quark and gluons

- hard matrix element
  perturbative computation
  Forde-Feynman rules

## *Deep Inelastic Scattering*

## *Introduce/Discuss/Study the PDFs*

# Process + kinematics




$$s = (e+p)^2$$

$$W^2 = (q+p)^2$$

$$Q^2 = -q^2 > 0$$

$$\nu = p.q = W^2 + Q^2$$

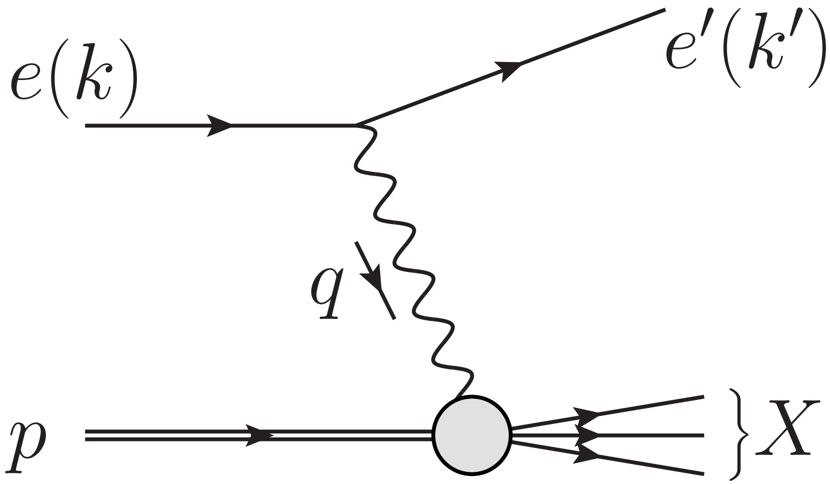
$$x = Q^2/(2\nu)$$

$$y = p.q/p.k = (W^2 + Q^2)/s$$

$ep \to eX$ with $\gamma$ exchange

- $Z$ and $W$ also possible as well as $\nu$ instead of $e$
- also more exclusive meas.: $ep \to ep$, $eXY$, $eYp$, *e.g.* jets, charm, vector-mesons, photons

# Process + kinematics

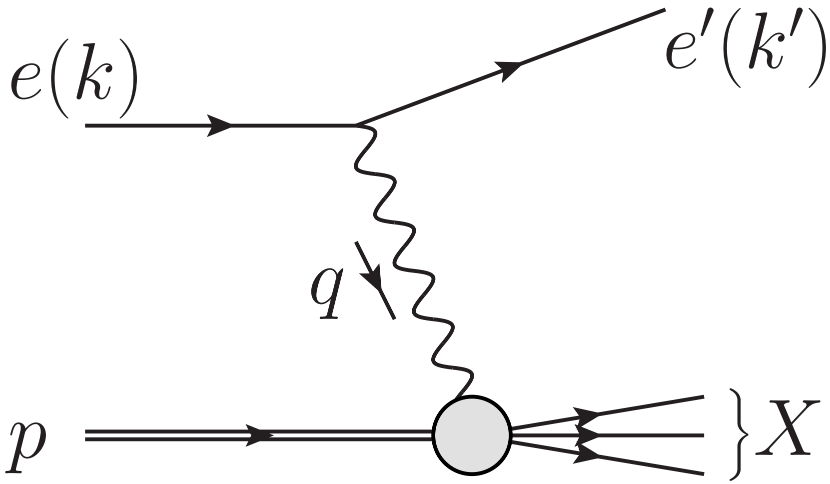



$$s = (e+p)^2$$
$$W^2 = (q+p)^2$$
$$Q^2 = -q^2 > 0$$
$$\nu = p.q = W^2 + Q^2$$
$$x = Q^2/(2\nu)$$
$$y = p.q/p.k = (W^2 + Q^2)/s$$

Experimentally: only the outgoing $e$ is needed to reconstruct the kinematics

$$Q^2 = 4EE' \cos^2(\theta_e/2) \qquad x = \frac{EE' \cos^2(\theta_e/2)}{P[E - E' \sin^2(\theta_e/2)]}$$

# Process + kinematics

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$$x = Q^2/(2\nu)$$

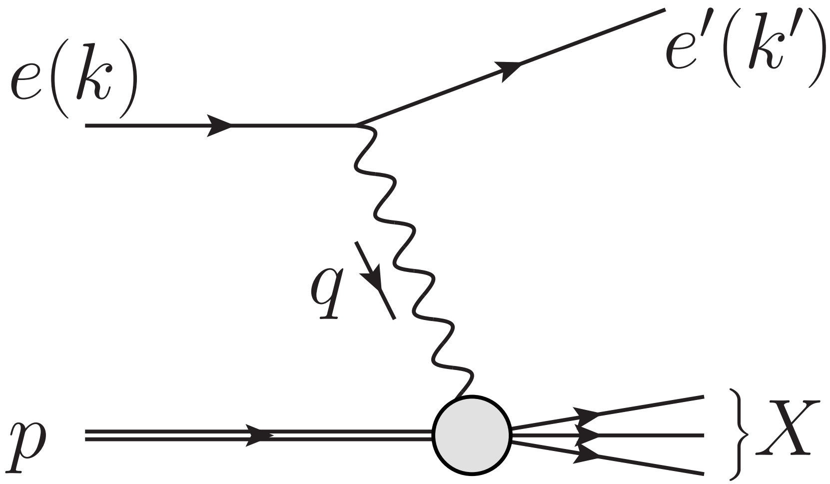
$$y = p.q/p.k = (W^2+Q^2)/s$$

Idea:

use the photon to probe the proton structure

$Q^2$ large $\Rightarrow$ small distance $\sim 1/Q$

# Process + kinematics

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$$Q^2 = -q^2 > 0$$

$$\nu = p.q = W^2 + Q^2$$

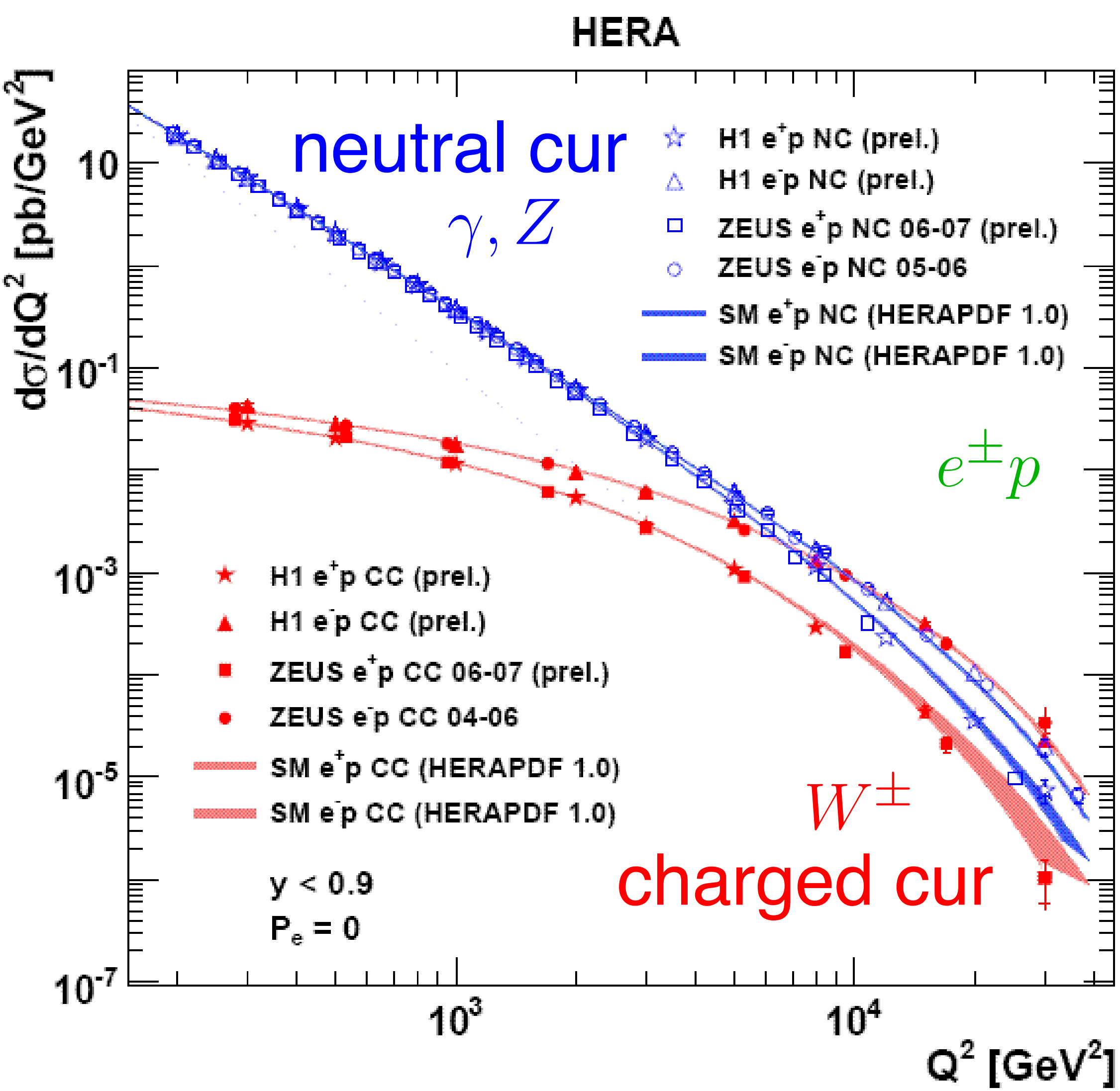
$$x = Q^2/(2\nu)$$

$$y = p.q/p.k = (W^2 + Q^2)/s$$

Experiments:
most important results recently from HERA at DESY
(H1 and ZEUS experiments)

# A crystal-clear example

Electroweak unification

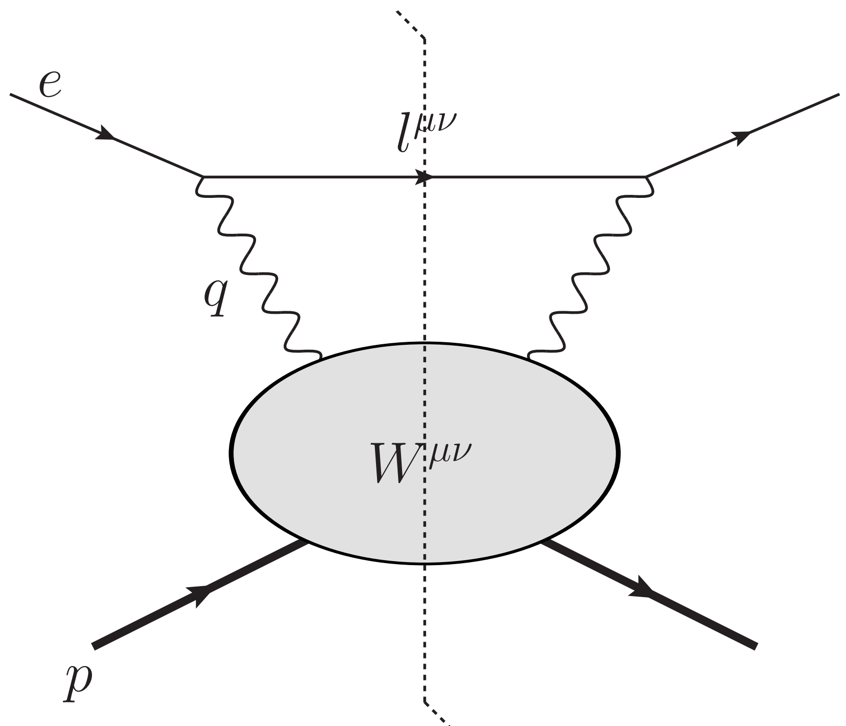



$e^{\pm}$ total x-sect differential in $Q^2$

Neutral currents
$ep \to eX$
via $\gamma, Z$

Charged currents
$ep \to \nu X$
via $W^{\pm}$

# *Process + kinematics*




Factorisation in a leptonic and hadronic part:

$$|\mathcal{M}|^2 = l_{\mu\nu} W^{\mu\nu} \qquad l^{\mu\nu} = 4e^2(k^\mu k'^\nu + k^\nu k'^\mu - g^{\mu\nu} k.k')$$

$\longrightarrow$ study the hadronic tensor $W^{\mu\nu}(W^2, Q^2)$
(or $W^{\mu\nu}(x, Q^2)$)

# *Hadronic tensor*

Most generic structure for $W^{\mu\nu}(x, Q^2)$

$$W^{\mu\nu} = Ag^{\mu\nu} + Bp^{\mu}p^{\mu} + Cq^{\mu}q^{\nu} + Dp^{\mu}q^{\nu} + Eq^{\mu}p^{\nu}.$$

Constraints:

$W^{\mu\nu} = W^{\nu\mu}$ and $q_{\mu}W^{\mu\nu} = 0$ (gauge inv.)

Implying

$$W^{\mu\nu} = -\left(g^{\mu\nu} + \frac{q^{\mu}q^{\nu}}{Q^2}\right)F_1 + \frac{2x}{Q^2}\left(p^{\mu} + \frac{q^{\mu}}{2x}\right)\left(p^{\nu} + \frac{q^{\nu}}{2x}\right)F_2$$

$F_1, F_2(x, Q^2)$: proton structure functions

# *Structure functions*

(inclusive) proton interaction fully parametrised by the 2 structure functions $F_1$ and $F_2(x, Q^2)$

- dimensionless
- $F_L = F_2 - 2xF_1$ (longitudinally-polarized $\gamma^*$)
- For charged currents: additional $F_3(x, Q^2)$

# Parton model

Useful to consider a frame where the proton is highly boosted ($P \gg 1$, $p$ looks like a pancake)

$$p^\mu \equiv (0, 0, P, P)$$

$$n^\mu \equiv (0, 0, \frac{-1}{2P}, \frac{1}{2P}) \qquad (n^2 = 0,\ n.p = 1)$$

$$q^\mu \equiv q_\perp^\mu + \nu n^\mu \qquad (n.q = 0, \vec{q}_\perp^{\,2} = Q^2)$$

We obtain

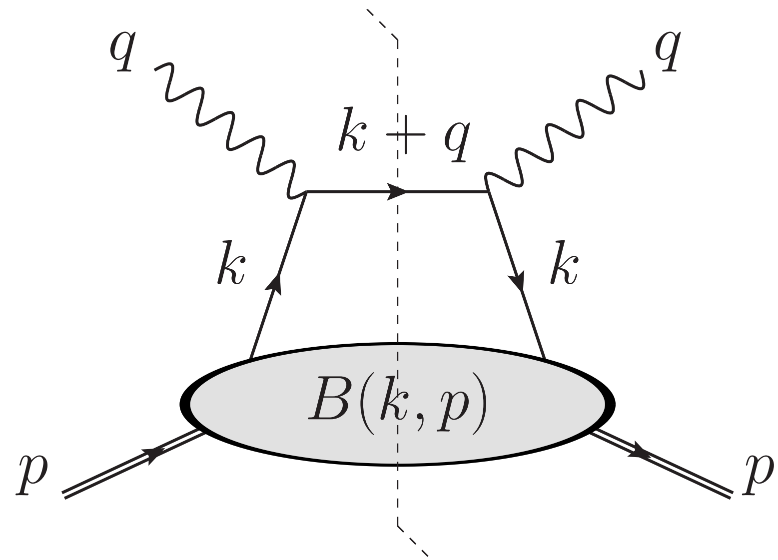
$$F_2 = \nu n^\mu n^\nu W_{\mu\nu}$$

$$F_L = \frac{4x^2}{\nu} p^\mu p^\nu W_{\mu\nu}$$

# Parton model

Bag model

The photon resolves
a quark inside the proton




$$k^\mu = \xi p^\mu + \frac{k^2 + k_\perp^2}{2\xi} n^\mu + k_\perp^\mu$$

$$W^{\mu\nu} = e_q^2 \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\left(\gamma^\mu (\not{k} + \not{q})\gamma^\nu B(k,p)\right) \delta\left((k+q)^2\right)$$

# Parton model

Bag model

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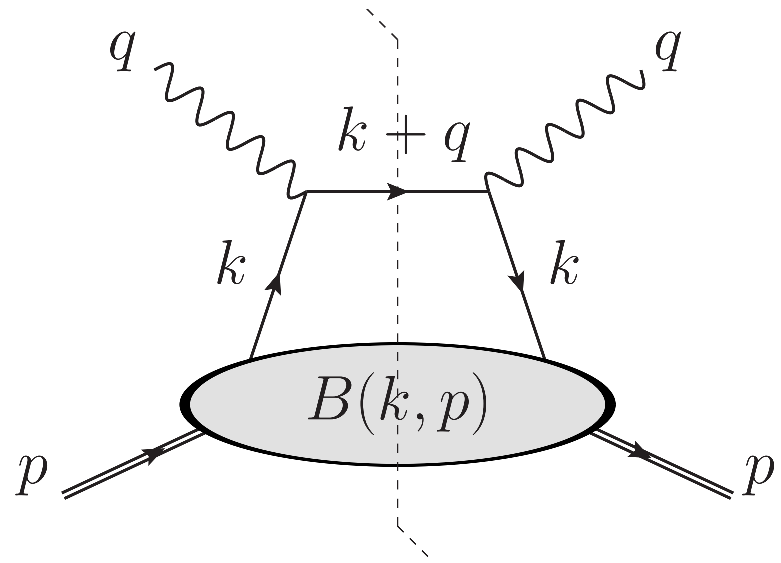


$$k^\mu = \xi p^\mu + \frac{k^2 + k_\perp^2}{2\xi} n^\mu + k_\perp^\mu$$

$$F_2 = \nu e_q^2 \int \frac{d^4 k}{(2\pi)^4} \mathrm{tr}\left(\not{n}\,(\not{k} + \not{q}\,)\not{n}\,B(k,p)\right)\delta\left((k+q)^2\right)$$

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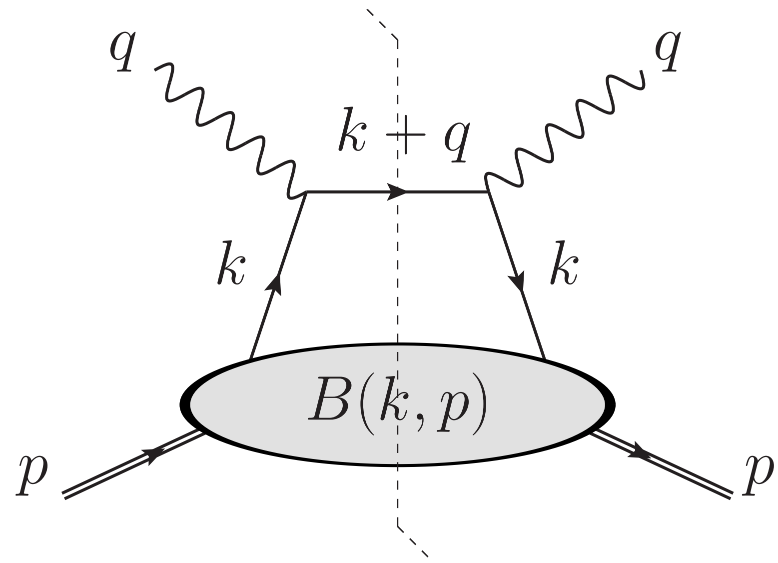


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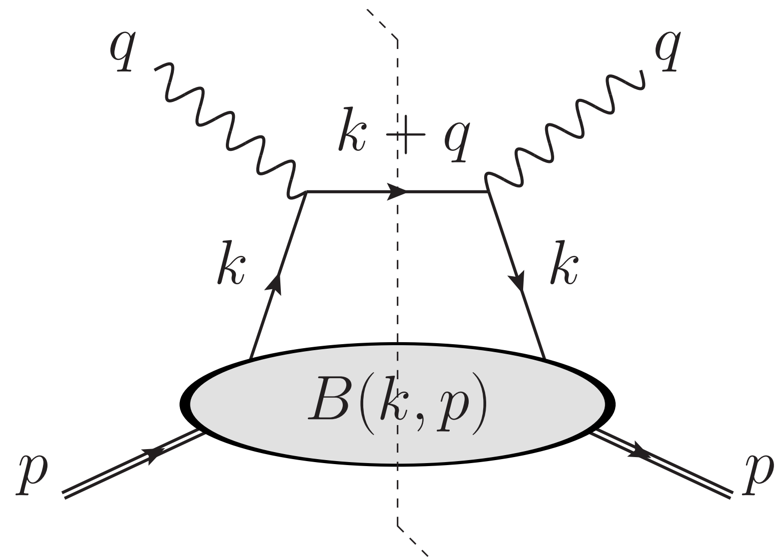
$$\mathrm{tr}\left(\not{n}\,(\not{k} + \not{q})\not{n}\,B(k,p)\right) = 2\xi\,\mathrm{tr}(\not{n}\,B(k,p))$$

# Parton model

Bag model

The photon resolves
a quark inside the proton




$$k^\mu = \xi p^\mu + \frac{k^2 + k_\perp^2}{2\xi} n^\mu + k_\perp^\mu$$

$$F_2 = \nu e_q^2 \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\left(\not{n}\,(\not{k} + \not{q})\,\not{n}\,B(k,p)\right)\delta\left((k+q)^2\right)$$

$$\delta\left((k+q)^2\right) = \delta\left(k^2 - Q^2 + 2\xi\nu - 2\vec{k}_\perp^{\,2}.\vec{q}_\perp^{\,2}\right)$$

$$\overset{Q^2 \gg}{\simeq} \delta(2\nu\xi - Q^2) \simeq \frac{1}{2\nu}\delta(2\nu\xi - Q^2)$$

# Parton model

Putting everything together:

$$F_2 = x e_q^2 \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\left(\not{n}\, B(k,p)\right) \delta(x-\xi)$$

i.e.

$$F_2 = x e_q^2 q(x) \qquad \textbf{with} \qquad q(x) = \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\left(\not{n}\, B(k,p)\right) \delta(x-\xi)$$

with a sum over flavours

$$\boxed{\boxed{F_2 = \sum_q x\, e_q^2 [q(x) + \bar{q}(x)]}}$$

**$q(x)$: parton distribution function (PDF)**

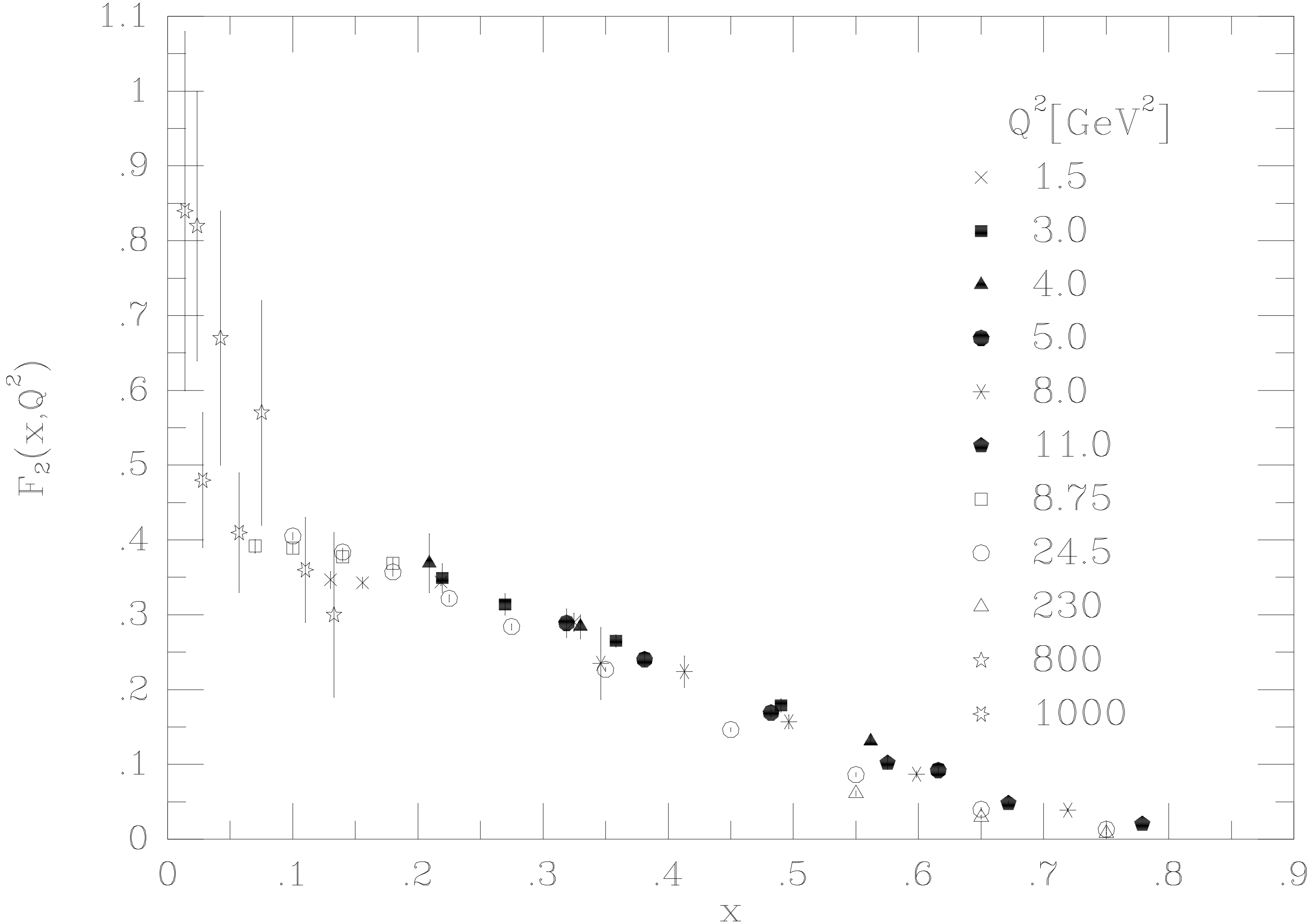
# Parton model

$$\boxed{F_2 = \sum_q x e_q^2 [q(x) + \bar{q}(x)]} \qquad q(x) \equiv \text{ PDF}$$

- interpreted as the probability density to find a quark carrying a fraction $x$ of the proton's momentum (universal!!)
- $F_2(x, Q^2) = F_2(x)$: $Q^2$-independent. Bjorken scaling
- $F_L$ suppressed by $1/Q^2$ compared to $F_2$
  $F_2 = 2xF_1$. Calan-Gross relation: spin 1/2 for q
- charged currents: different quark combinations

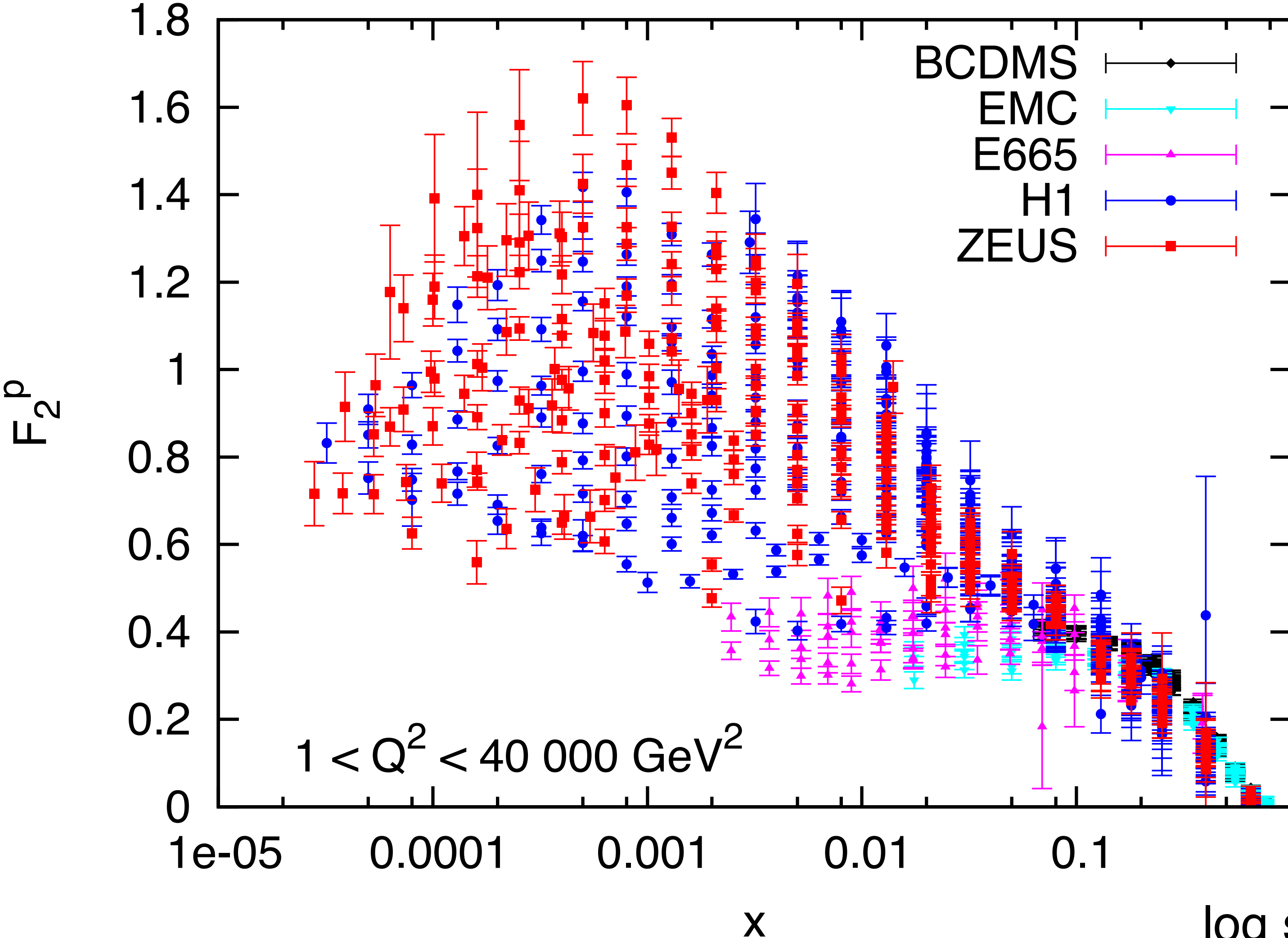
# Bjorken scaling

## $F_2$ from BCDMS, SLAC, NMC, H1 and ZEUS ($\sim 1990$)



# Bjorken scaling violations

HERA measurements ($\sim 1993 - 2007$)

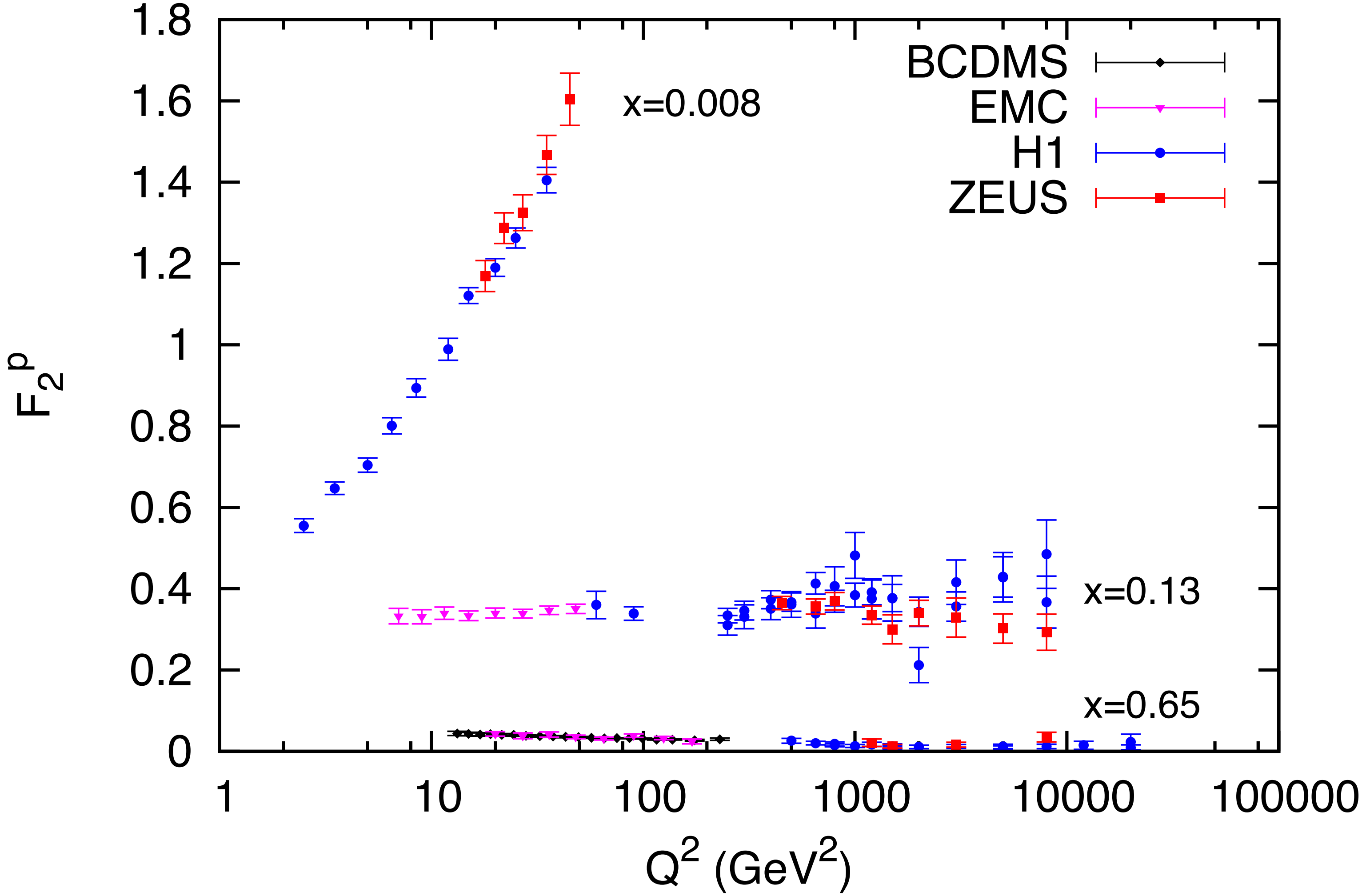



log scale!

Scaling violations!!!

# Bjorken scaling violations

A closer look for 3 bins in $x$



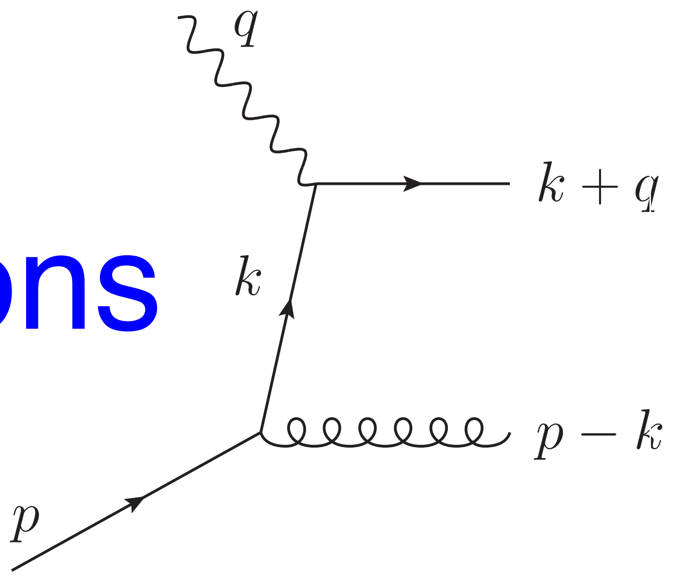

decrease at large $x$ (strong) rise at small $x$

# *Bjorken scaling violations*

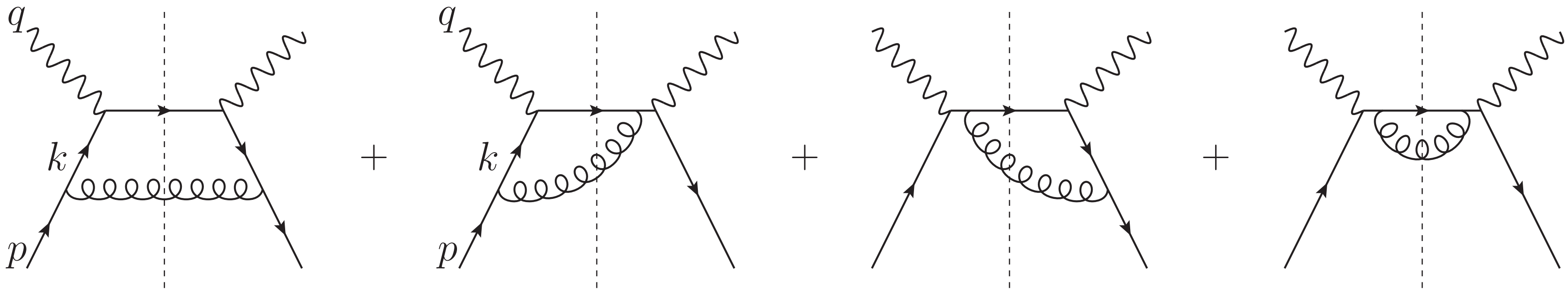
Can we describe the scaling violations in QCD?

Can we describe the scaling violations in QCD?

Idea: quarks can radiate gluons



# One-gluon emission




4 graphs to compute

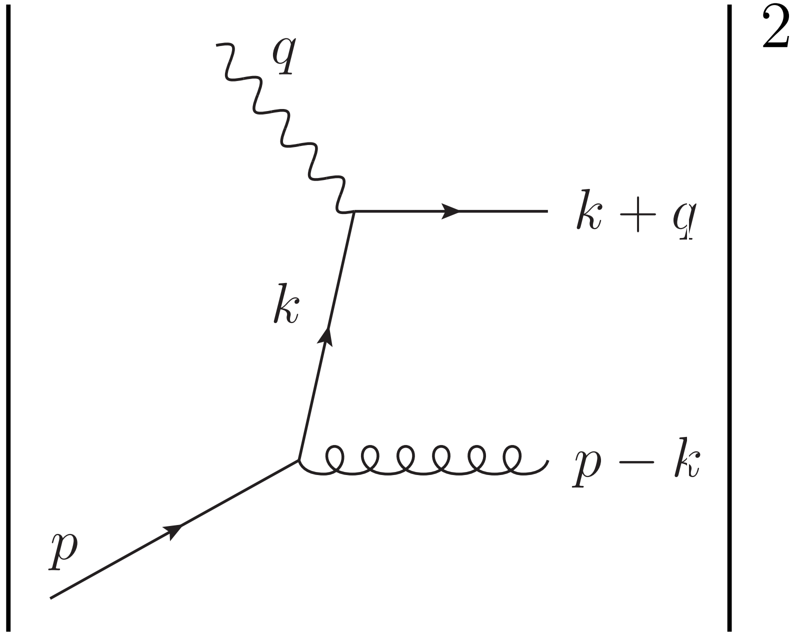
Work in an axial gauge $n.A = 0$ (recall $n^2 = 0$, $n.p = 1$, $n.q = 0$):
gluon of mom $k^\mu$ has propagator

$$d^{\mu\nu}(k) = \left(-g^{\mu\nu} + \frac{n^\mu k^\nu + k^\mu n^\nu}{n.k}\right)\frac{1}{k^2}$$

# One-gluon emission

$$k^\mu = \xi p^\mu + \frac{k_\perp^2 - |k^2|}{2\xi} n^\nu + k_\perp^\mu$$

$$p \equiv (0, 0, P, P)$$




$$n^\mu n^\nu \bar{\sum} |\mathcal{M}|^2 = \frac{1}{2N_c} e_q^2 g^2 \,\mathrm{tr}(t_a t^a)\, \frac{1}{k^4} \mathrm{tr}\left(\not{n}\,(\not{k} + \not{q}\,)\not{n}\,\not{k}\,\gamma^\alpha \not{p}\,\gamma^\beta \not{k}\,\right)$$

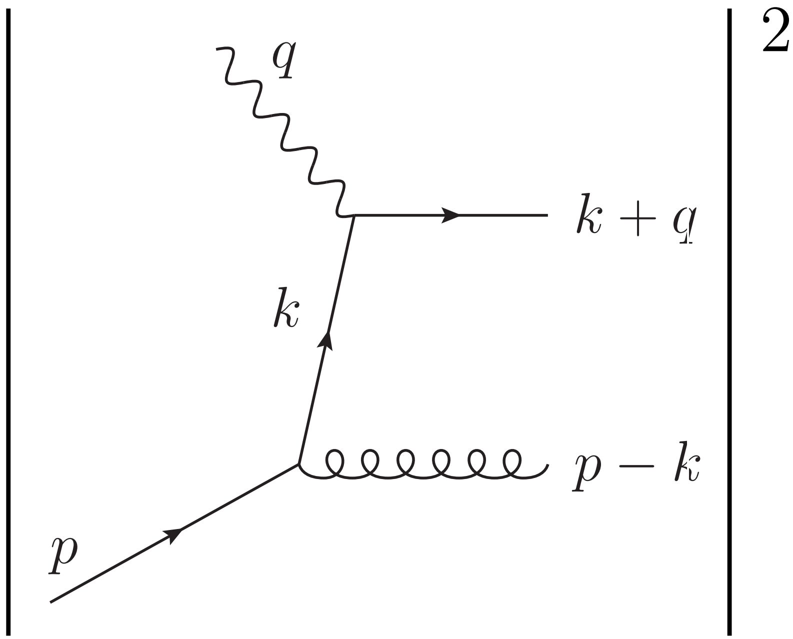
$$\left[-g^{\alpha\beta} + \frac{n^\alpha (p-k)^\beta + (p-k)^\alpha n^\beta}{n.(p-k)}\right]$$

$$= 32\pi\, e_q^2 \alpha_s \frac{\xi P(\xi)}{|k^2|} \qquad P(\xi) = C_F \frac{1+\xi^2}{1-\xi}$$

# One-gluon emission

$$k^\mu = \xi p^\mu + \frac{k_\perp^2 - |k^2|}{2\xi} n^\nu + k_\perp^\mu$$

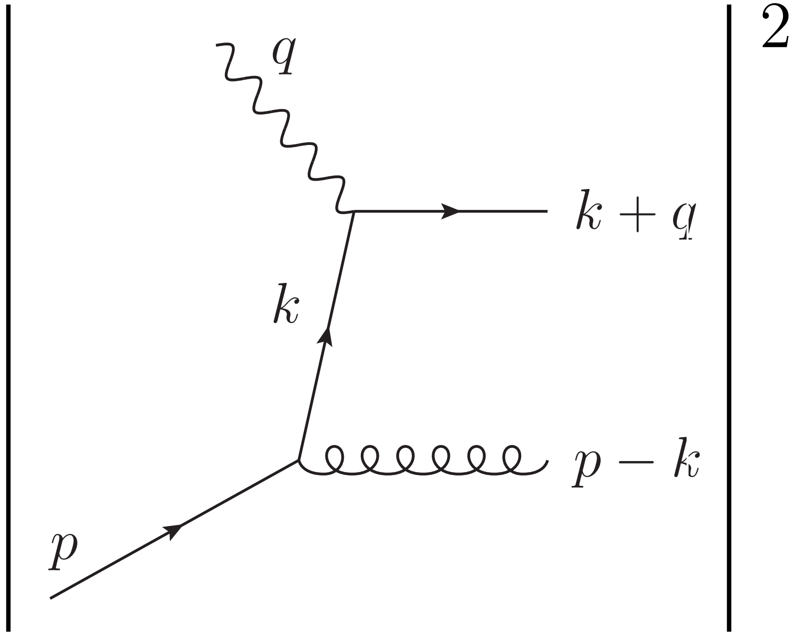
$$P(\xi) = C_F \frac{1+\xi^2}{1-\xi}$$




$$\hat{F}_2 \quad = \quad e_q^2 \frac{\alpha_s}{4\pi^2} \int d\xi \, \xi P(\xi) \int \frac{d|k^2|}{|k^2|} \, dk_\perp^2 \, d\theta \, \delta\left((p-k)^2\right) \delta\left((k+q)^2\right)$$

# One-gluon emission

$$k^\mu = \xi p^\mu + \frac{k_\perp^2 - |k^2|}{2\xi} n^\nu + k_\perp^\mu$$




$$P(\xi) = C_F \frac{1+\xi^2}{1-\xi}$$

$$\begin{aligned}
\hat{F}_2 &= e_q^2 \frac{\alpha_s}{4\pi^2} \int d\xi\, \xi P(\xi) \int \frac{d|k^2|}{|k^2|}\, dk_\perp^2\, d\theta\, \delta\left((p-k)^2\right) \delta\left((k+q)^2\right) \\
&= e_q^2 \frac{\alpha_s}{2\pi^2} \int_0^{2\nu} \frac{d|k^2|}{|k^2|} \int_{\xi_-}^{\xi_+} d\xi\, \frac{\xi P(\xi)}{\sqrt{(\xi_+ - \xi)(\xi - \xi_-)}}
\end{aligned}$$

with $\xi_\pm = x \pm \mathcal{O}(|k^2|/Q^2)$

# One-gluon emission

$$\hat{F}_2 = e_q^2 \frac{\alpha_s}{2\pi^2} \, xP(x) \int_0^{2\nu} \frac{d|k^2|}{|k^2|} = e_q^2 \frac{\alpha_s}{2\pi^2} \, xP(x) \int_0^{Q^2} \frac{d|k^2|}{|k^2|}$$

- other diagrams suppressed by powers of $Q$
- only kept the leading terms in $Q$
- $|k^2|$ integration DIVERGENT!!

# One-gluon emission

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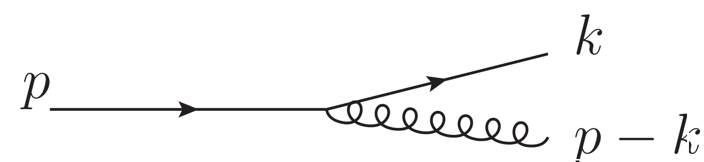
From $\delta((p-k)^2)$ we get $\vec{k}_\perp^{\,2} = (1-\xi)|k^2|$
Thus, $|k^2| \to 0 \Rightarrow \vec{k}_\perp \to 0$




This is thus a collinear divergence! The same as we already encountered in $e^+e^-$ collisions.

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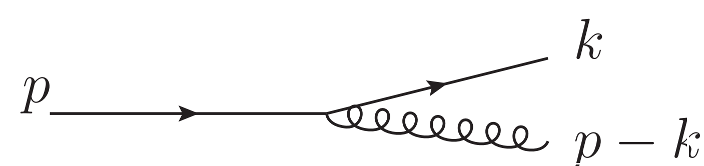


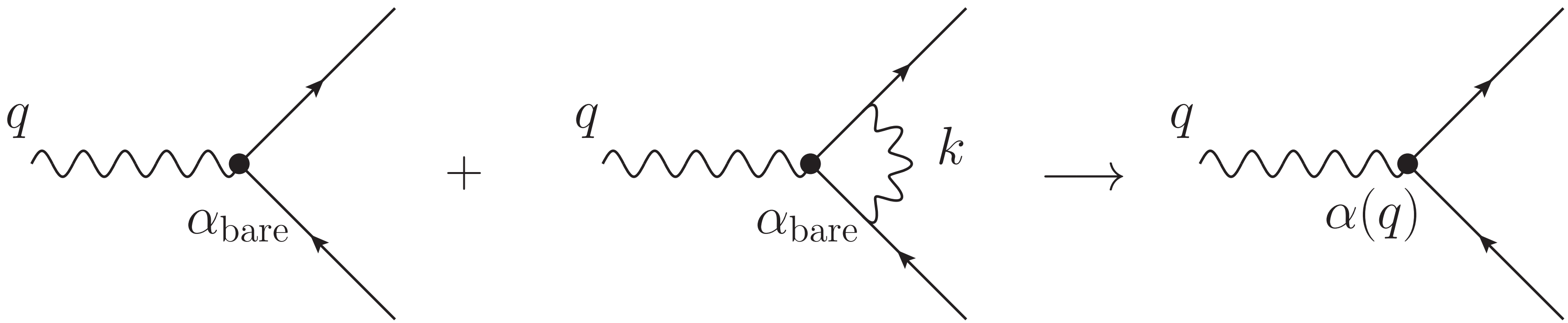

This is thus a collinear divergence! The same as we already encountered in $e^+e^-$ collisions.

Not cancelled by virtual corrections

Here: technique similar to renormalisation

# *Recall: renormalisation*

## Vertex correction in QED




$$\alpha + \beta_0\,\alpha^2 \int_0^{q^2} \frac{dk^2}{k^2} = \alpha + \beta_0\,\alpha^2 \int_0^{\mu^2} \frac{dk^2}{k^2} + beta\,\alpha^2 \int_{\mu^2}^{q^2} \frac{dk^2}{k^2}$$

$$\to \alpha(\mu^2) + \beta_0\,\alpha^2 \int_{\mu^2}^{q^2} \frac{dk^2}{k^2}$$

$$\to \alpha(\mu^2) + \beta_0\,\alpha^2(\mu^2) \int_{\mu^2}^{q^2} \frac{dk^2}{k^2}$$

$$\to \alpha(q^2)$$

# *Recall: renormalisation*

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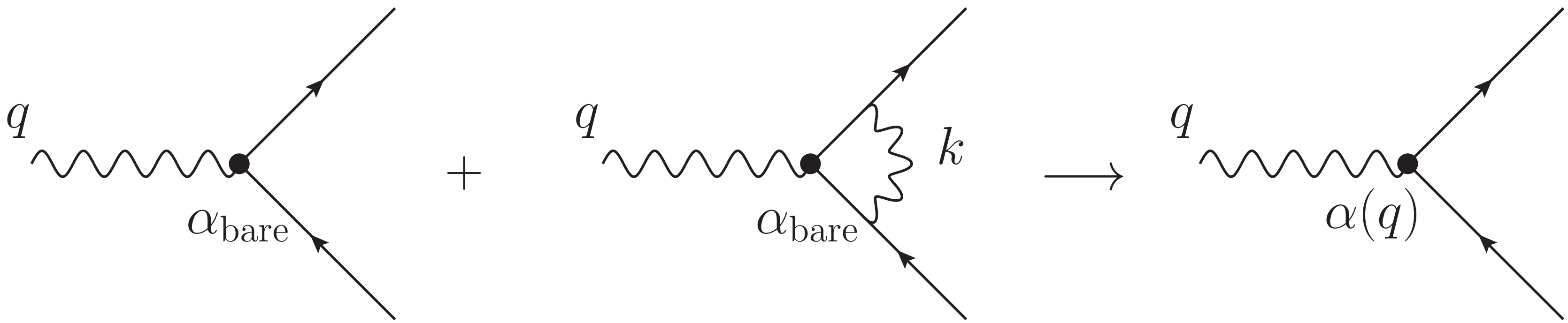


We have defined a scale-dependent coupling

$$\alpha(\mu^2) = \alpha + \beta_0 \, .\alpha^2 \int_0^{\mu^2} \frac{dk^2}{k^2}$$

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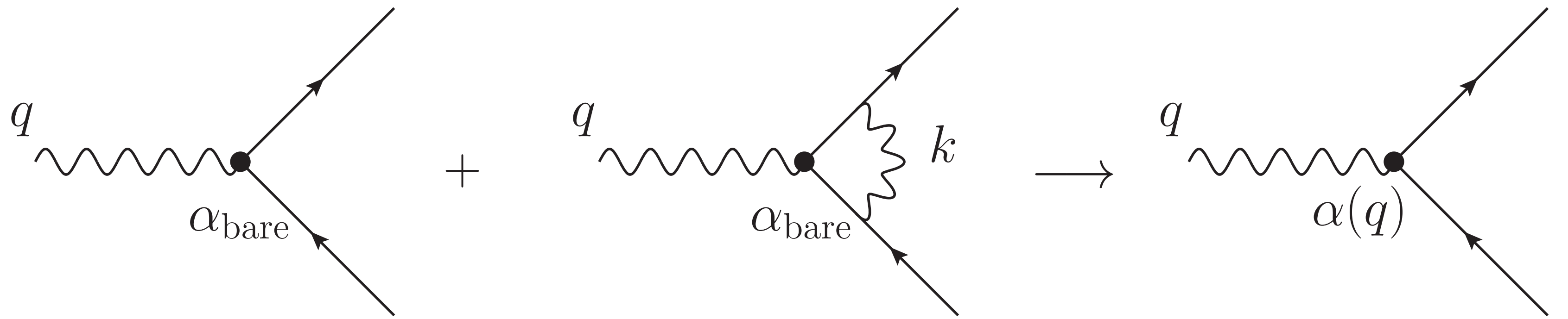



We have defined a scale-dependent coupling

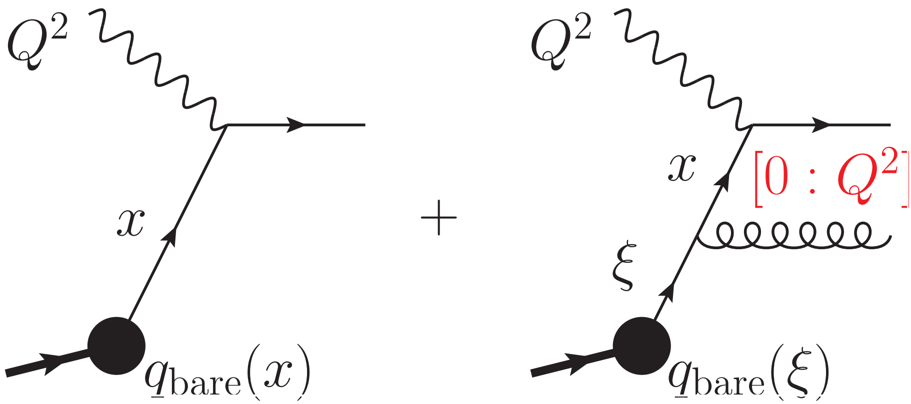
$$\alpha(\mu^2) = \alpha + \beta_0\,.\alpha^2 \int_0^{\mu^2} \frac{dk^2}{k^2}$$

$\mu^2$ is arbitrary *i.e.* physics should not depend on it

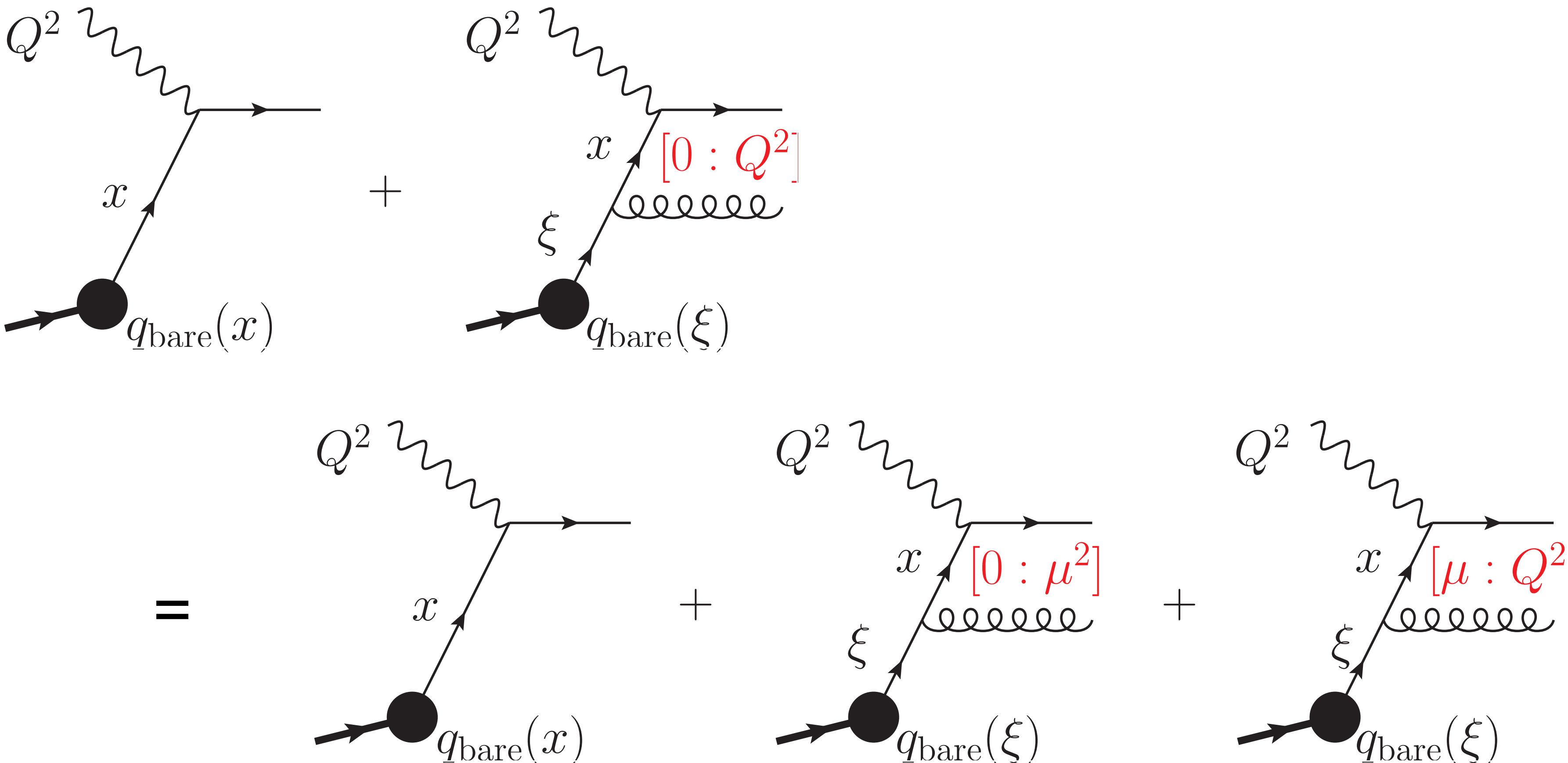
$$\boxed{\mu^2 \partial_{\mu^2} \alpha(\mu^2) = \beta_0 \alpha^2(\mu^2)}$$

renormalisation group equation

# Reabsorption of the collinear divergence



# *Reabsorption of the collinear divergence*



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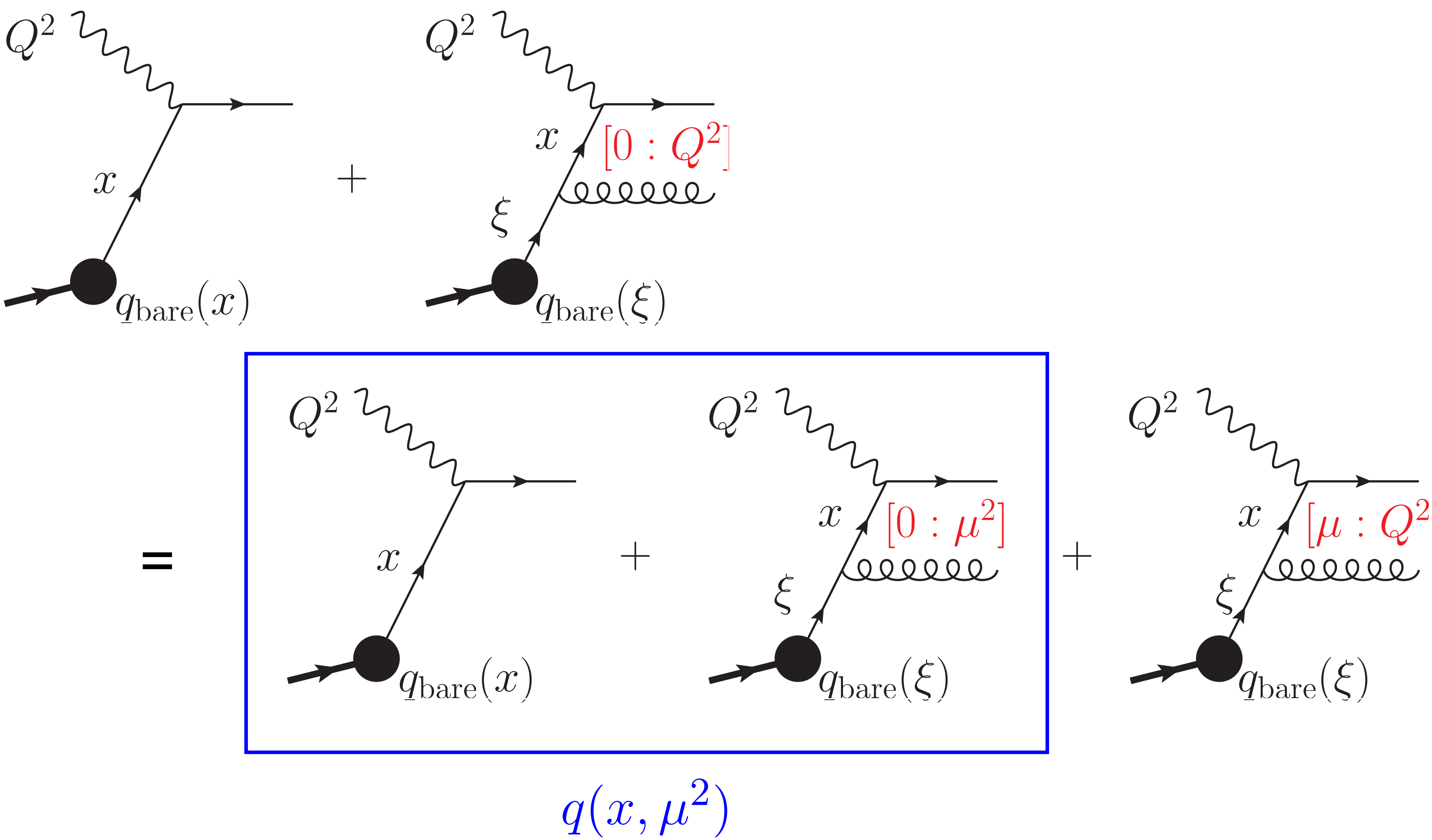

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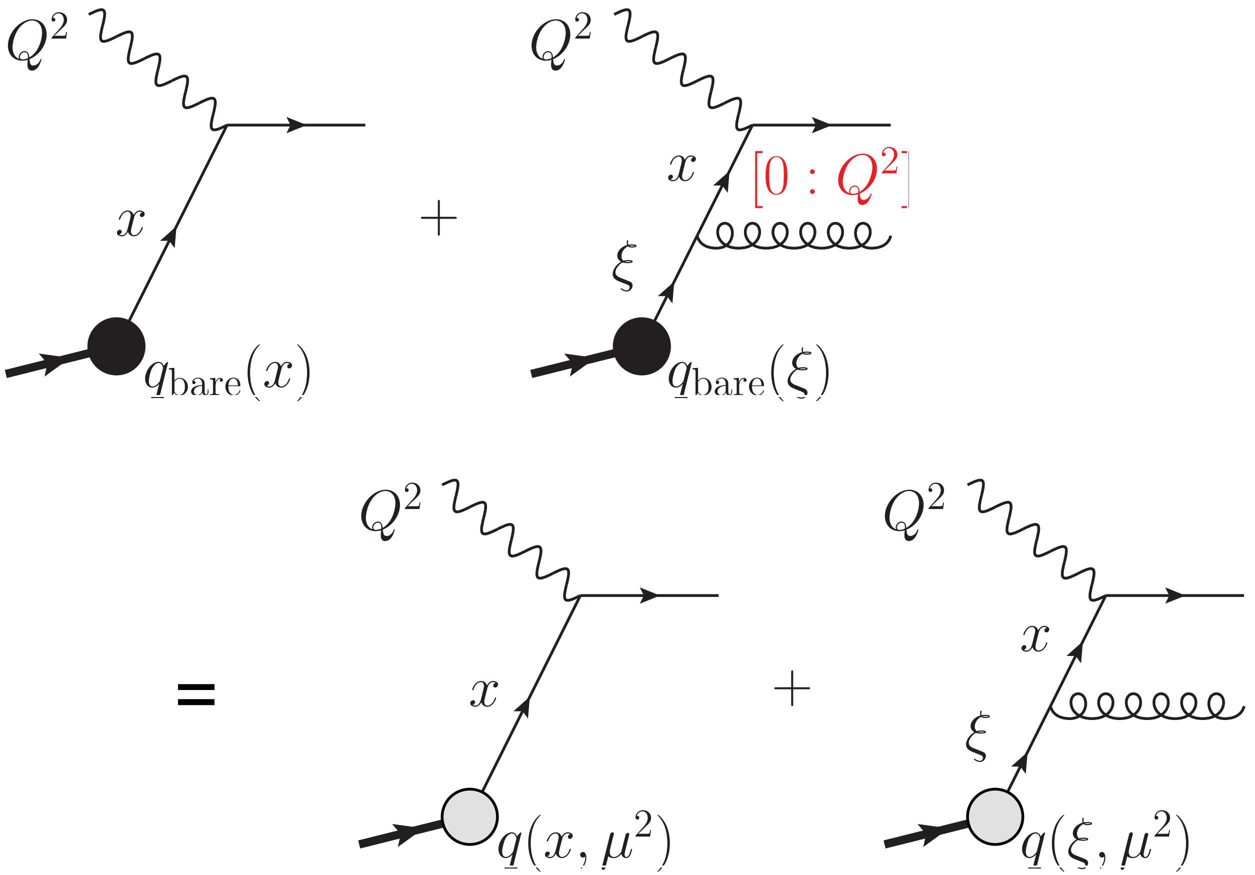

# Reabsorption of the collinear divergence

$$
\begin{aligned}
F_2(x,Q^2) &= xe_q^2\int_x^1 \frac{d\xi}{\xi}\left[\delta\left(1-\frac{x}{\xi}\right)+P\left(\frac{x}{\xi}\right)\int_0^{Q^2}\frac{d|k^2|}{|k^2|}\right]q_{\mathrm{bare}}(\xi)\\
&= xe_q^2\int_x^1 \frac{d\xi}{\xi}\left[\delta\left(1-\frac{x}{\xi}\right)+P\left(\frac{x}{\xi}\right)\int_0^{\mu^2}\frac{d|k^2|}{|k^2|}\right]q_{\mathrm{bare}}(\xi)\\
&\quad + xe_q^2\int_x^1 \frac{d\xi}{\xi}P\left(\frac{x}{\xi}\right)\int_{\mu^2}^{Q^2}\frac{d|k^2|}{|k^2|}q_{\mathrm{bare}}(\xi)\\
&= xe_q^2\int_x^1 \frac{d\xi}{\xi}\left[\delta\left(1-\frac{x}{\xi}\right)+P\left(\frac{x}{\xi}\right)\int_{\mu^2}^{Q^2}\frac{d|k^2|}{|k^2|}\right]q(\xi,\mu^2)\\
&= xe_q^2 q(\xi,Q^2)
\end{aligned}
$$

$P(x)=\frac{\alpha_s}{2\pi}C_F\frac{1+x^2}{1-x}$

# *Reabsorption of the collinear divergence*

We have defined

$$q(x,\mu^2) = q_{\rm bare}(x) + \int_x^1 \frac{d\xi}{\xi} P\left(\frac{x}{\xi}\right) \int_0^{\mu^2} \frac{d|k^2|}{|k^2|} q_{\rm bare}(\xi)$$

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Physics independent of the choice for $\mu^2$

$$\boxed{\mu^2 \partial_{\mu^2} q(x,\mu^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} P\left(\frac{x}{\xi}\right) q(\xi,\mu^2)}$$

DGLAP equation

# The DGLAP equation

$$\boxed{\boxed{Q^2\partial_{Q^2}q(x,Q^2) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}P\left(\frac{x}{\xi}\right)q(\xi,Q^2)}}$$

- DGLAP: Dokshitzer-Gribov-Lipatov-Altarelli-Parisi

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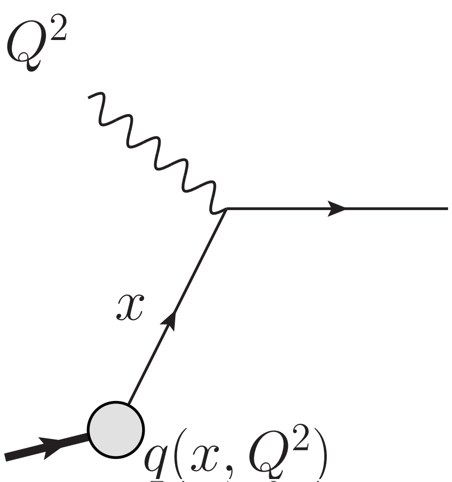
- $\mu$ called the factorisation scale
- Leading order computation in $\alpha_s \log(Q^2/\mu^2)$
- Actually resums all terms $\alpha_s^n \log^n(Q^2/\mu^2)$
  (recall: $\alpha_s \log(Q^2/\mu^2) \sim 1 \Rightarrow$ compute at all orders)

# The DGLAP equation: resummation

$$\boxed{Q^2 \partial_{Q^2} q(x, Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} P\left(\frac{x}{\xi}\right) q(\xi, Q^2)}$$



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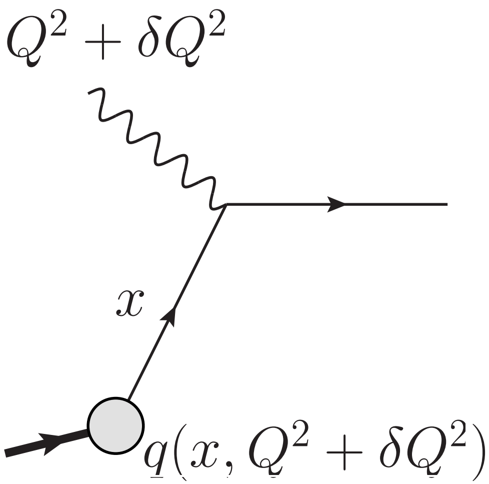

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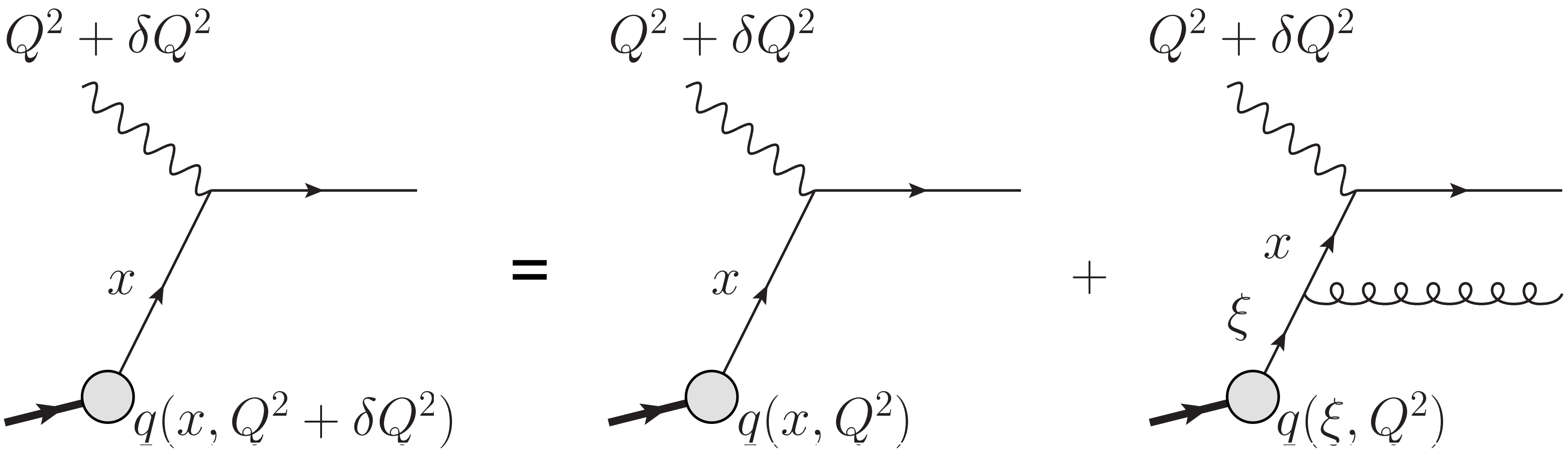

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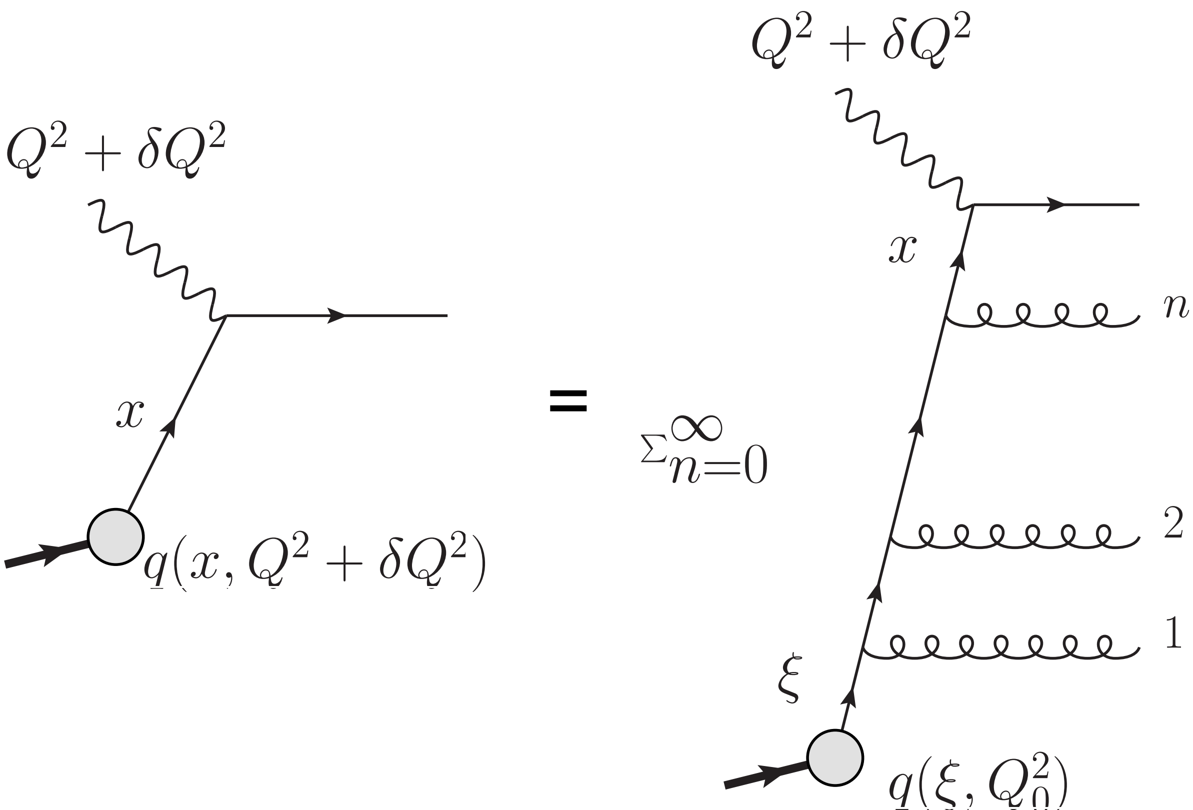


Resumming (leading) contributions $\alpha_s^n \log^n(Q^2/Q_0^2)$

# *The DGLAP equation: splitting function*

$$\boxed{Q^2 \partial_{Q^2} q(x, Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} P\left(\frac{x}{\xi}\right) q(\xi, Q^2)}$$

$P(\xi)$ called the splitting function:

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Correction due to virtual-gluon emission:

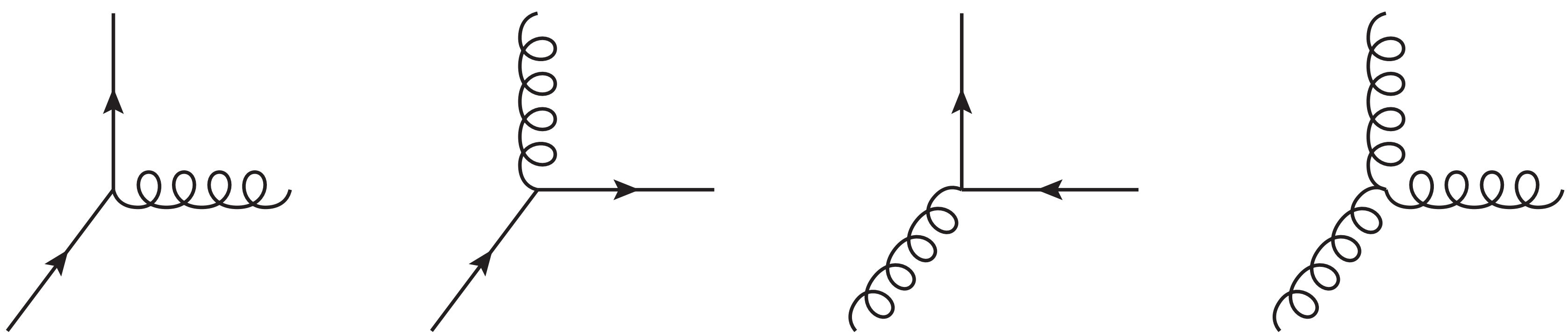
$$P(x) = C_F \left[\frac{1+x^2}{1-x}\right]_+$$

NB: the $1/(1-x)$ behaviour is the soft QCD divergence

# The DGLAP equation: splitting function

$$Q^2\partial_{Q^2}\begin{pmatrix} q(x,Q^2) \\ g(x,Q^2)\end{pmatrix} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg}\end{pmatrix}\left(\frac{x}{\xi}\right)\begin{pmatrix} q(\xi,Q^2) \\ g(\xi,Q^2)\end{pmatrix}$$

$P_{ab}(\xi)$ called the splitting function:



$P_{qq}$ $\quad$ $P_{gq}$ $\quad$ $P_{qg}$ $\quad$ $P_{gg}$

$P_{ab}(x)$ is the probability to obtain a parton of type $a$ carrying a fraction $x$ of the longitudinal momentum of a parent parton of type $b$

# *DGLAP and the factorisation theorem*

The result is more general: it holds at any order in perturbation theory

$$Q^2 \partial_{Q^2} q(x, Q^2) = \int_x^1 \frac{d\xi}{\xi} P\left(\frac{x}{\xi}\right) q(\xi, Q^2)$$

with

$$P(x) = \left(\frac{\alpha_s}{2\pi}\right) P^{(0)}(x) + \left(\frac{\alpha_s}{2\pi}\right)^2 P^{(1)}(x) + \left(\frac{\alpha_s}{2\pi}\right)^3 P^{(2x)}(x) + \dots$$

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- LO resums $\alpha_s^n \log^n(Q^2/\mu^2)$ (leading logarithms)
- NLO resums $\alpha_s^n \log^n(Q^2/\mu^2)$ and $\alpha_s^{n+1} \log^n(Q^2/\mu^2)$

Note: order refers to $P$; includes diagrams at all orders

Note: known up to NNLO since 2004 (Moch, Vermaseren, Vogt)

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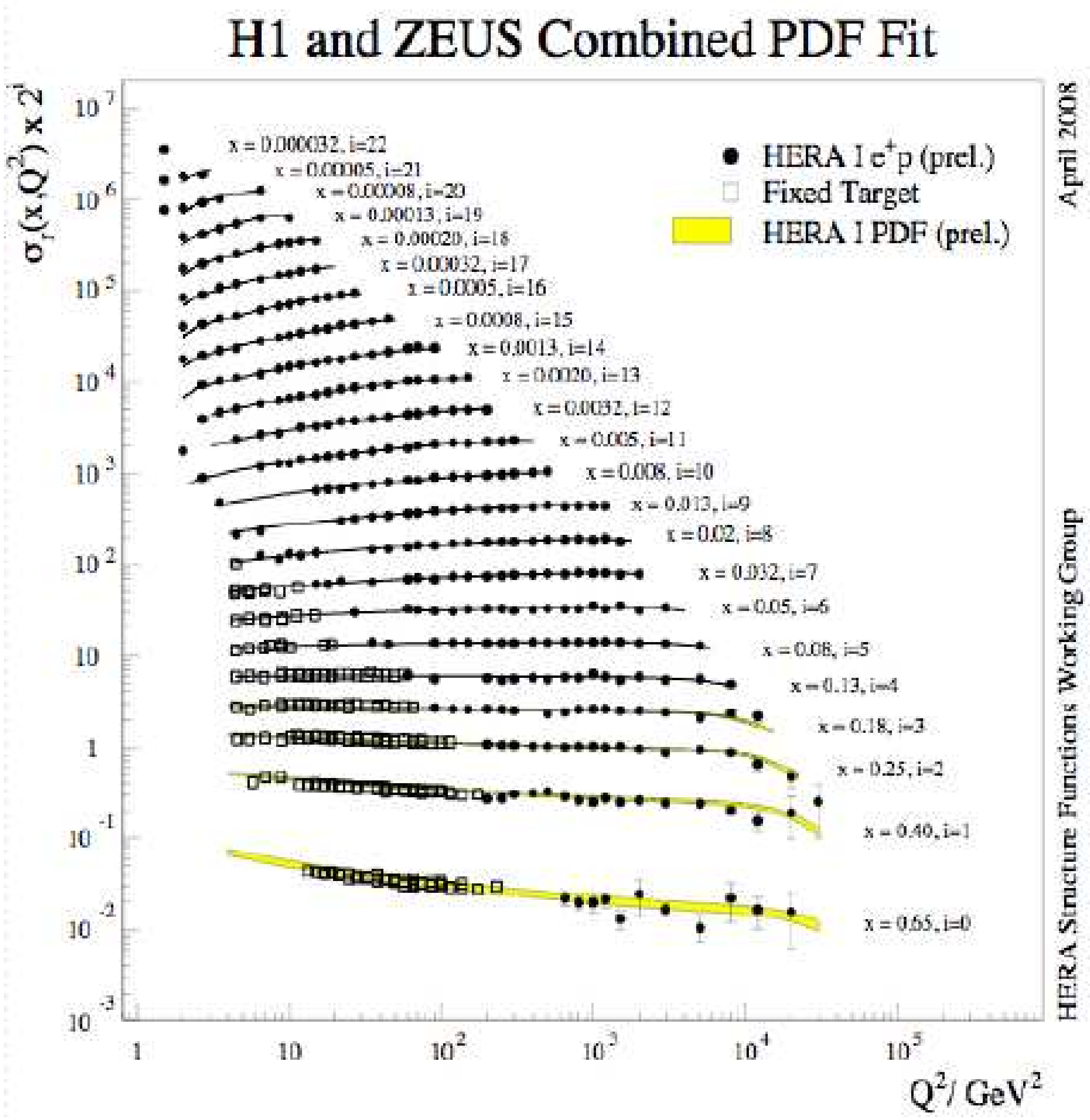
Fundamental result in QCD know as the <u>factorisation theorem</u>

Collinear divergences can be reabsorbed in the definition of the PDFs at all orders!

# DGLAP vs. data

Very nice description of the $Q^2$-dependence observed in the data



# *DGLAP vs. data*

DGLAP only gives the $Q^2$ evolution of the PDFs
One still needs an initial condition $f_a(x, \mu^2)$

Global PDF fit:

- Parametrise $q$ and $g$ at an initial scale $\mu^2$
  *e.g.* $q(x, \mu^2) = x^\lambda (1-x)^\beta (A + B\sqrt{x} + Cx)$
- Obtain the PDFs $f_a(x, Q^2)$ at all $Q^2$ using DGLAP
- Compute a series of observables (*e.g.* $F_2$)
- Fit the experimental measurements ($\chi^2$ minimisation)

# DGLAP vs. data

- Many teams: MSTW/MRST, CTEQ, NNPDF, HERA, H1, ZEUS, Alekhin, GRV
- Each with many updates

  e.g. CTEQ4l, CTEQ4m, CTEQ5l, CTEQ5m, CTEQ6, CTEQ6l, CTEQ6m, CTEQ61, CTEQ65, CTEQ66

  MRST98, MRST2001, MRST2002, MRST2003, MRST2004, MRST2006, MRST2007, MSTW2008

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  - Choice of initial scale
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  - Order of the fit (LO, NLO, NNLO)
  - Data selection (*e.g. cuts, old vs. new data*)
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  - List of observables (9)

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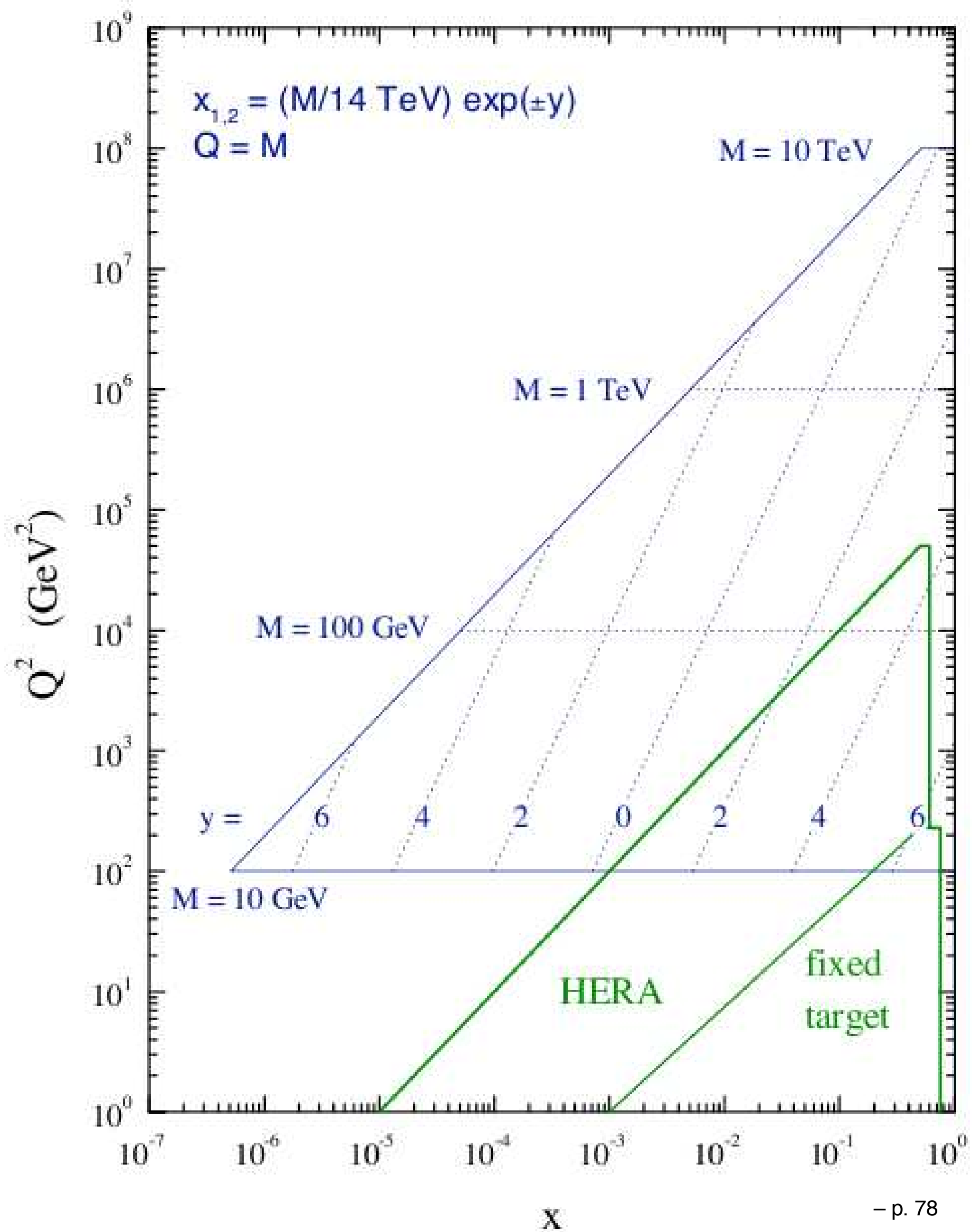
    $F_2^p$, $F_2^d$, $F_L$, $F_2^\nu$, $F_3^\nu$, $F_2^c$, $F_2^b$, Drell-Yan, Tev. jets

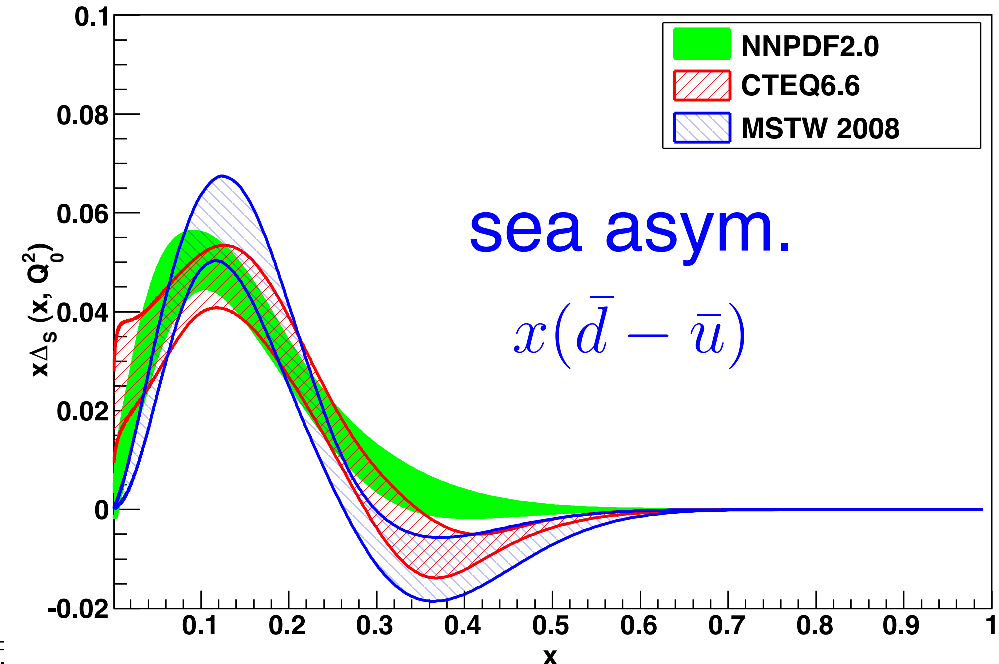
# Global fits

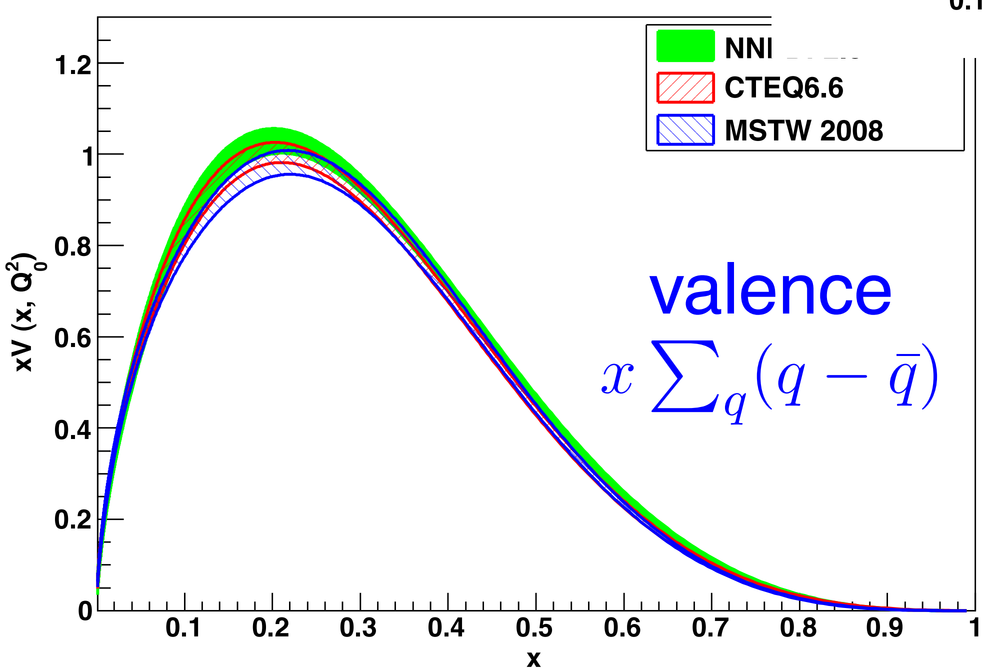
Global fits are important for LHC physics as they affect every perturbative computation



# Global fits

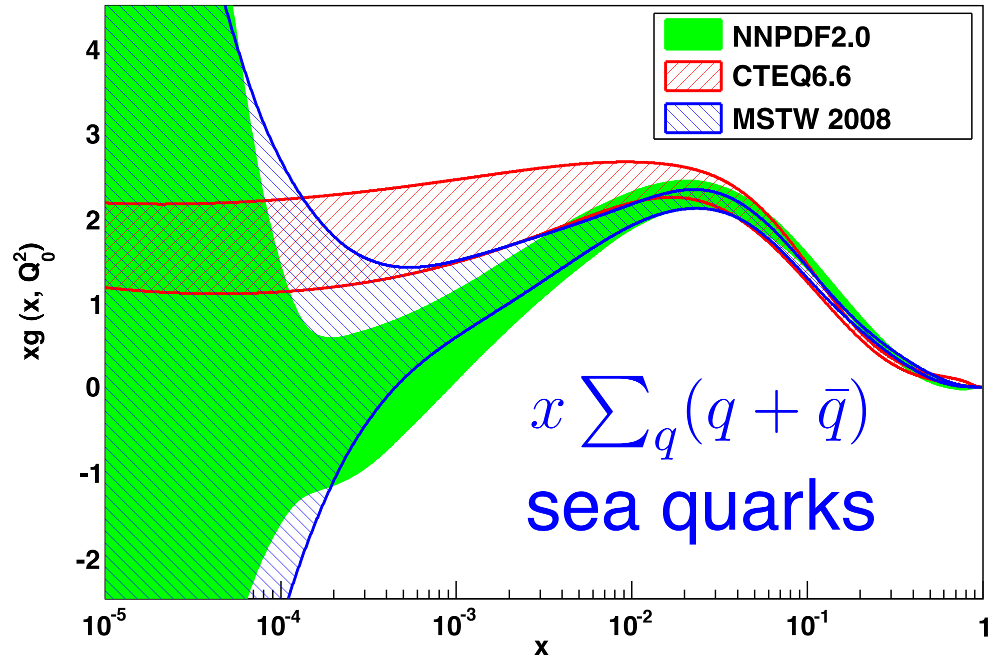
Initial distributions
$Q^2 = \mu^2 = 2$ GeV

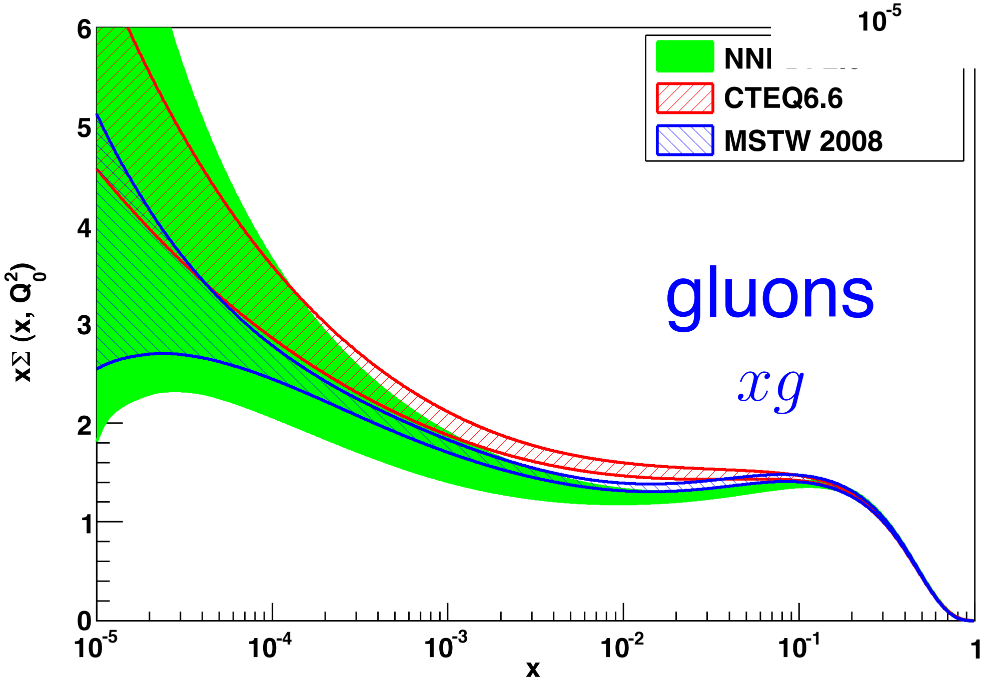


# Global fits

Initial 'flavour-singlet' distributions
$Q^2 = \mu^2 = 2$ GeV

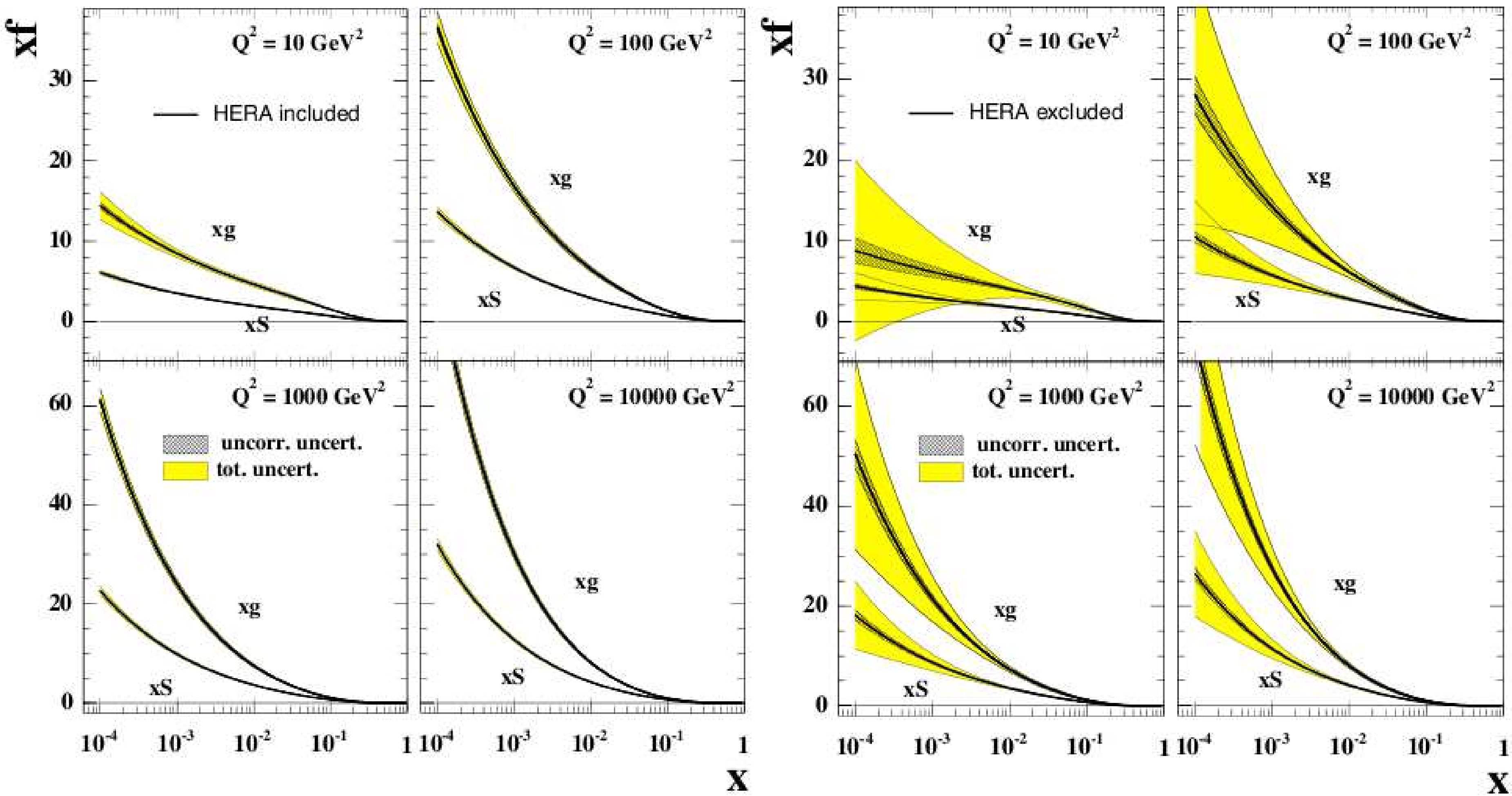


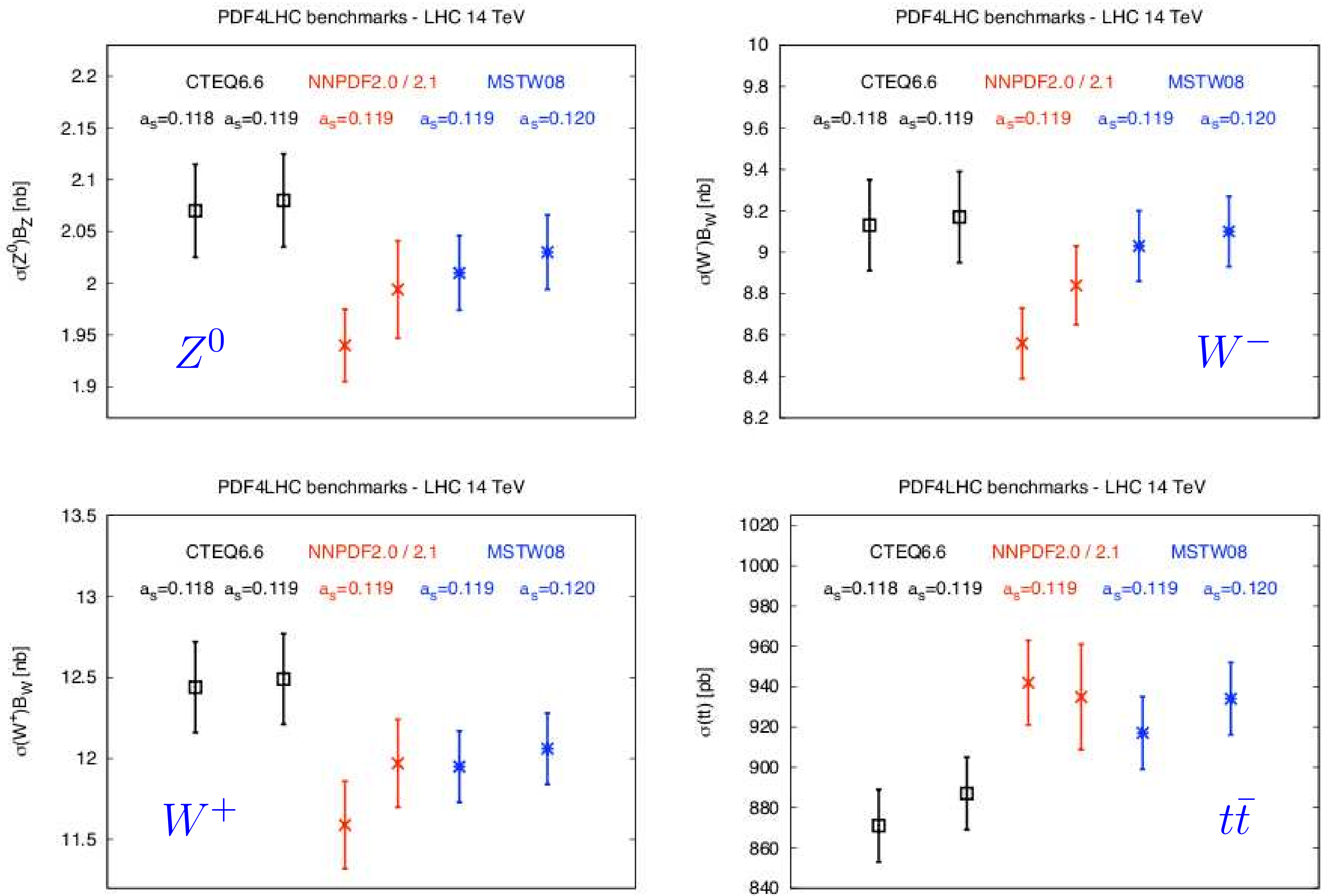
# Global fits

## Impact of HERA measurements

With HERA | Without HERA



# Global fits



PDF4LHC benchmarks - LHC 14 TeV
CTEQ6.6
NNPDF2.0 / 2.1
MSTW08
a_s=0.118 a_s=0.119 a_s=0.119 a_s=0.119 a_s=0.120
σ(Z⁰)B_Z [nb]
Z⁰
σ(W⁻)B_W [nb]
W⁻
σ(W⁺)B_W [nb]
W⁺
σ(tt) [pb]
tt̄

# DIS: summary

DIS: $\gamma^* p$ scattering with highly virtual $\gamma$ $(Q^2 \gg \Lambda^2_{\mathrm{QCD}})$

- Parton model
  - directly probes partons inside the proton
  - Bjorken scaling

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- Parton model
  - directly probes partons inside the proton
  - Bjorken scaling
- QCD collinear divergences
  - Violations of Bjorken scaling
  - Factorisation theorem/DGLAP equation (fundamental result/prediction of QCD)
  - Parton Distribution Functions (PDF)
  - Global fits for the PDF determination of the PDFs: mandatory for precision at the LHC

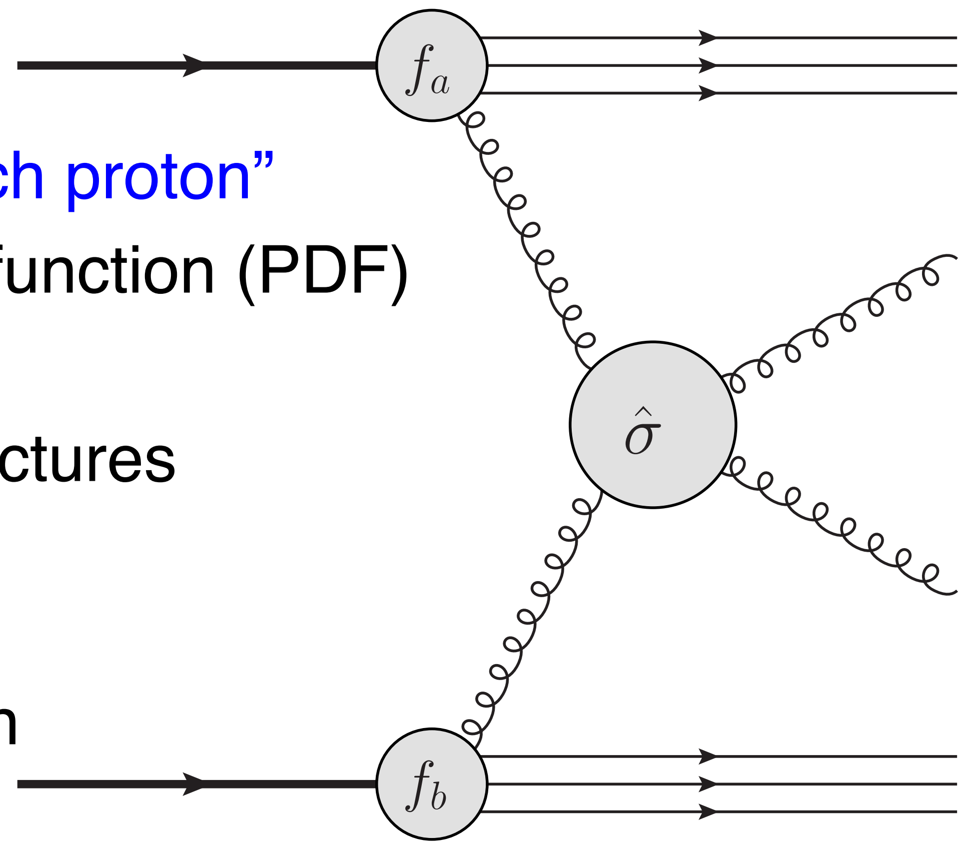
*Time for questions!*

# $pp$ collisions

**(at last!)**

# The very fundamental collision

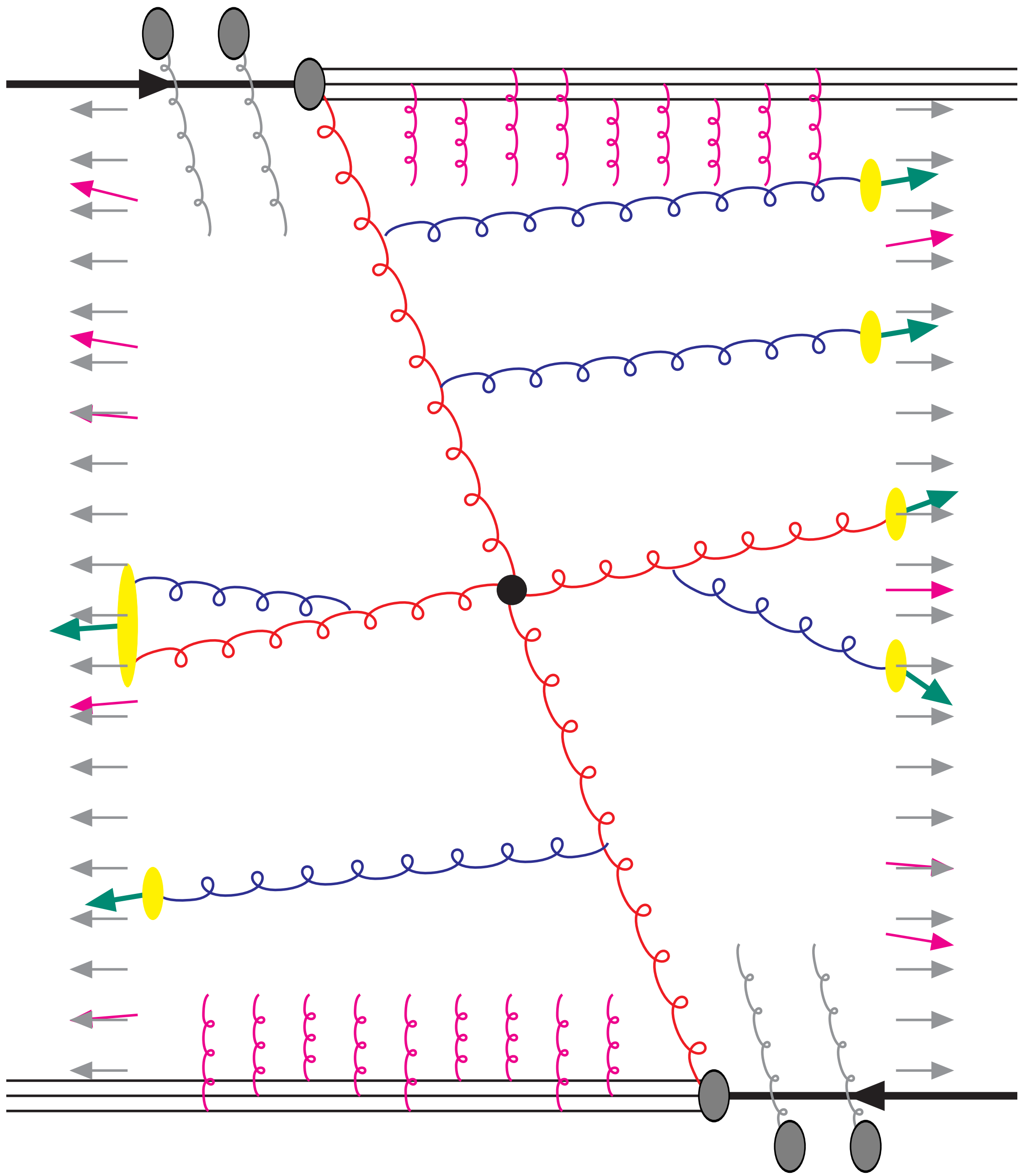
$$\sigma = f_a \otimes f_b \otimes \hat{\sigma}$$




- “take a parton out of each proton”
  $f_a \equiv$ parton distribution function (PDF)
  for quark and gluons
  a big chapter of these lectures

- hard matrix element
  perturbative computation
  Forde-Feynman rules

# The more realistic version



- Hard ME

  perturbative

- Parton branching

  initial+final state radiation

- Hadronisation

  $q, g \to$ hadrons

- Multiple interactions

  Underlying event (UE)

- Pile-up

  $\lesssim 25$ $pp$ at the LHC

# Plan

- A few generic considerations
  - kinematics (done)
  - Monte-Carlo

- Processes one-by-one
  - Drell-Yan
  - Jets (done)
  - $W/Z$ (+jets)
  - top
  - $H$
  - SUSY (?)

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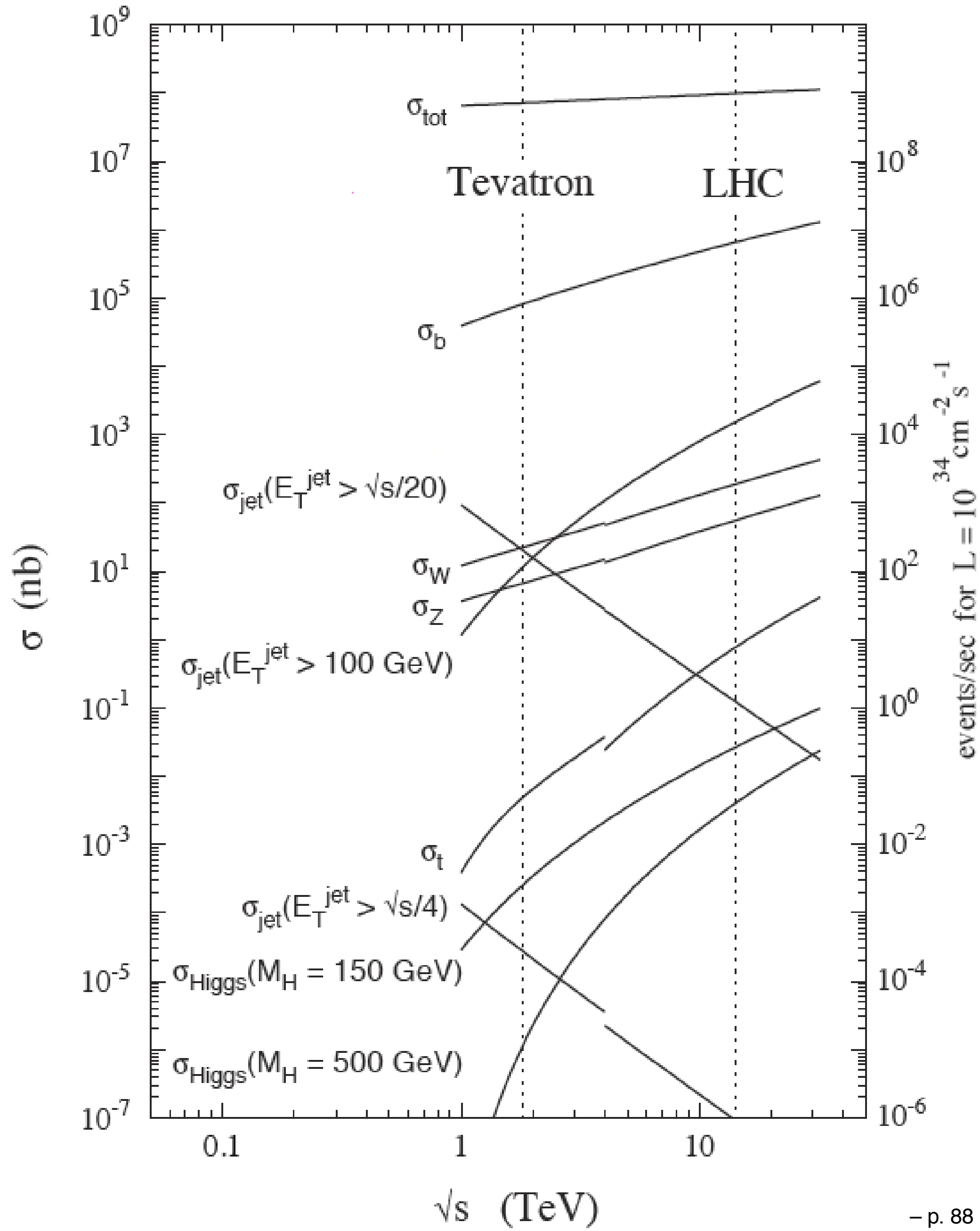


# *Parton luminosities*

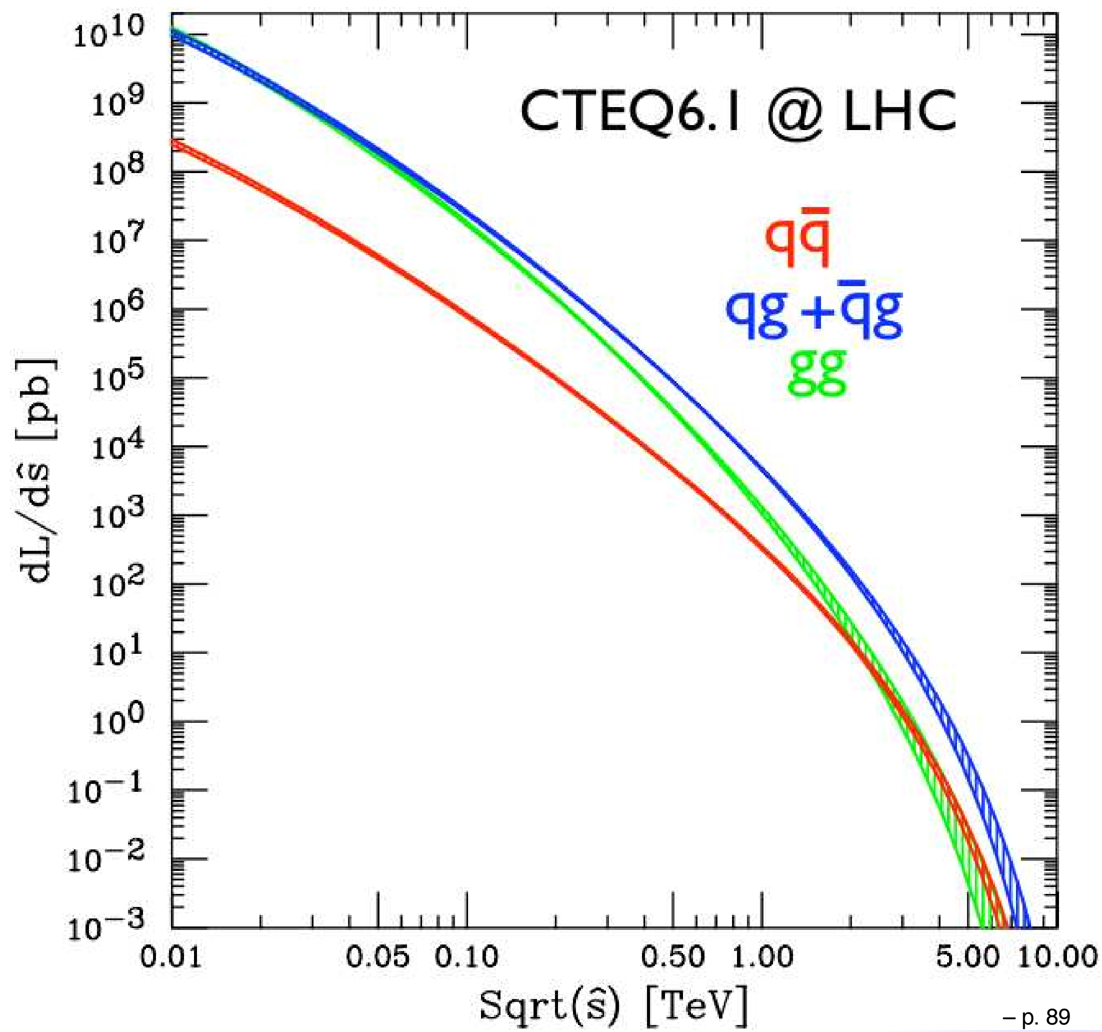
Vary $\sqrt{s} \Rightarrow$ same ME, only PDF vary

$$\sigma = \sum \int dx_1\, dx_2\, f_a(x_1) f_b(x_2)\, \hat{\sigma}$$

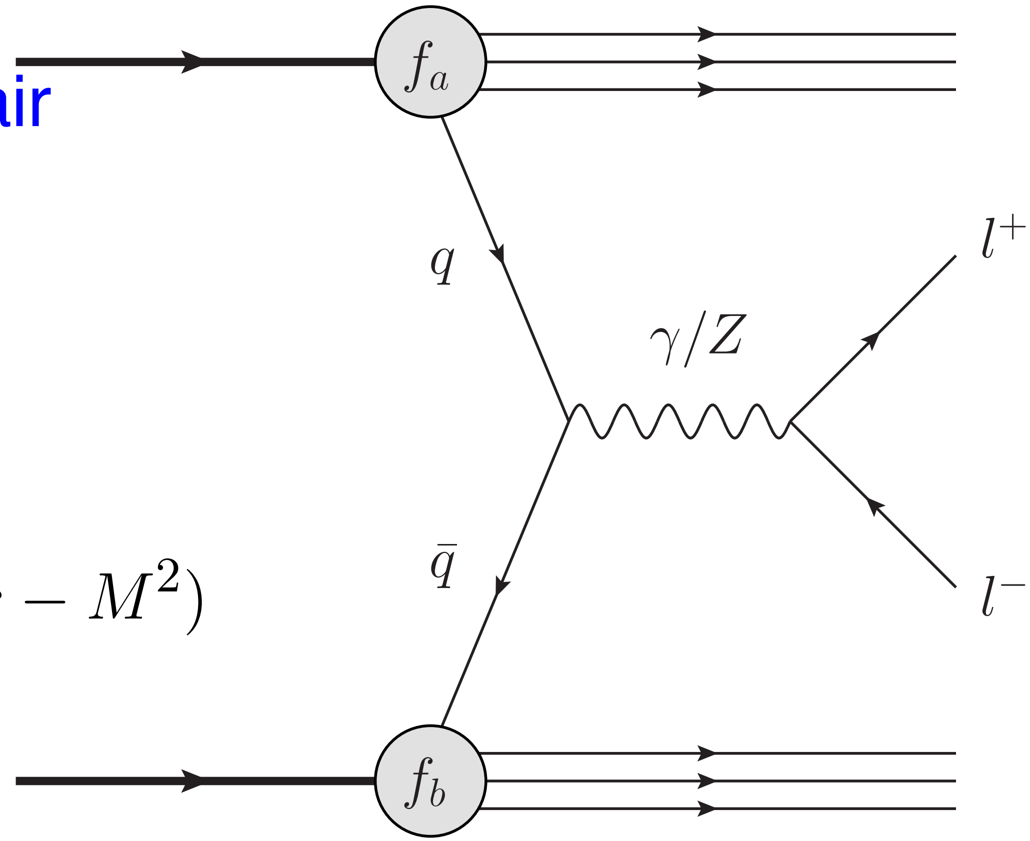
$$= \sum_{ij} \int d\hat{s}\, \frac{dL_{ij}}{d\hat{s}}\, \hat{\sigma}(\hat{s})$$

NB: Tevatron: $p\bar{p}$

LHC: $pp$



# Drell-Yan




Production of a lepton pair
(of mass $M$)

Hard matrix element:

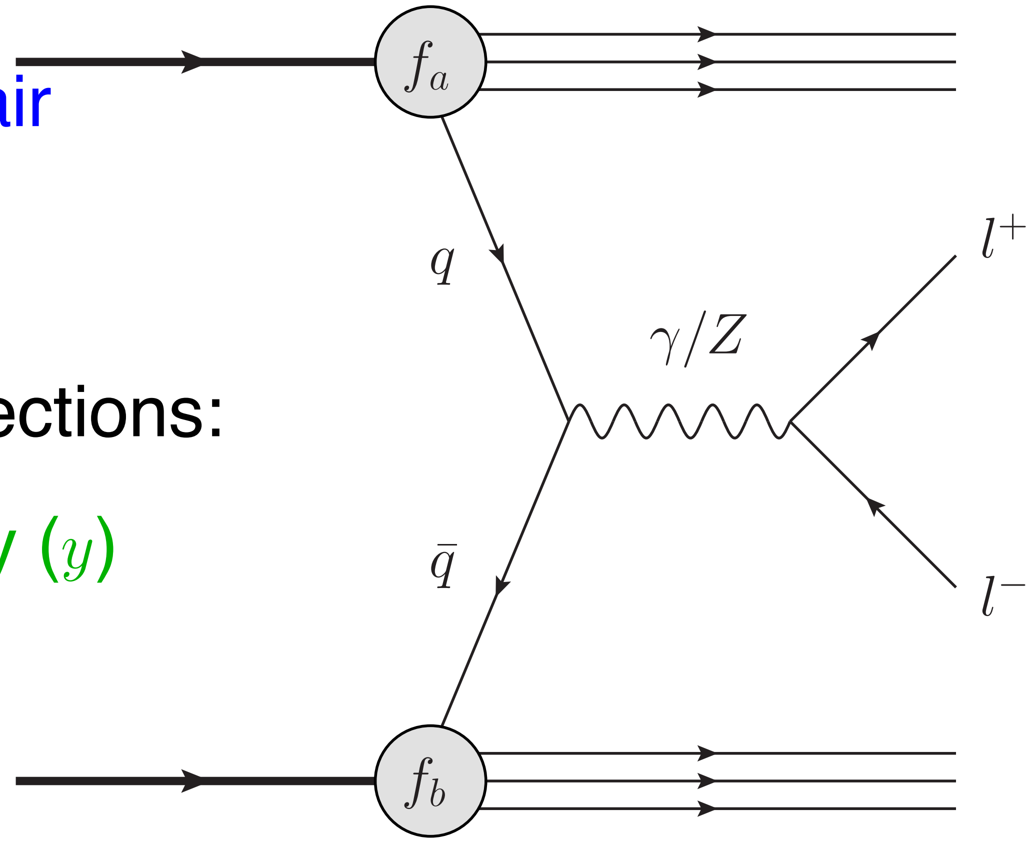
$$\frac{d\hat{\sigma}}{dM^2} = \frac{e_q^2 N_c}{N_c^2} \frac{4\pi\alpha^2}{3M^2} \delta(x_1 x_2 s - M^2)$$

Lowest order (PDF$_1 \otimes$PDF$_2 \otimes$ME)

$$\frac{d\sigma}{dM^2} = \int dx_1\, dx_2 \sum_q [q(x_1, M^2)\bar{q}(x_2, M^2) + (1 \leftrightarrow 2)] \frac{d\hat{\sigma}}{dM^2}$$

# *Drell-Yan*

Production of a lepton pair
(of mass $M$)




More differential cross-sections:

Ex. 1: lepton-pair rapidity ($y$)

$\Rightarrow \delta(x_1 x_2 s - M^2)$
$\delta(y - \frac{1}{2}\log(x_1/x_2))$

$$\frac{d^2\sigma}{dM^2 dy} = \sum_q \frac{4\pi e_q^2 \alpha^2}{3N_c M^2 s}\left[q(\frac{M}{\sqrt{s}}e^y, M^2)\bar{q}(\frac{M}{\sqrt{s}}e^{-y}, M^2) + (y \leftrightarrow -y)\right]$$

# Drell-Yan

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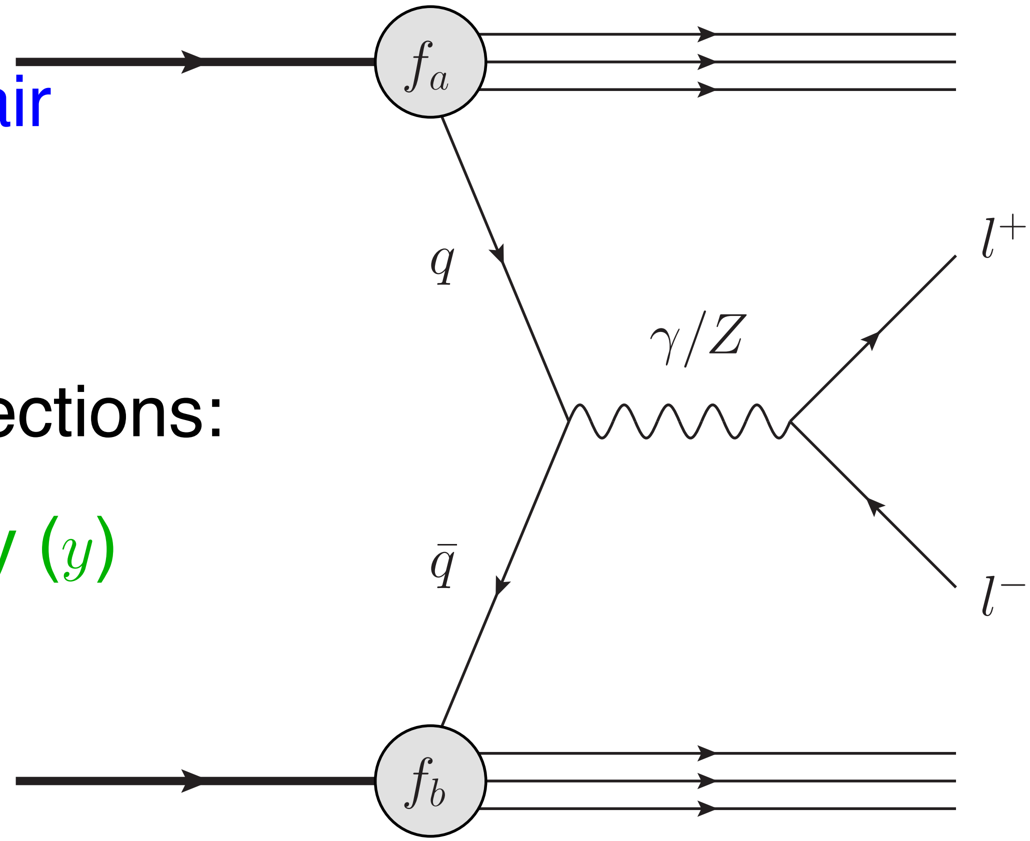



More differential cross-sections:

Ex. 1: lepton-pair rapidity ($y$)

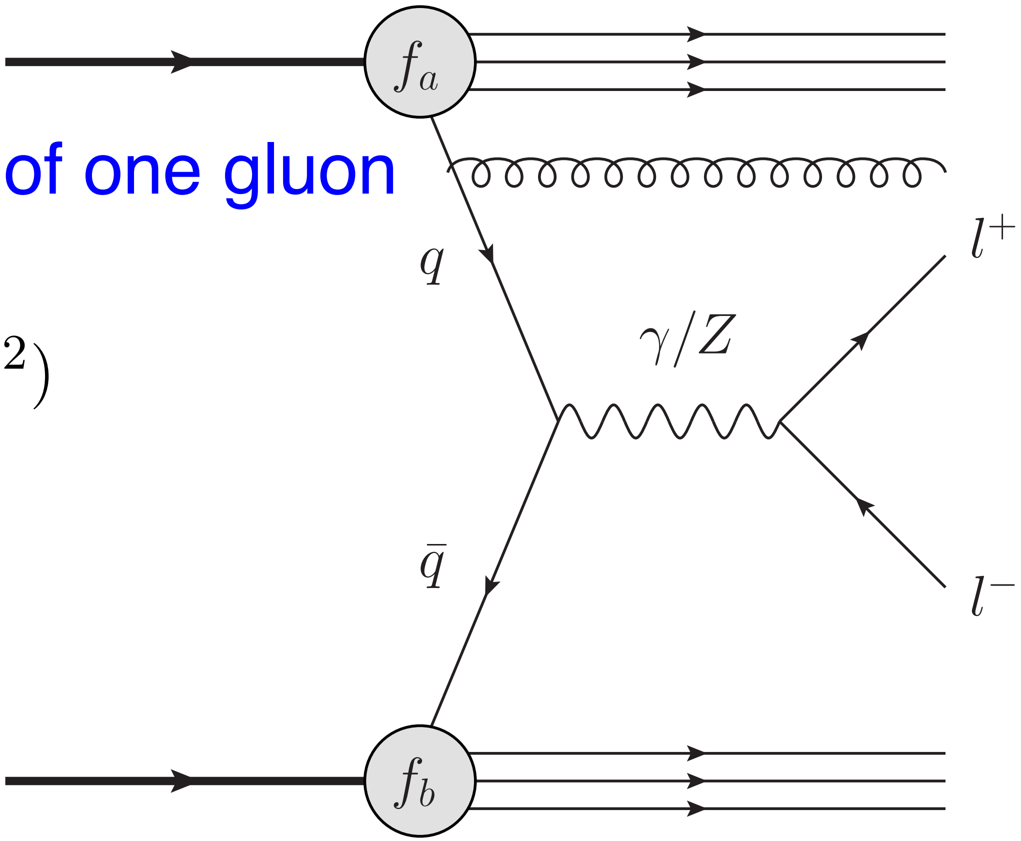
$$\Rightarrow \delta(x_1 x_2 s - M^2)$$
$$\delta(y - \tfrac{1}{2}\log(x_1/x_2))$$

Ex. 2: Feynman $x$ ($x_F$)

$x_F = \frac{2}{\sqrt{s}}(p_{z,l^+} - p_{z,l^-}) \stackrel{\mathrm{LO}}{=} x_1 - x_2$: also 2 $\delta$'s

# Drell-Yan

- Next order: emission of one gluon
  - real and virtual
  - depends on $g(x, M^2)$
  - $p_{t,\gamma/Z} \neq 0$



# Drell-Yan

- Next order: emission of one gluon
- factorisation proven at ANY order

$$\frac{d\sigma}{dM^2} = \int dx_1\, dx_2\, dz_1\, dz_2 \sum_f f_a(x_1, M^2) f_b(x_2, M^2) D_{ab}(z_1/x_1, z_2/x_2) \frac{d\hat{\sigma}}{dM^2}(z_1, z_2; M^2)$$

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$$\frac{d\hat{\sigma}}{dM^2}(z_1, z_2; M^2)$$

- ONLY case where the factorisation
  $\mathrm{PDF}_1 \otimes \mathrm{PDF}_2 \otimes \mathrm{ME}$ is proven,
  otherwise it's just a “reasonable assumption”

# Monte-Carlo generators

Parton cascades, hadronisation, Underlying Event, pileup: a realistic event is complicated!

$\Rightarrow$ Use of (Monte-Carlo) event generators to simulate full events

# *Monte-Carlo generators: fixed order*

Perturbative computations are the base of everything
But are often hard/impossible to compute analytically
(especially for exclusive measurements)

$\Rightarrow$ use a fixed-order Monte-Carlo genrator

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- Aim: provide signals and backgrounds for LHC studies (usually needed at NLO)
  See the LesHouche list of completed/wanted processes, *e,g,*
  - many jets
  - W+jets
  - H+jets
  - top ($t\bar{t}$ and single top)
  - SUSY

# Monte-Carlo generators: fixed order

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$\Rightarrow$ use a fixed-order Monte-Carlo genrator

- Aim: provide signals and backgrounds for LHC studies (usually needed at NLO)
- Generate matrix elements + phase-space
- 2 big categories:
  LO (many legs) or NLO (includes virtual corrections)
- Tendency to automate!
- Plenty of them: Alpgen, MadGraph, NLOJet, MCFM, BlackHat, Golem,...

# *Monte-Carlo generators: full event*

For full-event simulation, Monte-Carlo generators are a cornerstone

- parton cascade: collinear splittings (DGLAP-like)
  As seen in $e^+e^-$, they have the form

$$\frac{d^2P}{d\theta dz} = \alpha_s P(z)\frac{1}{\theta}$$

Leading terms ($\alpha_s^n \log^n(1/\theta)$) have angular ordering
$\theta_1 > \theta_2 > \cdots > \theta_n$

Watch out: LO collinear branchings!!!
*e.g.* Multi-jet processes hardly reliable

(alternatives like virtuality ordered but always LO

# *Monte-Carlo generators: full event*

For full-event simulation, Monte-Carlo generators are a cornerstone

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- hadronisation: non-perturbative *per se*!
  *e.g.* Lund string fragmentations (form strings based on colour connections and fragment them)

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For full-event simulation, Monte-Carlo generators are a cornerstone

- parton cascade: collinear splittings (DGLAP-like)
- hadronisation: non-perturbative *per se*!
- Multiple interactions/Underlying Event: hadronic beams carry colour *i.e.* interact strongly
  - Modelling
  - Then tuning to Tevatron (and LHC) data

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- Progress towards NLO generator
- Most commonly used: Pythia, Herwig, Sherpa... but others available
- more in the tutorials

# $W/Z$ production

- Production:
  - $q\bar{q}' \to W^{\pm}$
  - $q\bar{q} \to Z$
  - 14 TeV $\sigma_W \approx 20$ nb *i.e.* 200 $W$/s ($\mathcal{L} = 10^{34}$ cm$^2$/s)

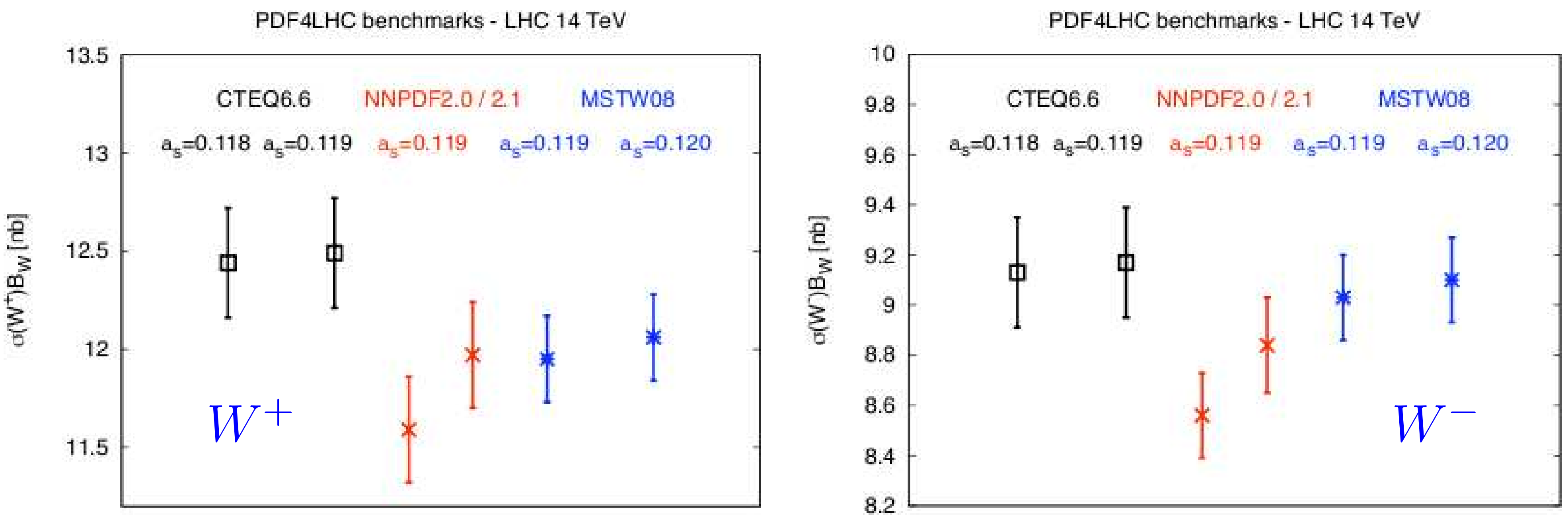
- Decay:
  - $W \to q\bar{q} \to 2$ jets (BR$\approx 2/3$)
    $W \to \ell\nu_\ell$ (BR$\approx 1/3$)
  - $Z \to q\bar{q} \to 2$ jets (BR$\approx 70\%$)
    $Z \to \ell\bar{\ell}$ (BR$\approx 10\%$)
    $Z \to \nu\bar{\nu}$ (BR$\approx 20\%$)
  - leptonic channel most convenient
    hadronic important for statistics!

# $W/Z$ physics

- not really a discovery channel...
- ... but important in many respects
  - often $W/Z$+jets
  - standard model tests/MC calibration
  - background to many searches
    *e.g.* top ($\to Wb$) or SUSY ($\not{E}_t$)
- $W$ cross-section as a standard candle for luminosity measurements

# $W$ for lumi measurement

$W$ cross-section as a standard candle for luminosity measurements




PDF main source of uncertainty

# *top physics*

- Production:
  - Mostly $gg \to t\bar{t}$
  - Tevatron: $\sigma_t \approx 4$ pb: discovery!
  - LHC: $\sigma_t \approx 1$ nb: $\approx 10/$s LHC$\equiv$ top factory
- Decay:
  - Mostly $t \to Wb$
    $t \to q\bar{q}b$ ($\approx 66\%$) or $t \to \ell\nu_\ell b$ ($\approx 33\%$)
  - for $t\bar{t}$: 3 options
    - leptonic: not-so-easy because 2 neutrinos
    - semi-leptonic: $\ell$, 4 jets (2$b$) and $\not{E}_t$
      (the most convenient)
    - hadronic: 6 jets *i.e.* technical to reconstruct
      but $\approx$ 45% of the stat!

# *top physics*

top very important at the LHC

- precision mass measurement
- many new physics scenario involve the top (mostly because of its large mass)

$\Rightarrow$ need to reconstruct as many tops as possible top very important at the LHC

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- many new physics scenario involve the top (mostly because of its large mass)

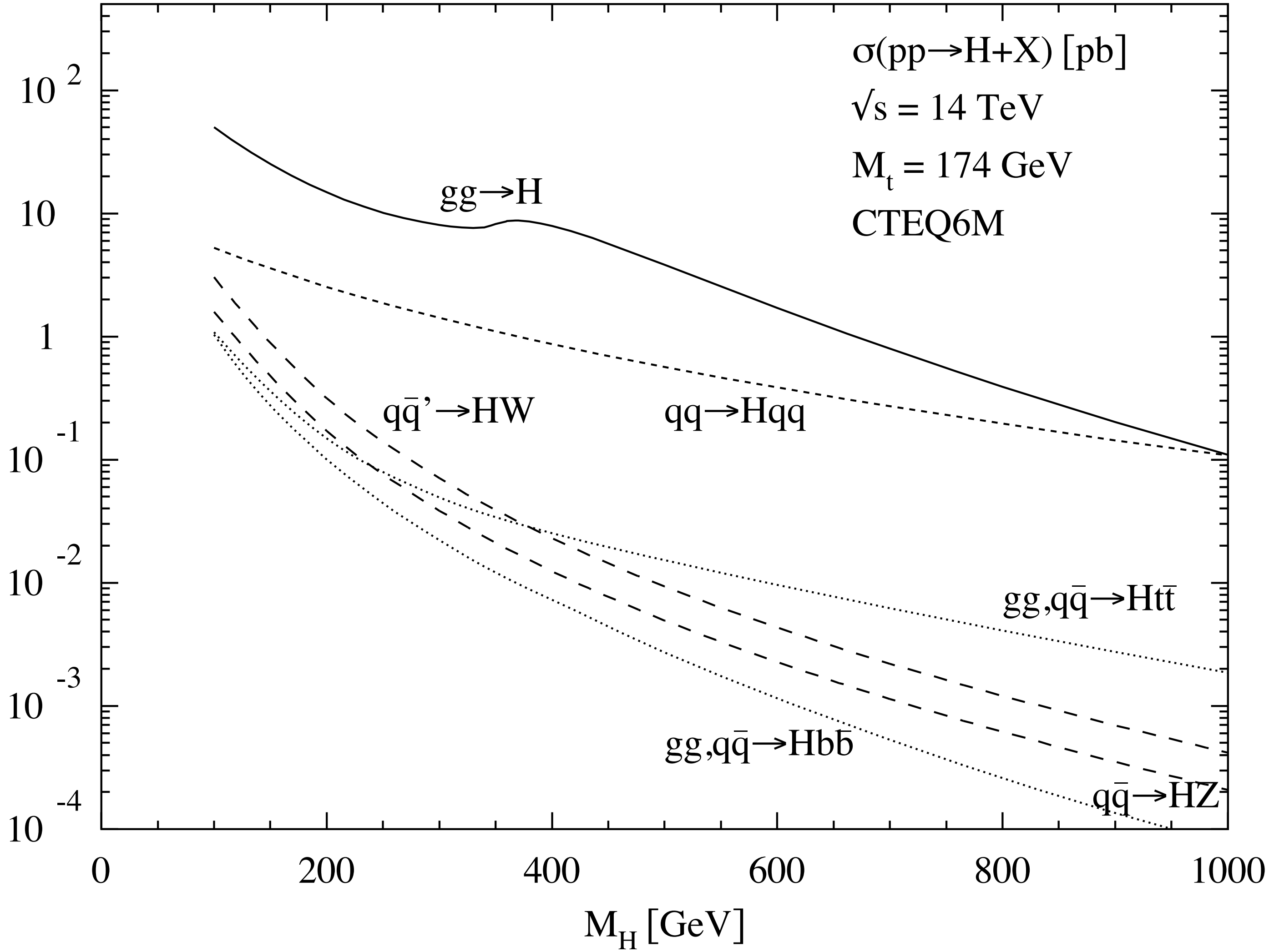
$\Rightarrow$ need to reconstruct as many tops as possible

Issues:

- $W$+jets background
- $b$ mis-tagging
- combinatorial background (especially for full hadr.)
- efforts *e.g.* in boosted-top reconstruction

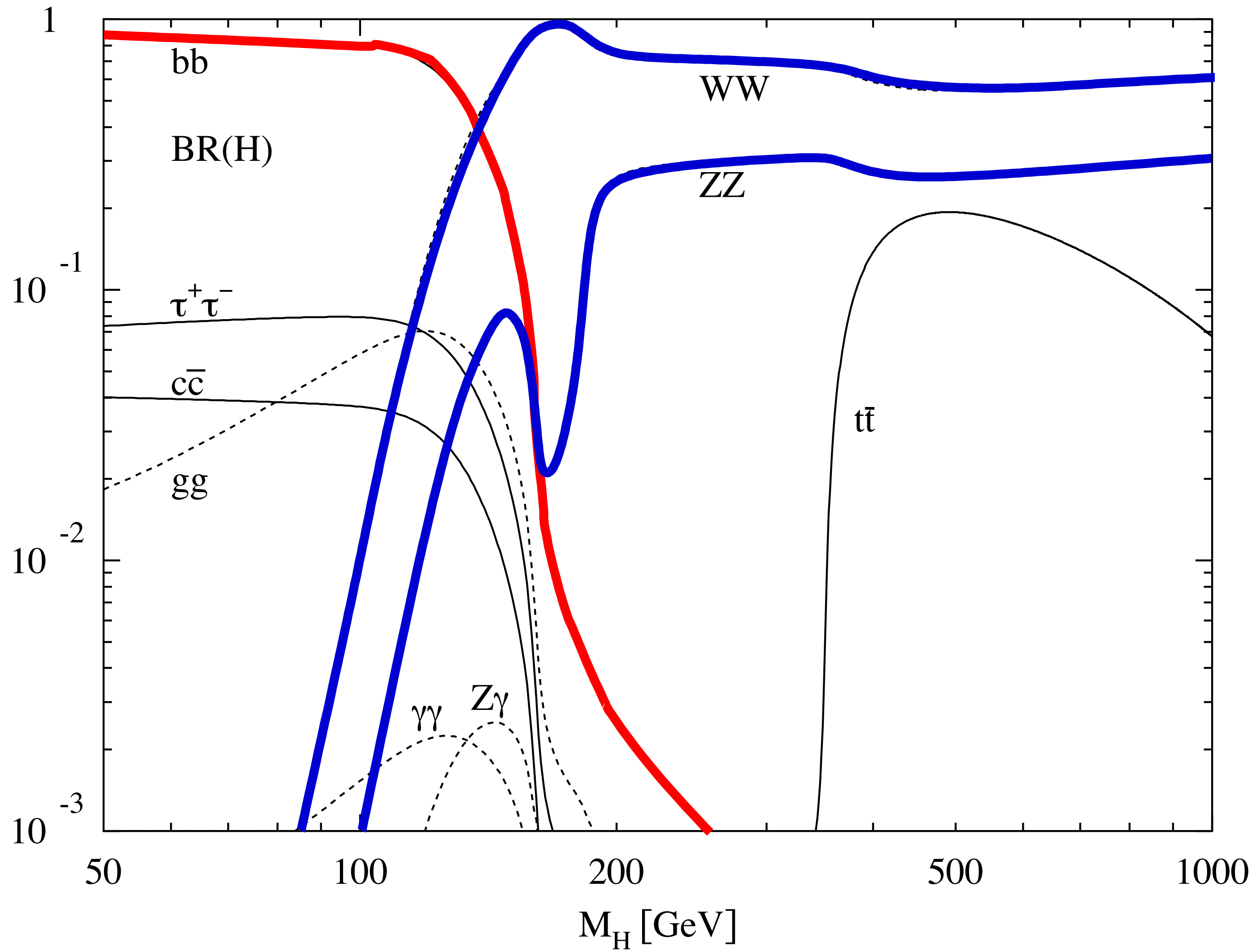
# *Higgs: production*

Production at the LHC: mostly $gg$ fusion (through top loop)




$m_H = 120$ GeV $\Rightarrow \sigma_H^{(\mathrm{L0})} \approx 21$ pb (vs $0.3$ at the Tevatron)

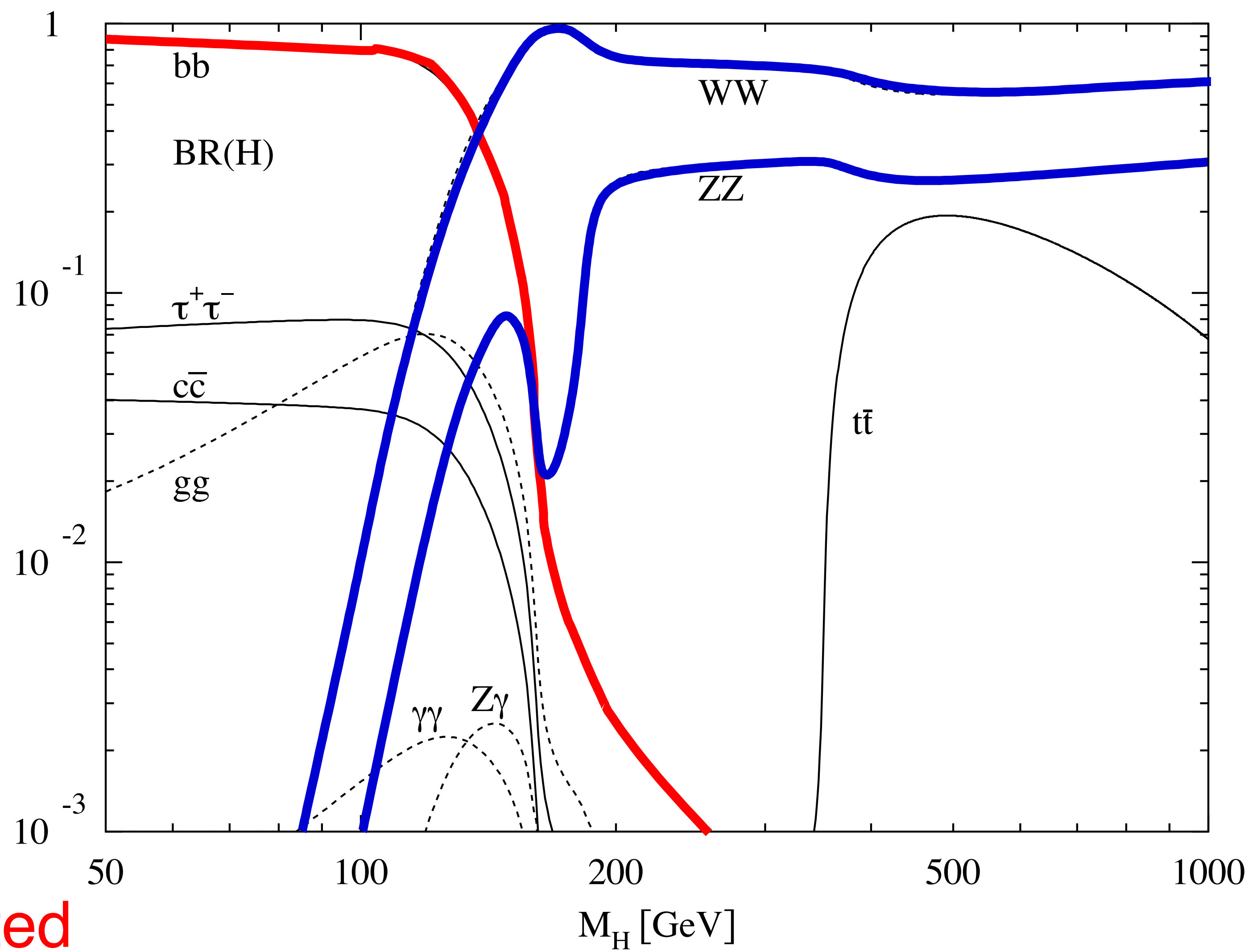
# Higgs: decay




Heavy higgs
$(m \gtrsim 2m_W)$:

mostly $H \to WW^{(*)}$ or $H \to ZZ$
the easiest situation (see *e.g.* Tevatron)

# Higgs: decay

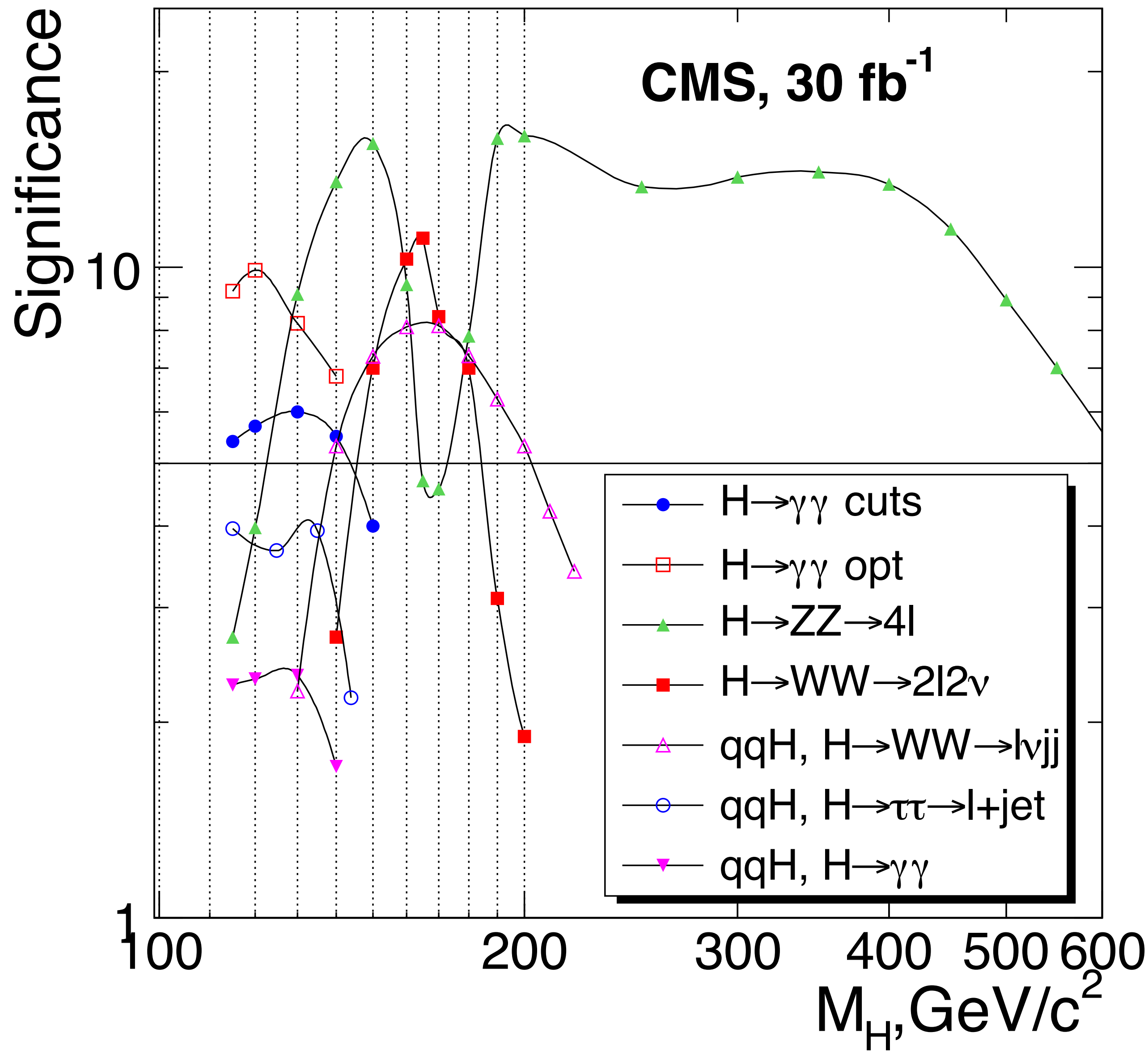



Light higgs ($m < 2m_W$): more complicated

- $bb$ →jets dominant but buried in the QCD bkgd
- $\gamma\gamma$ clean but only 0.1-0.3% of the events

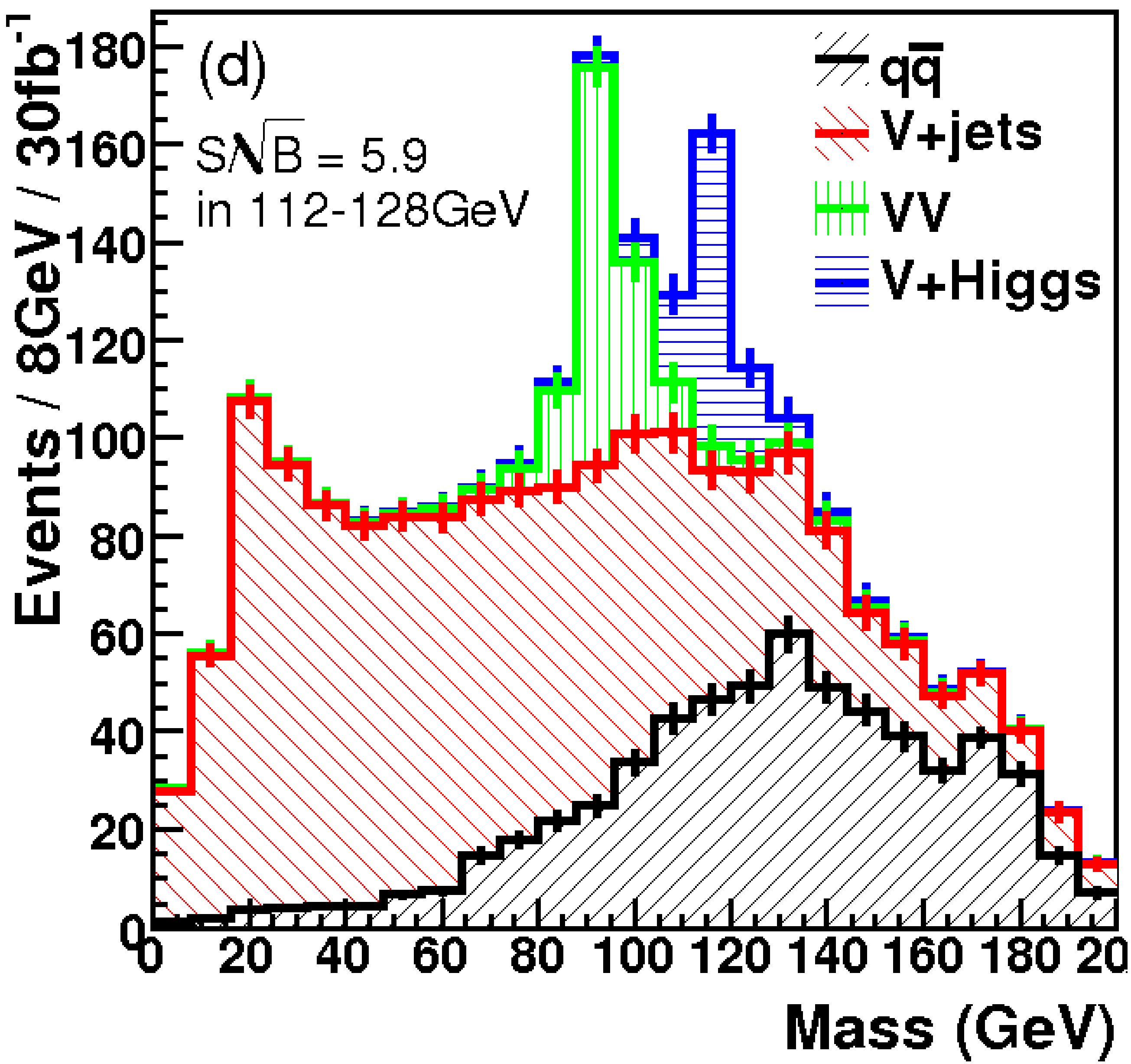
# *Higgs: discovery*

$\sim$ 30 fb$^{-1}$
needed for
$5\sigma$ discovery



# Higgs: additional comments

- $H \to b\bar{b}$ may be visible/helpful for boosted $H + W/Z$



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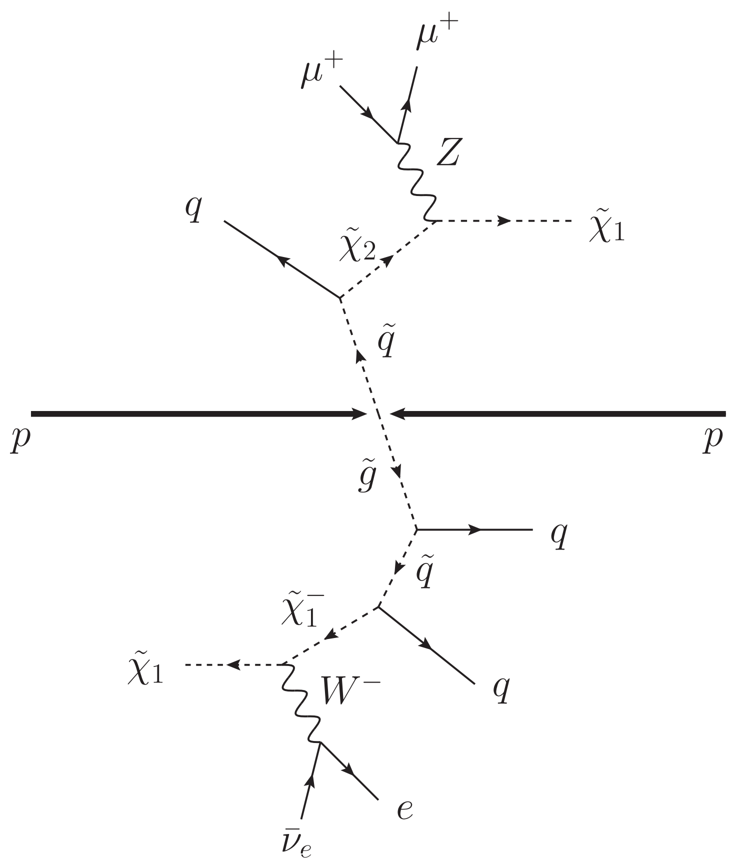
- Not the end of the story:
  also need to verify Higgs properties/couplings.
  - *e.g.* $t\bar{t}H$ may help
  - need for luminosity!

# SUSY

Typical SUSY process:

- production of a pair of supersymmetric particles
- decay: SM particles + lightest SUSY particle (LSP)



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Typical issues

- Need good determination of $\not{E}_t$
- Control the multi-jet background at large $p_t$

***Time for questions!***