# Vector Boson Scattering: Status and Prospects for the Large Hadron Collider and Beyond Vrije Universiteit Brussel - Seminar 

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Thank you for the invitation!

## A real cosmic muon $(\mu)$ passing through the CMS detector at the LHC


$\square$ CMS Experiment at the LHC, CERN
Data recorded: 2022-Mar-11 08:17:42.214016 GMT
Run / Event / LS: 348683 / 35407138 / 1771


Since $|\vec{B}|=4 \mathrm{~T}$ and radius $\neq 0, \infty \Longrightarrow \mu$ is massive and charged!

## Particle Physics: Then and Now

Since the late 20th, a chief goal of particle physics has been to establish the spectrum of particles, their structures, and their properties
possible with many tools, e.g., production at colliders, tabletop measurements of fundamental symm., and rare decays

## Particle Physics: Then and Now

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possible with many tools, e.g., production at colliders, tabletop measurements of fundamental symm., and rare decays

## The Standard Model (SM)

 of particle physics- position indicates quantum numbers/ charges
(just like in chemistry!)
- e.g., spin, weak isospin, color, electromagnetic, weak hyper charge

credit: I. Bigaran

Today's goals include understanding the origin of the SM itself

Undoubtedly, the SM is incredibly successful...

incredible TH \& EX agreement ... but not perfect (we will return to this point!)
first a few ingredients

## Nuclear $\beta$ decay ${ }^{1}$


${ }^{1}$ For non-experts: Action $=\mathcal{S}=\int d t L=\int d^{4} \times \mathcal{L} . \leftarrow$ HEP uses Lagrangian density with four-vectors $x^{\mu}, k^{\mu}$

Inverting $\nu_{e}$ leg $\Longrightarrow$ inverse $\beta$ decay ( $\nu$-nucleus scattering!)

$-i \mathcal{M}\left(\nu_{e} \mathcal{N} \rightarrow e^{-\mathcal{P}}\right) \sim G_{F}\left[\bar{u}\left(k_{\mathcal{P}}\right) \gamma^{\mu} P_{L} u\left(k_{\mathcal{N}}\right)\right] \cdot\left[\bar{u}\left(k_{e}\right) \gamma_{\mu} P_{L} u\left(k_{\nu_{e}}\right)\right] \sim G_{F} E^{2}$
$\Longrightarrow \sigma\left(\nu_{e} \mathcal{N} \rightarrow e^{-\mathcal{P}}\right) \sim \frac{1}{(\text { flux })} \oint_{\text {dof }}$ (phase space) $\times|\mathcal{M}|^{2} \sim G_{F}^{2} \frac{E^{4}}{\pi E^{2}}$

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$\Longrightarrow$ scatt. rate $(\sigma)$ grows with scatt. energy but without limit
$\Longrightarrow$ violation of unitarity in scattering theory, i.e.,$\sum($ prob $) \leq 1$

Inverse $\beta$ decay is a charged-current interaction!


Fermi thry is the low-energy manifestation of the electroweak thry

$$
\left(\frac{g_{W}}{\sqrt{2}}\right)^{2} \times\left(\frac{g_{\mu \nu}-\frac{q_{\mu} q_{\nu}}{M_{W}^{2}}}{q^{2}-M_{W}^{2}+i \Gamma W M_{W}}\right) \xrightarrow{\left|q^{2}\right| \ll M_{W}^{2}} \frac{-g_{W}^{2}}{2 M_{W}^{2}}=-2 \sqrt{2} G_{F}
$$

$\Longrightarrow \sigma\left(\nu_{e} \mathcal{N} \rightarrow e^{-} \mathcal{P}\right) \sim \frac{g_{W}^{4}}{\pi} \frac{E^{2}}{\left(E^{2}-M_{W}^{2}\right)^{2}} \quad \leftarrow$ high- $E$ behavior is regulated (finite)

## Rotating graph $\Longrightarrow W^{ \pm}$boson production



Electroweak sector of Standard Model is powerful:

- explains $\beta$ decay
- explains inverse $\beta$ decay
- predicts $W^{ \pm}$production in pp collisions

- some inputs needed, e.g., $G_{F}, M_{W}$

Transverse mass distribution for all $\mathrm{W} \rightarrow \mathrm{e} \nu$ events recorded by UA1

A little surgery with diagrams $\Longrightarrow W^{+} W^{-}$pair production
(why make one $W^{ \pm}$when you can make $W^{+} W^{-}$pairs?)

$$
-i \mathcal{M}\left(e^{-} e^{+} \xrightarrow{\nu} W^{+} W^{-}\right) \sim g_{W}^{2} \times E \times\left(\frac{-E}{E^{2}}\right) \times\left(\frac{E}{M_{W}}\right)^{2} \sim-g_{W}^{2} \frac{E^{E^{4}}}{E^{2} M_{W}^{2}}
$$

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$\Longrightarrow$ violation of unitarity in scattering theory!

A little surgery with diagrams $\Longrightarrow W^{+} W^{-}$pair production
(why make one $W^{ \pm}$when you can make $W^{+} W^{-}$pairs?)


Delicate (structural) cancellations when all particles are included!

## Diagram fun $\Longrightarrow Z$ boson production

predicted by Glashow, Weinberg, Salam ('68)
3 ('79); discovered by UA1,UA2('83); ('84)



Electroweak sector of Standard Model is powerful:

- explains $\beta$ decay

Invariant mass distribution of all $\mathrm{e}^{+} \mathrm{e}^{-}$pairs recorded by UA1

- explains inverse $\beta$ decay
- predicts $Z$ production in pp collisions
- some inputs needed, eg, $G_{F}, M_{W}, M_{Z}$


## The Standard Model toolbox

- $W^{ \pm}, Z, \gamma$ all exist!
- effective field theories break down at high energies :
- unitarity violation $=$ bad ©
- breakdown of theory $\Longrightarrow$ unitarity violation $)^{(\cdot)}$
- missing contributions $\Longrightarrow$ unitarity violation ${ }^{(\cdot)}$
- small mis-cancellations from new contributions
$\Longrightarrow E$-enhanced scattering rates $)^{-}$
vector boson scattering (VBS) / fusion (VBF)

Cut, rotate, glue, etc. sub-graphs $\Longrightarrow W^{+} W^{+} \rightarrow W^{+} W^{+}$scattering (why make $W^{+} W^{-}$pairs when you can scatter them?)


Just one of many examples:

- $W^{+} W^{-}, W^{ \pm} Z, W^{ \pm} \gamma, \gamma \gamma, Z Z, Z \gamma$ scattering are all possible
- $W^{+} W^{-} \rightarrow Z Z, W^{ \pm} \gamma \rightarrow W^{ \pm} Z$, etc, are also possible

Cut, rotate, glue, etc. sub-graphs $\Longrightarrow W^{+} W^{+} \rightarrow W^{+} W^{+}$scattering
(why make $W^{+} W^{-}$pairs when you can scatter them?)

$\Longrightarrow$ scattering amplitude $(\mathcal{M})$ grows with scattering energy!
$\Longrightarrow$ violation of unitarity in scattering theory!

Cut, rotate, glue, etc. sub-graphs $\Longrightarrow W^{+} W^{+} \rightarrow W^{+} W^{+}$scattering
(why make $W^{+} W^{-}$pairs when you can scatter them?)

$$
\begin{aligned}
& -i \mathcal{M}\left(W^{+} W^{+} \xrightarrow{h} W^{+} W^{+} W^{+}\right) \sim\left(\frac{E}{M_{W}}\right)^{4} \times\left(\frac{1}{E^{2}}\right) \times\left(g_{W} M_{W}\right)^{2} \sim \frac{+g_{W}^{2} E^{2}}{M_{W}^{2}}
\end{aligned}
$$

Delicate (structural) cancellations when all particles are included!
Lee, Quigg, and Thacker ('77×2); Chanowitz and Gaillard ('84,'85)

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Delicate (structural) cancellations when all particles are included!
Lee, Quigg, and Thacker (' $77 \times 2$ ); Chanowitz and Gaillard (' 84, ' 85 )
$\Longrightarrow$ modified $h-V-V$ couplings can disrupt cancellations

Too many contributions?
It is possible that Higgs with $m_{h}=125 \mathrm{GeV}$ is one of several in nature
add'I scalars appears in Two Higgs Doublet Models, Supersymmetry, scalar-singlet dark matter, composite Higgs

$$
\underbrace{\left|h_{\mathrm{SM}}\right\rangle}_{\text {interaction eigenstate }}=\underbrace{\cos \psi\left|h_{125 \mathrm{GeV}}\right\rangle}_{\text {mass eigenstate }}+\underbrace{\sin \psi\left|H_{\text {several }} \mathrm{TeV}\right\rangle}_{\text {mass eigenstate }}
$$



$$
-i \mathcal{M}\left(W^{+} W^{+} \xrightarrow{h / H} W^{+} W^{+}\right) \sim \frac{g_{W}^{2} E^{4}}{M_{W}^{2}\left(E^{2}-\not 巾_{h}^{2}\right)} \underbrace{\cos ^{2} \psi}_{\mathcal{O}(1)}+\frac{g_{W}^{2} E^{4}}{M_{W}^{2}\left(\mathbb{F}^{2}-m_{H}^{2}\right)} \underbrace{\sin ^{2} \psi}_{\ll 1}
$$

$\Longrightarrow \mathcal{M}$ grows with scattering energy for $E_{(\sim 1 \mathrm{TeV})} \ll m_{H^{(\text {several } \mathrm{TeV})} \text { ! }}$

# big idea: studying VBS $=$ studying Higgs sector 

The LHC is the largest, etc. hadron collider ( $p p, p A, A A$ ) at $\sqrt{s}=13.6 \mathrm{TeV}$, with a broad particle and nuclear physics program


The ATLAS and CMS detectors at the LHC were designed to study VBS

LHC's long-term plan includes using VBS to measure SM physics with high precision and search for new phenomena

Buarque (ed.), Gallinaro (ed.), RR (ed.), et al, Rev. Physics ('22) [arXiv:2106.01393]

ATLAS $W^{+} W^{+} \rightarrow W^{+} W^{+}$candidate event $\left(p p \rightarrow e^{+} \nu_{e} \mu^{+} \nu_{\mu j} j\right)$

(8)ATLAS

## Run: 302956

Event: 1297610851
2016-06-29 09:25:24 CEST
$\mathrm{m}_{\mathrm{jj}}=3.8 \mathrm{TeV}$

ATLAS $W^{+} W^{+} \rightarrow W^{+} W^{+}$candidate event $\left(p p \rightarrow e^{+} e^{+} \nu_{e} \nu_{e} j j\right)$

[PRL('19)]

Plotted: in $p p \rightarrow \ell_{1}^{ \pm} \ell_{2}^{ \pm} \nu \nu j j$, invariant mass of (L) (jj)-system, (R) ( $\ell_{1} \ell_{2}$ )-system




VBS observed for first time during LHC's Run II [CMS('18),AtLAS('19)]

- VBS at the LHC probes multi-TeV energy scales
- First measurements of VBS within $20 \%$ of SM predictions


# polarization 

The $W_{\lambda}^{ \pm}, Z_{\lambda}$ bosons are massive, spin- 1 objects

Plotted: angle of outgoing $W^{-}$in

$$
p p \rightarrow W^{+} W_{\lambda}^{-} j j \text { via VBS }
$$

- 2 transverse polarizations (L,R)
- 1 longitudinal polarization (0)

polarizations of vector bosons


ATLAS [ATL-PHYS-PUB-2018-023]

The $W_{\lambda}^{ \pm}, Z_{\lambda}$ bosons are massive, spin-1 objects

- 2 transverse polarizations (L,R) - 1 longitudinal polarization (0)

polarizations also imprint on kinematics of decay products!

Plotted: angle of outgoing $\ell^{-}$in $p p \rightarrow W^{+} W_{\lambda}^{-} j j \rightarrow W^{+} \ell^{-} \overline{\nu_{\ell}} j j$ via VBS



## First measurement of polarization

The $W_{\lambda}^{ \pm}, Z_{\lambda}$ bosons are massive, spin- 1 objects

- 2 transverse polarizations (L,R) - 1 longitudinal polarization (0)
in $W^{ \pm} W^{ \pm}$scattering
CMS (PLB'20)

polarizations also imprint on kinematics of decay products!
uncertainties sizable but will improve with time


The $W_{\lambda}^{ \pm}, Z_{\lambda}$ bosons are massive, spin- 1 objects

First measurement of polarization fractions $\left(f_{\lambda}\right)$ in $W^{ \pm} Z$ scattering

ATLAS [PLB('23)]

- 2 transverse polarizations $(L, R) \leftarrow 0.6$
- 1 longitudinal polarization (0)

polarization also imprints on kinematics of decay products!



## calculating scattering rates for helicity-polarized particles

Calculating helicity-polarized cross sections is delicate business
loss of Lorentz invariance, etc!

$$
\Pi_{\mu \nu}^{V}(q)=\frac{-i\left(g_{\mu \nu}-q_{\mu} q_{\nu} / M_{V}^{2}\right)}{q^{2}-M_{V}^{2}+i M_{V} \Gamma_{V}}=\sum_{\lambda \in\{0, \pm 1, A\}} \eta_{\lambda}\left(\frac{-i \varepsilon_{\mu}(q, \lambda) \varepsilon_{\nu}^{*}(q, \lambda)}{q^{2}-M_{V}^{2}+i M_{V} \Gamma_{V}}\right)
$$

Different treatments of weak boson propagators

- double-pole approximation/on-shell projection
- spin-truncated propagator [MadGraph]
w/ Buarque Franzosi, Mattelaer, Shil [1912.01725]
- full NLO in QCD+EW for $V V^{\prime}$ production and decay [PowHEG] Denner, Pelliccioli [2107.06579]; Pelliccioli, et al [2311.16031], others
- partial NLO in QCD for arbitrary processes [Sherpa]


## calculating scattering rates for helicity-polarized particles

Calculating helicity-polarized cross sections is delicate business

$$
\frac{-i \varepsilon_{\mu}(q, \lambda) \varepsilon_{\nu}^{*}(q, \lambda)}{q^{2}-M_{V}^{2}}=\bigcap_{V_{\lambda}(q)}
$$



- NEW! helicity polarization as a Feynman rule w/ Javurkova, et al [2401.17365]

Projected sensitivity to pol. for (L) $g g \rightarrow Z_{\lambda} Z_{\lambda^{\prime}}$ and (R) $V V \rightarrow Z_{\lambda} Z_{\lambda^{\prime}}$



## singly and doubly charged scalars

## New Higgs bosons?

It is possible that Higgs with $m_{h}=125 \mathrm{GeV}$ is one of several in nature
add'I scalars appears in Two Higgs Doublet Models, Supersymmetry, scalar-singlet dark matter, composite Higgs


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It is also possible new Higgses are not arbitrary but belong to a group
e.g., Type II Seesaw model for neutrino masses: $\Delta \mathcal{L}=y \bar{L} \cdot \Delta L^{c} \rightarrow\left(y v_{2}\right) \bar{\nu}_{L} \nu_{L}^{c}$
$\Phi_{\mathrm{SM}}=\frac{1}{\sqrt{2}}\binom{i \sqrt{2} G^{+}}{v_{1}+h_{\mathrm{SM}}^{0}+i G^{0}}$

## New Higgs bosons?

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$\Phi_{\mathrm{SM}}=\frac{1}{\sqrt{2}}\binom{i \sqrt{2} G^{+}}{v_{1}+h_{\mathrm{SM}}^{0}+i G^{0}} \quad \Delta_{\mathrm{Type} \mathrm{II}}=\frac{1}{\sqrt{2}}\left(\begin{array}{cc}H^{+} & \sqrt{2} H^{++} \\ v_{2}+H^{0}+i \xi^{0} & -H^{+}\end{array}\right)$
$\Longrightarrow$ if $h^{0}$ and $H^{0}$ are accessible with VBS, then also $H^{ \pm}$and $H^{ \pm \pm}$

Singly $\left(H^{ \pm}\right)$and doubly $\left(H^{ \pm \pm}\right)$charged scalars are predicted in several popular models

Plotted: invariant mass of ( $W Z$ )-system in $p p \rightarrow W^{ \pm}(\rightarrow j j) Z\left(\rightarrow \ell^{+} \ell^{-}\right) j j$

ATLAS [PRL('15)]


Searches for $H^{ \pm}$in $W^{ \pm} Z$ scattering with early Run II data gave suggestive hints of something new $\odot$ !


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 popular models

Searches for $H^{ \pm}$in $W^{ \pm} Z$ scattering with early Run II data gave suggestive hints of something new © $\odot$

Plotted: excluded upperlimit on scattering rate of $p p \rightarrow W^{ \pm} Z_{j j}$ via $H^{ \pm}$as a function of $m_{H}^{ \pm}$

ATLAS [PRL('15)]


Singly $\left(H^{ \pm}\right)$and doubly $\left(H^{ \pm \pm}\right)$charged scalars are predicted in several popular models


Searches for $H^{ \pm}$in $W^{ \pm} Z$ scattering with all Run II data shows "bump" just a statistical fluctuation $)^{(3}$

Plotted: excluded upperlimit on scattering rate of $p p \rightarrow W^{ \pm} Z_{j j}$ via $H^{ \pm}$as a function of $m_{H}^{ \pm} \quad$ CMs [EPJC('21)]


## heavy (Majorana) fermions ${ }^{2}$

[^0]
## for non-experts: adding $\nu_{R}$ to the SM (1 slide)

To generate Dirac masses for $\nu$ like other SM fermions, we need $\nu_{R}$

$$
\begin{aligned}
\mathcal{L}_{\nu} \text { Yuk. }=-y_{\nu} \bar{L} \tilde{\Phi}_{\nu_{R}}+H . c .= & -y_{\nu}\left(\overline{\nu_{L}} \overline{\ell_{L}}\right)\binom{\langle\Phi\rangle+h}{0} \nu_{R}+H . c . \\
& =\underbrace{-y_{\nu}\langle\Phi\rangle \overline{\nu_{L}} \nu_{R}+H . c .+\ldots}_{=m_{D}}
\end{aligned}
$$

$\nu_{R}$ do not exist in the SM, so pretend that they do and $\nu_{R}=\nu_{R}^{c}$ :

$$
\Longrightarrow \mathcal{L}_{\text {mass }}=\frac{-1}{2} \underbrace{\left(\begin{array}{ll}
\overline{\nu_{L}} & \overline{\nu_{R}^{c}}
\end{array}\right)}_{\text {chiral state }} \underbrace{\left(\begin{array}{cc}
0 & m_{D} \\
m_{D} & \mu_{k}
\end{array}\right)}_{\text {mass matrix (chiral basis) }}\binom{\nu_{L}}{\nu_{R}^{c}}
$$

(sizes of $m_{D} \& \mu_{\mathcal{L}}$ have major impact on pheno; see Pascoli, et al [1712.07611])
After diagonalizing the mass matrix, identify $\nu_{L}$ (chiral eigenstate) in the SM as a linear combination of mass eigenstates:

$$
\left|\nu_{L}\right\rangle=\cos \theta|\nu\rangle+\sin \theta|N\rangle
$$

chiral state light mass state heavy mass state (this is a prediction!)

Plotted: Normalized production rate $\left(\sigma /|V|^{2(4)}\right)$ vs $m_{N}$

$\gamma W^{ \pm}$and $W^{ \pm} W^{ \pm}$scattering drive high-mass scattering rates!
w/ Fuks, Neundorf, Peters, Saimpert [PRD('21)]

Search for $W^{ \pm} W^{ \pm} \rightarrow \ell^{ \pm} \ell^{\prime \pm}$ quickly adopted by LHC groups!


ATLAS t-channel
this work
$\mathcal{L}=140 \mathrm{fb}^{-1}$
ATLAS s-channel
JHEP 10 (2019) 265
$\mathcal{L}=35.9 \mathrm{fb}^{-1}$
ATLAS s-channel
JHEP 07 (2015) 162
$---\sqrt{s}=8 \mathrm{TeV}$
$\mathcal{L}=20.3 \mathrm{fb}^{-1}$
ATLAS displaced
__ arXiv:2204.11988
$\mathcal{L}=139 \mathrm{fb}^{-1}$
CMS t-channel
_-- arXiv:2206.08956
$\mathcal{L}=139 \mathrm{fb}^{-1}$
CMS s-channel
_-- JHEP 01 (2019) 122 $\mathcal{L}=35.9 \mathrm{fb}^{-1}$
CMS displaced JHEP 07 (2021) 081 $\mathcal{L}=139 \mathrm{fb}^{-1}$

ATLAS ('23) [2305.14931]
ee/e $\mu$ channels at Moriond
$\leftarrow$ CMS ('22) [2206.08956]

## a future beyond the LHC

Many physics and technical discussions are taking place over the successor of the LHC (beyond '30s-'40s)


Multi-stage $100 \mathrm{TeV} e^{+} e^{-} p p$ collider at CERN (FCC program) and $14-30 \mathrm{TeV} \mu^{+} \mu^{-}$at Fermilab are most supported

European Strategy for Particle Physics [1910.11775, CERN-ESU-013, Mid-term review ('24)];
Black (ed.), Jindariani (ed.), Li (ed.), F. Maltoni (ed.), et al, [2209.01318]

Why? ${ }^{3}$ Situation where scattering formalism is theoretically interesting


Partonic collisions at $Q \sim \mathcal{O}(10) \mathrm{TeV}$ explore when electroweak (EW) symmetry is nearly restored, i.e., $\left(M_{W / Z / H}^{2} / Q^{2}\right) \rightarrow 0$

When momentum transfers reach $Q \sim \mathcal{O}(10) \mathrm{TeV}$, vector boson scattering (VBS/VBF) acts a bit... funny
w/ A. Costantini, et al [JHEP('20)]

[^1]
## some examples of VBS at higher energies

## Quick interlude: s-channel annihilation vs VBS

(

More legs $\Longrightarrow$ more propagators $\Longrightarrow \int d k^{2} /\left(k^{2}-M_{W}^{2}\right) \sim \log \left(\Lambda^{2} / M_{W}^{2}\right)$ Larger $s \Longrightarrow \operatorname{larger}\left(M_{W W}^{2} / M_{W}^{2}\right) \Longrightarrow$ collinear $V$ compensate for $g$

## Higgs production



## cross sections $(\sigma)$ vs $\sqrt{s}$ for s-channel annihilation (dash) vs VBS (solid)



- Eventually, $\sigma^{V B F}>\sigma^{s-c h a n n e l}$ since
- $\sigma^{\text {s-channel }} \sim 1 / s$
- $\sigma^{V B F} \sim \log ^{2}\left(M_{V V}^{2} / M_{V}^{2}\right) / M_{V V}^{2}$ due to forward emission of $V \equiv W l_{\underline{\underline{1}}} Z_{\bar{E}}$

Top production




- Do you notice a pattern?


## Many-boson production ${ }^{4}$



[^2]

- VBF is the dominant production vehicle for many processes


## Evidence for trend that VBS rates will always exceed s-ch. rates

Is this obvious? (not to me at first!) Is there intuition for this? (yes!)
w/ A. Costantini, et al [JHEP('20,'21)]

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idea: increasing $\sigma^{\mathrm{VBS}}$ is manifestation of growing partonic content

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w/ A. Costantini, et al [JHEP('20,'21)]
Idea: crudely compare the production of $X$ by writing generically

$$
\begin{gathered}
\sigma^{s-c h .} \sim \frac{\left(s-M_{X}^{2}\right)}{\left(s-M_{V}^{2}\right)^{2}} \sim \frac{\left(s-M_{X}^{2}\right)}{s^{2}} \leftarrow \text { assumes } s \gg M_{V}^{2} \\
\frac{d \sigma^{V B F}}{d z_{1} d z_{2}} \sim \underbrace{}_{{ }^{V} \mu \mathrm{PDFs}} \sim \underbrace{f_{V}\left(z_{1}\right) f_{V^{\prime}}\left(z_{2}\right)}_{M_{V V^{\prime}}^{2}=z_{1} z_{2} s \gg M_{V}^{2}} \underbrace{\frac{\left(M_{V V^{\prime}}^{2}-M_{X}^{2}\right)}{\left(M_{V V^{\prime}}^{2}-M_{V}^{2}\right)^{2}}} \sim f_{V}\left(z_{1}\right) f_{V^{\prime}}\left(z_{2}\right) \frac{\left(z_{1} z_{2} s-M_{X}^{2}\right) \sigma^{s-c h .}}{\left(z_{1} z_{2}\right)^{2}\left(s-M_{X}^{2}\right)}
\end{gathered}
$$

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\frac{d \sigma^{V B F}}{d z_{1} d z_{2}} \sim \underbrace{f_{V}\left(z_{1}\right) f_{V^{\prime}}\left(z_{2}\right)}_{" \mu \text { PDFs" }} \underbrace{\frac{\left(M_{V V^{\prime}}^{2}-M_{X}^{2}\right)}{\left(M_{V V^{\prime}}^{2}-M_{V}^{2}\right)^{2}}}_{M_{V V^{\prime}}^{2}=z_{1} z_{2} s \gg M_{V}^{2}} \sim f_{V}\left(z_{1}\right) f_{V^{\prime}}\left(z_{2}\right) \frac{\left(z_{1} z_{2} s-M_{X}^{2}\right) \sigma^{s-c h .}}{\left(z_{1} z_{2}\right)^{2}\left(s-M_{X}^{2}\right)}
\end{gathered}
$$

PDFs are largest when $z=E_{V} / E_{\mu} \ll 1$ but $E_{V} \sim \sqrt{s} \gg M_{V}$
$\Longrightarrow f_{V}\left(z_{i}\right) \sim \frac{g_{V}^{2}}{4 \pi} \frac{1}{z_{i}} \log \left(\frac{s}{M_{V}^{2}}\right) \leftarrow$ crude approximation

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\frac{d \sigma^{V B F}}{d z_{1} d z_{2}} \sim \underbrace{f_{V}\left(z_{1}\right) f_{V^{\prime}}\left(z_{2}\right)}_{" \mu \text { PDFs" }} \underbrace{\frac{\left(M_{V V^{\prime}}^{2}-M_{X}^{2}\right)}{\left(M_{V V^{\prime}}^{2}-M_{V}^{2}\right)^{2}}}_{M_{V V^{\prime}}^{2}=z_{1} z_{2} s \gg M_{V}^{2}} \sim f_{V}\left(z_{1}\right) f_{V^{\prime}}\left(z_{2}\right) \frac{\left(z_{1} z_{2} s-M_{X}^{2}\right) \sigma^{s-c h .}}{\left(z_{1} z_{2}\right)^{2}\left(s-M_{X}^{2}\right)}
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$$

PDFs are largest when $z=E_{V} / E_{\mu} \ll 1$ but $E_{V} \sim \sqrt{s} \gg M_{V}$
$\Longrightarrow f_{V}\left(z_{i}\right) \sim \frac{g_{V}^{2}}{4 \pi} \frac{1}{z_{i}} \log \left(\frac{s}{M_{V}^{2}}\right) \leftarrow$ crude approximation
Observation: $\sigma^{V B F}=\sigma^{s-c h .} \times \int d z_{1} d z_{2} \ldots$ is solvable for $M_{V V^{\prime}} \gg M_{X}$ !

## Universal behavior: when production of $X$ by VBS and annihilation are

 driven by same physics, VBS dominates when $\sqrt{s}$ satisfies$$
\frac{\sigma^{\mathrm{VBF}}}{\sigma^{s-c h .}} \sim \mathcal{S}\left(\frac{g_{W}^{2}}{4 \pi}\right)^{2}\left(\frac{s}{M_{X}^{2}}\right) \log ^{2} \frac{s}{M_{V}^{2}} \log \frac{s}{M_{X}^{2}}>1
$$

Scaling estimate not so bad if $M_{X} \gg M_{V}$. Difference is about $\mathcal{O}(10 \%)$

| mass ( $M_{X}$ ) [TeV] | $S Z$ (Singlet) | $H_{2} Z(2 \mathrm{HDM})$ | $t^{\prime} \overline{t^{\prime}}$ (VLQ) | $\overline{t \bar{t}}$ (MSSM) | $\tilde{\chi}^{0} \tilde{\chi}^{0}$ (MSSM) | $\tilde{\chi}^{+} \tilde{\chi}^{-}$(MSSM) | Scaling (Eq. 7.7) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 400 GeV | 2.1 TeV | 2.1 TeV | 11 TeV | 2.9 TeV | 3.2 TeV | 7.5 TeV | $1.0(1.7) \mathrm{TeV}$ |
| 600 GeV | 2.5 TeV | 2.5 TeV | 16 TeV | 3.8 TeV | 3.8 TeV | 8.1 TeV | $1.3(2.4) \mathrm{TeV}$ |
| 800 GeV | 2.8 TeV | 2.8 TeV | 22 TeV | 4.3 TeV | 4.3 TeV | 8.5 TeV | $1.7(3.1) \mathrm{TeV}$ |
| 2.0 TeV | 4.0 TeV | 4.0 TeV | $>30 \mathrm{TeV}$ | 7.8 TeV | 6.9 TeV | 11 TeV | $3.7(6.8) \mathrm{TeV}$ |
| 3.0 TeV | 4.8 TeV | 4.8 TeV | $>30 \mathrm{TeV}$ | 10 TeV | 9.0 TeV | 13 TeV | $5.3(9.8) \mathrm{TeV}$ |
| 4.0 TeV | 5.5 TeV | 5.5 TeV | $>30 \mathrm{TeV}$ | 13 TeV | 11 TeV | 15 TeV | $6.8(13) \mathrm{TeV}$ |

Table 9. For representative processes and inputs, the required muon collider energy $\sqrt{s}[\mathrm{TeV}]$ at which the VBF production cross section surpasses the $s$-channel, annihilation cross section, as shown in figure 17. Also shown are the cross over energies as estimated from the scaling relationship in equation (7.7) assuming a mass scale $M_{X}\left(2 M_{X}\right)$.

# summary and outlook 



Vector boson scattering is a powerful probe of the Standard Model and new phenomena

Long-predicted but observed first during Run I/II of LHC!

- With Run II data, first measurements of VBS and key test of SM
- Run III (now-'25): VBS as new probe of new phenomena
- Run IV ('30-'40) legacy precision measurements + novel exploration of SM at high energies Buarque (ed.), Gallinaro (ed.), RR (ed.), et al, Rev. Physics ('22) [arXiv:2106.01393]


## Thank you!

## backup

## Diagram games $\Longrightarrow h$ boson production



Electroweak sector of Standard Model is powerful:

- explains $\beta$ decay


Invariant mass distribution of all $\mathrm{e}^{+} \mathrm{e}^{-}$pairs recorded by UA1

- explains inverse $\beta$ decay
- explains masses of $W^{ \pm}, Z$, e, others
- inputs needed, eg, $G_{F}, M_{W}, M_{Z}, m_{h}$


## neutrino masses

## For the experts (1 slide)

To generate $m_{\nu}$ via the Higgs mechanism, we need $\nu_{R}$

$$
\begin{aligned}
\mathcal{L}_{\nu} \text { Yuk. }=-y_{\nu} \bar{L} \tilde{\Phi}_{\nu_{R}}+\ldots & =-y_{\nu}\left(\overline{\nu_{L}} \overline{\ell_{L}}\right)\binom{\langle\Phi\rangle+h}{0} \nu_{R}+\ldots \\
& =\underbrace{-y_{\nu}\langle\Phi\rangle}_{\equiv m_{\nu}} \overline{\nu_{L}} \nu_{R}+\ldots
\end{aligned}
$$

$\nu_{R}$ do not exist in the SM , so $m_{\nu}=0$ !

Dilemma: postulating $\nu_{R}$ requires either new conservation laws or violation of lepton number and/or lepton flavor number symmetries
(expected but no evidence! suggestive that there is more to the picture)
neutrinoless $\beta \beta$ decay (at $d=5$ ) at the LHC



The helicity amplitude for the $0 \nu \beta \beta$ process $q \overline{q^{\prime}} \rightarrow \ell_{1}^{+} \ell_{2}^{+} \bar{f} f^{\prime}$ is
$\mathcal{M}_{L N V}=J_{f_{1} f_{1}}^{\mu} J_{f_{2} f_{2}^{\prime}}^{\nu} \Delta_{\mu \alpha}^{W} \Delta_{\nu \beta}^{W} \underbrace{T_{L N V}^{\alpha \beta} \mathcal{D}\left(p_{\nu}\right)}_{\text {lepton current }}$

Difficult to simulate since Weinberg op. modifies propagator of $\nu_{\ell}$
modern Monte Carlo tools work in mass basis and do not like the idea of modifying $\langle 0| \overline{\nu_{\ell^{\prime}}} \nu_{\ell}|0\rangle$


Solution: Treat vertex as a particle! Invent unphysical Majorana fermion with (small) mass $m_{\ell \ell}$ that couples to all lepton flavors

$$
T_{L N V}^{\alpha \beta} \mathcal{D}\left(p_{\nu}\right) \propto \gamma^{\alpha} P_{L} \frac{i\left(/ p+m_{\ell \ell^{\prime}}\right)}{p^{2}-m_{\ell \ell^{\prime}}^{2}} \gamma^{\beta} P_{R}=\gamma^{\alpha} P_{L} \frac{i m_{\ell \ell^{\prime}}}{p^{2}} P_{L} \gamma^{\beta} \times\left[1+\mathcal{O}\left(\left|\frac{m_{\ell \ell^{\prime}}^{2}}{p^{2}}\right|\right)\right]
$$

Plotted: Normalized production rate $\left(C_{5}=1\right)$ vs scale ( $\Lambda$ )
w/ Fuks, Neundorf, Peters, Saimpert [2012.09882]
Full $2 \rightarrow 4$ calculation at $\mathrm{NLO}(+\mathrm{PS})$ in QCD is more involved

Driven by $W_{0}^{+} W_{0}^{+}$scattering $\hat{\sigma}\left(W^{+} W^{+} \rightarrow \ell^{+} \ell^{+}\right) \sim \frac{\left|C_{5}^{\ell \ell}\right|^{2}}{18 \pi \Lambda^{2}}$

Once $\sigma$ is obtained for a "high" scale, i.e., $C_{5}^{\ell \ell^{\prime}}=1, \Lambda=200 \mathrm{TeV}$, rescale for other $\Lambda / C_{5}$.
$C_{5}^{e e} / \Lambda$ is heavily constrainted. What can the LHC say about $C_{5}^{\ell \ell^{\prime}}$ ?


Plotted: "hadronic energy / lepton energy" for different signal categories in $p p \rightarrow \mu^{ \pm} \mu^{ \pm} j j$ via $W^{ \pm} W^{ \pm} \rightarrow \mu^{ \pm} \mu^{ \pm}$



For the first time collider searches for Weinberg operator constrains

$$
\Lambda / C_{5}^{\mu \mu} \gtrsim 5 \mathrm{TeV}
$$

w/ Fuks, Neundorf, Peters, Saimpert [PRD( '21, '21)] CMS [PRL('22)]


[^0]:    ${ }^{2}$ For reviews at colliders, see Cai, Han, Li, RR [1711.02180] and Pascoli, RR, Weiland [1812.08750]

[^1]:    ${ }^{3}$ Many motivations, e.g., Al Ali, et al. [2103.14043]; R\&D progress as reported in the European Strategy Update (Delahaye, et al) [1901.06150], muoncollider.web.cern.ch; Snowmass (on-going this week)

[^2]:    ${ }^{4}$ My favorite! I find these processes really neat!

