

Recent progress in *ab-initio* studies of light nuclei and few-nucleon reactions

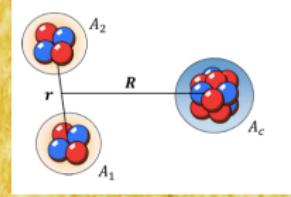


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INFN-Pisa



Exploring low-energy nuclear properties:
latest advances on reaction mechanisms with light nuclei

Workshop in honor of Pierre Descouvemont



Bruxelles, June 1, 2023

Outline

- **Introduction**

- Microscopic *ab-initio* approach
- Chiral effective field theory (χ EFT) framework
- The Hyperspherical Harmonics (HH) *ab-initio* method

- **Selected results**

- $A = 2$ reactions: pp weak capture, muon capture on deuteron
- $A = 4$ reactions of interest for Big Bang Nucleosynthesis (BBN)

- **Outlook**

Introduction: microscopic *ab-initio* studies

Nuclear observable X

- Microscopic →

- Nucleus = system of A nucleons
 - interacting among themselves → structure
 - interacting with external electroweak probes → reactions

- Microscopic → *ab-initio*

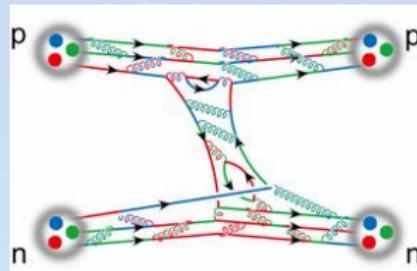
- realistic description of nuclear interactions
- realistic description of electroweak currents
- exact¹ (*ab-initio*) method to solve the quantum-mechanical problem

⇒ True predictions for observable X

Ideal case: robust procedure to estimate the theoretical error

¹ exact \equiv no uncontrolled approximations

The nuclear Hamiltonian: $H = T + V$



Nuclear interaction: $V = V_{NN} + V_{NNN}$

Until $\simeq 20\text{--}30$ years ago: **phenomenological potentials**

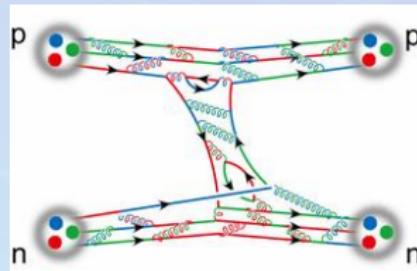
- $V_{NN} + V_{NNN}$ semi-phenomenological
- V_{NN} with $\simeq 40$ parameters fitted to $A = 2$ data
 $\rightarrow \chi^2/\text{datum} \simeq 1$
- V_{NNN} with 2-3 parameters fitted to $B(A = 3, 4)$

Very common models: [AV18+UIX](#), [AV18+Illinois](#)

Very successful, but

- **many parameters**
- no connection with QCD
- no estimate of theoretical uncertainty

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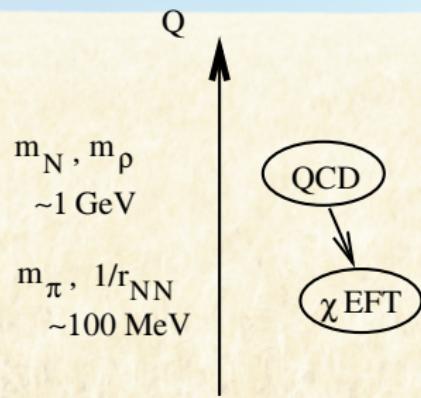
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\Rightarrow **Chiral Effective Field Theory (χ EFT)**



Chiral Effective Field Theory (χ EFT): a very short summary

- QCD → quarks and gluons ("high-energy" d.o.f.)
- Nuclear physics → nucleons and pions ("low-energy" d.o.f.)
- EFT → processes with $E \simeq p \simeq m_\pi \ll \Lambda_{\text{QCD}} \sim 1 \text{ GeV}$
 - ★ keep the "l-e" d.o.f.: π and N (and sometimes Δ 's - $m_\Delta = m_N \sim 300 \text{ MeV}$)
 - ★ Lagrangians describing the interactions of $\pi - N(\pi - \Delta)$ are expanded in powers of $O(p/\Lambda_{\text{QCD}})^\nu \rightarrow$ perturbative expansion
 - ★ "h-e" d.o.f. integrated out → contact interactions with "l-e" d.o.f. and low-energy constants (LECs) obtained from experiment
- χ EFT → EFT with spontaneous breaking of QCD's χ -symmetry
- Regularization of short-range terms with cutoff function → $\Lambda \simeq 400 - 600 \text{ MeV}$

Disadvantage: limited to processes with $E \leq [2 \div 3] m_\pi$

Advantages

- nuclear force "hierarchy" → accurate $V_{NN} + V_{NNN}$
- consistent framework for interactions + currents (just add electroweak field as d.o.f.)
- possibility to estimate the theoretical uncertainty (perturbative expansion)

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χ EFT potentials

- **Idaho potentials:** N3LO-Idaho ($\Lambda = 500$ MeV) \rightarrow EMN
[D. Entem *et al.*, Front. Phys. **8**, 57 (2020)]
- **Norfolk potentials (NV)**
[M. Piarulli and I. Tews, Front. Phys. **7**, 245 (2019)]
- **N2LOsim potentials**
[B.D. Carlsson *et al.*, Phys. Rev. X **6**, 011019 (2016)]
- **SMS-RS:** semi-local regularization scheme (local for TPE and non-local for contact part)
[P. Reinert, H. Krebs, E. Epelbaum, Eur. Phys. J. A **54**, 86 (2018)]

For instance:

Name	DOF	O_χ	(R_S, R_L) or Λ	E range	Space
NVIa	π, N, Δ	N3LO	(0.8, 1.2) fm	0–125 MeV	r
NVIb	π, N, Δ	N3LO	(0.7, 1.0) fm	0–125 MeV	r
NVIIa	π, N, Δ	N3LO	(0.8, 1.2) fm	0–200 MeV	r
NVIIb	π, N, Δ	N3LO	(0.7, 1.0) fm	0–200 MeV	r
EMN450	π, N	up to N4LO	450 MeV	0–300 MeV	p
EMN500	π, N	up to N4LO	500 MeV	0–300 MeV	p
EMN550	π, N	up to N4LO	550 MeV	0–300 MeV	p

A. Gnech, L.E. Marcucci, M. Viviani, arXiv:2305.07568

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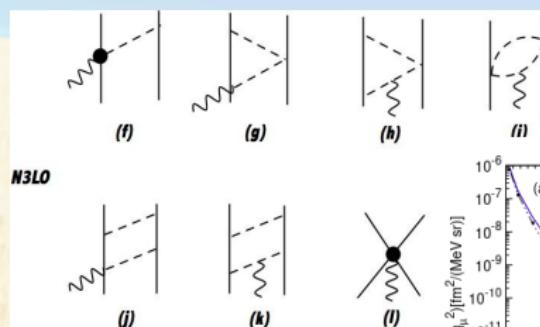
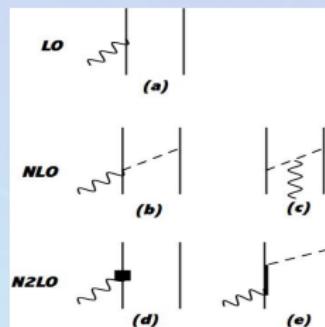
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$V_{NNN} \rightarrow$ see later

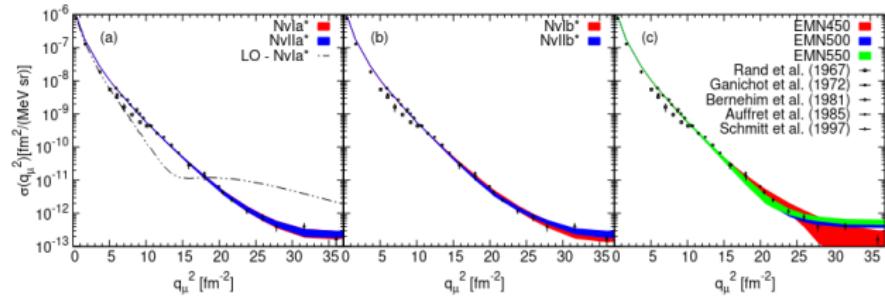
Electromagnetic current in χ EFT



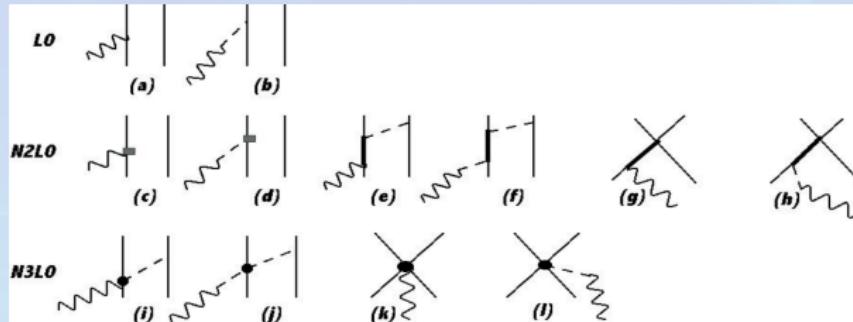
- $j_\Delta^{\text{N}2\text{LO}}(\mathbf{q})$ in panel (e) absent in Δ -EFT
- not included the Δ intermediate states at N3LO
- $j_{\text{OPE}}^{\text{N}3\text{LO}}(\mathbf{q}) \rightarrow d_2^S, d_2^V; d_3^V$
- $j_{\text{MIN}}^{\text{N}3\text{LO}}(\mathbf{q}) \rightarrow$ from πN scattering
- $j_{\text{NM}}^{\text{N}3\text{LO}}(\mathbf{q}) \rightarrow d_1^S; d_1^V$

FIT: all 5 LECs fitted to $A = 2, 3$ magnetic moments, d magnetic form factor and $d(e, e')pn$ at threshold

A. Gnech and R. Schiavilla, Phys. Rev. C **106**, 044001 (2022)



Axial current in χ EFT



A. Baroni *et al.*, Phys. Rev. C **98**, 044003 (2018)

- Ignore pion-pole terms [(b), (d), (f), (h), (j), (l)]
- diagrams (g) and (h) vanish; diagram (e) $\rightarrow c_3^\Delta ; c_4^\Delta$ (similar to $c_3 ; c_4$ of diagram (i))
- CTs in (i) and (k)

$$\begin{aligned} j_{5,a}^{N3LO}(\mathbf{q}; CT) &= z_0 e^{i\mathbf{q} \cdot R_{ij}} \frac{e^{-(r_{ij}/R_S)^2}}{\pi^{3/2}} (\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)_a (\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j) \\ z_0 &= \frac{g_A}{2} \frac{m_\pi^2}{f_\pi^2} \frac{1}{(m_\pi R_S)^3} \left[-\frac{1}{4} \frac{m_\pi}{g_A \Lambda_\chi} c_D + \frac{m_\pi}{3} (c_3 + 2c_4 + c_3^\Delta + 2c_4^\Delta) + \frac{m_\pi}{6m} \right] \end{aligned}$$

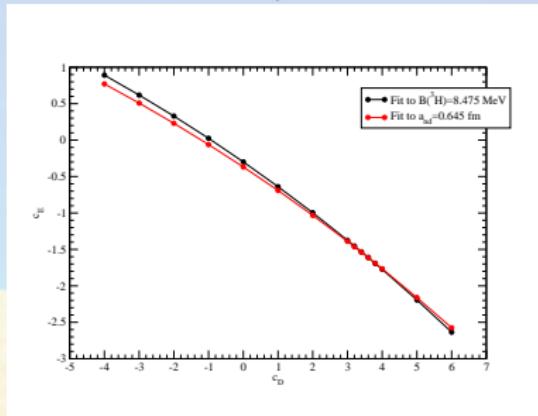
$z_0/d_R \leftrightarrow c_D$ (LEC in V_{NNN}) $\longrightarrow GT^{exp}$ in 3H β -decay

Factor $-1/4$ missing in many calculation (error spread in 2012-2018)

Interplay potential-current in χ EFT

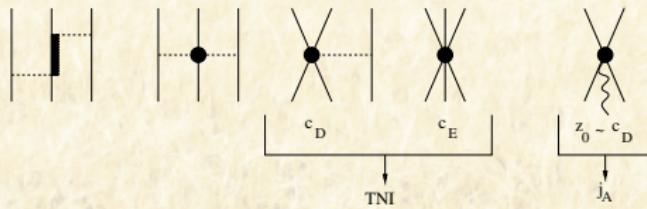
- NV2+3/nY: fit c_D & c_E to $B(^3\text{H})$ and $a_{nd}^{Exp} = (0.645 \pm 0.010) \text{ fm}$

M. Piarulli *et al.*, Phys. Rev. Lett. **120**, 052503 (2018)



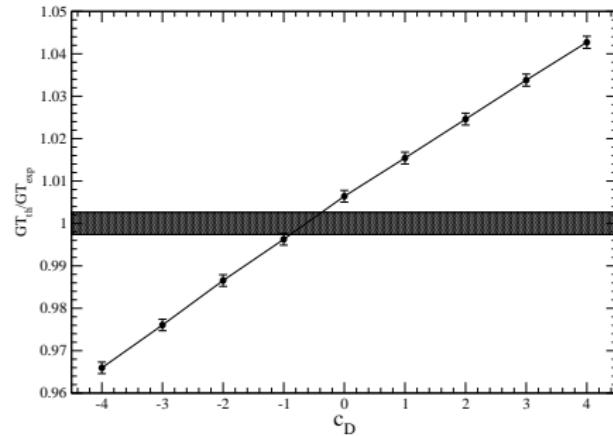
→ correlation $B(^3\text{H})/a_{nd}$

- Use $B(^3\text{H})$ and GT^{exp} of ^3H β -decay → NV2+3/nY*



	NV2+3/Ia	NV2+3/Ib
c_D	3.666	-2.061
c_E	-1.638	-0.982
GT	0.9885	0.9730
	NV2+3/Ia*	NV2+3/Ib*
c_D	-0.635	-4.71
c_E	-0.09	0.55

$$GT^{exp} = 0.9511 \pm 0.0013$$



$A = 3, 4$ HH binding energies and scattering lengths

Model	B(^3H)	B(^3He)	B(^4He)	$a_{nd}^{(2)}$	$a_{nd}^{(4)}$
NV2/Ia	8.718	7.090	25.15	1.119	6.326
NV2/Ib	7.599	6.885	23.96	1.307	6.327
NV2+3/Ia	8.475	7.735	28.33	0.645	6.327
NV2+3/Ib	8.475	7.737	28.30	0.645	6.327
NV2+3/Ia*	8.477	7.727	28.30	0.638	6.326
NV2+3/Ib*	8.469	7.724	28.21	0.650	6.327
Exp.	8.475	7.725	28.30	0.645(10)	6.35(2)

The Hyperspherical Harmonics (HH) method

Bound states

$$\Psi^{JJ_z} = \sum_{\mu} c_{\mu} \Psi_{\mu}$$

- $\Psi_{\mu} \rightarrow$ known functions (spin-isospin HH functions)
- Rayleigh-Ritz var. principle:
 $\delta_c \langle \Psi^{JJ_z} | H - E | \Psi^{JJ_z} \rangle = 0$
⇒ Solve for E and c_{μ}

Scattering states

$$\Psi_{LSJ} = \Psi_{core}^{LSJ} + \Psi_{asym}^{LSJ}$$

- $\Psi_{core}^{LSJ} = \sum_{\mu} c_{\mu} \Psi_{\mu}$
- $\Psi_{asym}^{LSJ} \propto \Omega_{LS}^R + \sum_{L'S'} R_{LL',SS'} \Omega_{L'S'}^I$
- Kohn var. principle:
 $[R_{LL',SS'}] = R_{LL',SS'} - \langle \Psi_{L'S'J} | H - E | \Psi_{LSJ} \rangle$
⇒ Solve for c_{μ} and $R_{LL',SS'} \rightarrow$ phase-shifts and mixing angles

Strength

and

weakness

- very accurate
- both r - and p -space
- both bound and scattering states
- at present limited to $A = 6$
- in prospective $A = 8$
- not much more ...

SELECTED RESULTS

- $A = 2$ reactions: pp and $\mu - d$ weak captures

The pp fusion in χ EFT: an update

B. Acharya, L.E. Marcucci, L. Platter, arXiv:2304.03327

- updated constants (especially $g_A = 1.2754$)
- correct the $-1/4$ factor
- better techniques (Bayesian methods) to estimate the theoretical error
- benchmark of two approaches (Var. Method and Lippmann-Schwinger)
- various χ EFT potentials

Model	Method	$1/m_N^2$ term	c_D	Goal
SMS-RS	LS	excluded	from nd scatt.	$\Delta(\chi)$
N2LOsim	LS	excluded	from GT^{exp}	update
LO ... N2LO [†]	LS	excluded	from GT^{exp}	$\Delta(c_D)$
N3LO-Idaho	VM/LS	included/excluded	from GT^{exp}	update + benchmark

[†] Bayesian analysis of S. Wesolowski *et al.*, Phys. Rev. C **104**, 064001 (2021)

Order-by-order convergence (SMS-RS)

Order	$S(0) \times 10^{-23} \text{ MeV fm}^2$	$S'(0)/S(0) \text{ MeV}^{-1}$	$S''(0)/S(0) \text{ MeV}^{-2}$	$S'''(0)/S(0) \text{ MeV}^{-3}$
LO	4.143	10.75	306.75	-5150
NLO	4.094	10.81	312.78	-5370
NNLO [N3LO]	4.100	10.83	313.72	-5382

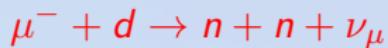
Benchmark VM vs. LS (N3LO-Idaho)

ft_3H -value s^{-1}	Method	$S(0) \times 10^{-23} \text{ MeV fm}^2$	$S'(0)/S(0) \text{ MeV}^{-1}$	$S''(0)/S(0) \text{ MeV}^{-2}$	$S'''(0)/S(0) \text{ MeV}^{-3}$
1134.6(3.1)	VM	4.115(4)	10.60	347.1	-6908
	LS	4.101(4)	10.83	313.8	-5382
1129.6(3.0)	VM	4.118(4)	10.60	347.1	-6907
	LS	4.104(4)	10.83	313.8	-5381
1132.1(4.3)	VM	4.117(4)	10.60	347.1	-6908
	LS	4.104(4)	10.83	313.8	-5382

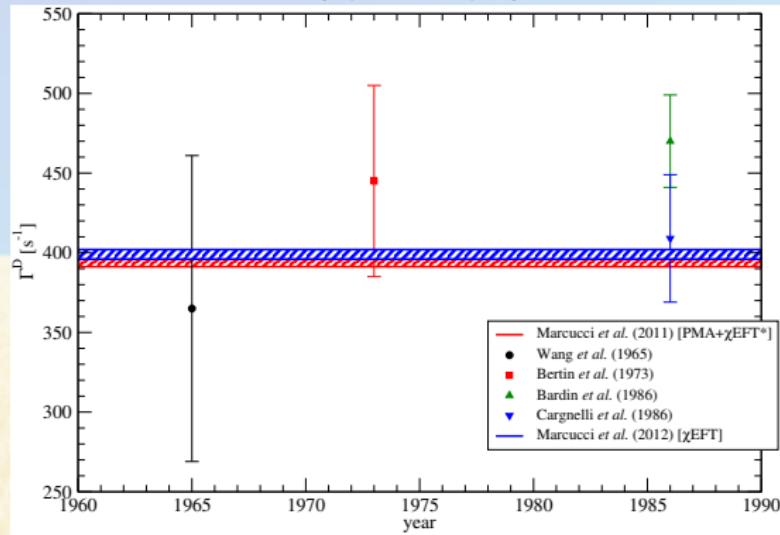
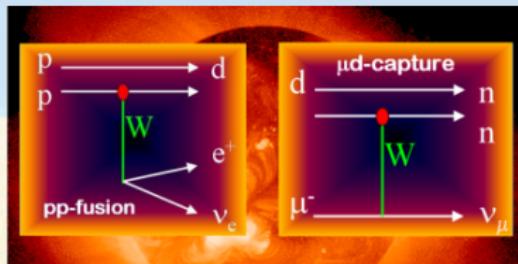
$$S(0) = [4.100 \pm 0.024(\text{syst}) \pm 0.013(\text{stat}) \pm 0.008(g_A)] \times 10^{-23} \text{ MeV fm}^2$$

The muon capture on deuteron in χ EFT (I)

A. Gnech, L.E. Marcucci, M. Viviani, arXiv:2305.07568



Two hyperfine states ($1/2$ & $3/2$) $\Rightarrow \Gamma^D$ & Γ^Q



MuSun Collab. at PSI $\rightarrow 1.5\%$ exp. error

The muon capture on deuteron in χ EFT (II)

$$\Gamma(E'_1) = \frac{G_V'^2}{\pi} |\psi_{1s}(0)|^2 E_1 p_1 \int d \cos \theta_1 \frac{E_2 k_\nu^2}{E_2 + k_\nu + p_1 \cos \theta_1} \sum_{s_1 s_2 h_\nu} \sum_{f_z} |M_{fi}(f_z, s_1, s_2, h_\nu; p_1, \cos \theta_1)|^2$$
$$\Gamma = \int_0^{E'_1 \text{ max}} dE'_1 \Gamma(E'_1)$$

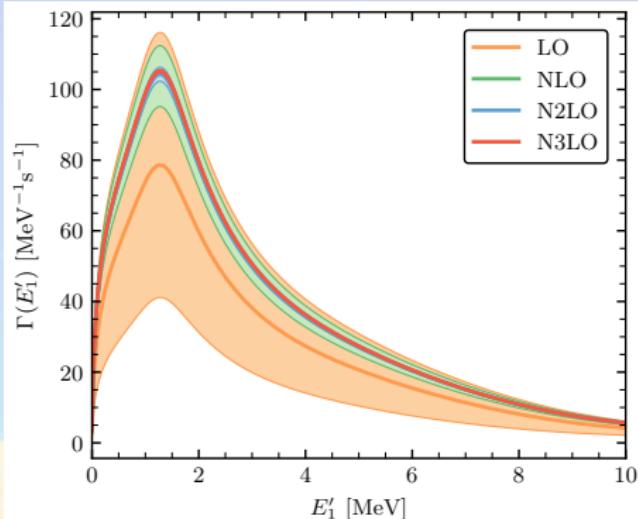
with $\cos \theta_1 = \mathbf{q} \cdot \mathbf{p}_1$

- update previous work with most recent potentials and currents
- provide $\Gamma(E'_1)$ to experimentalists (rather than $\Gamma(p)$)
- robust estimate of theoretical uncertainties

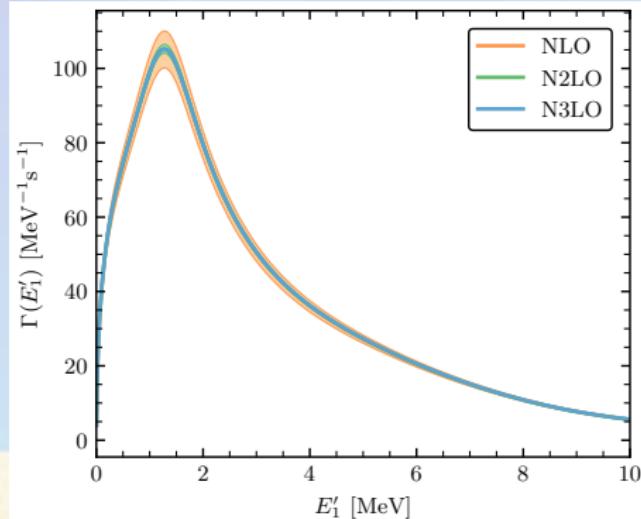
Theoretical uncertainties from:

- $g_A(q^2) = g_A \left(1 - \frac{1}{6} r_A^2 q^2\right)$ with $r_A^2 = 0.46(16) \text{ fm}^2$
R.J. Hill *et al.*, Rep. Prog. Phys. **81**, 096301 (2018)
- **chiral truncation** of interaction and current (Bayesian analysis)
- **model dependence**

Chiral truncation error for the current (EMN550 at N3LO)



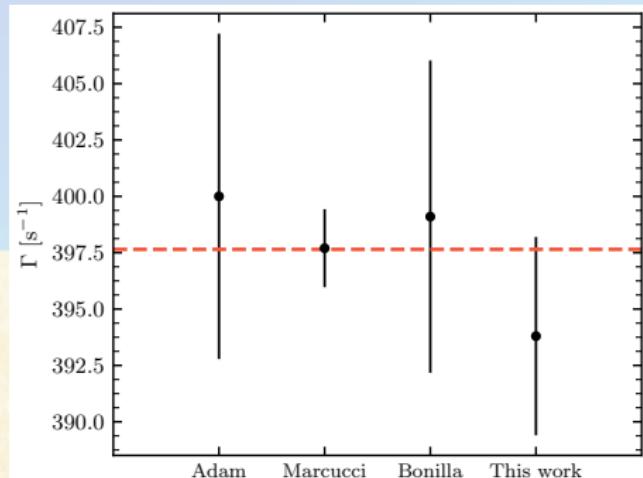
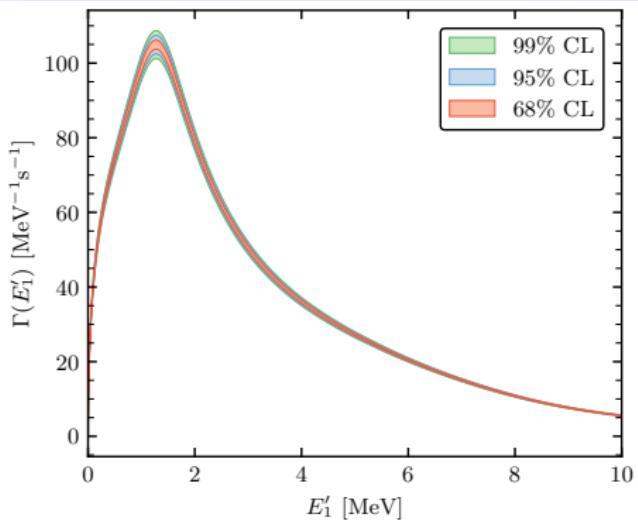
Chiral truncation error for the interaction (EMN550)



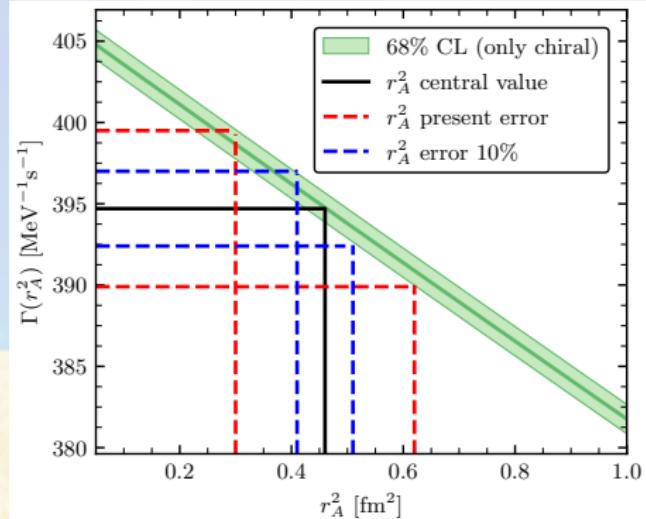
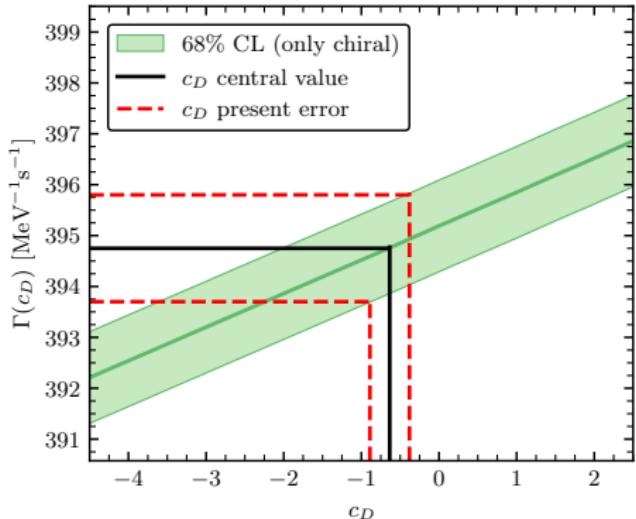
Bands= 2σ truncation error

Inter.	$\Gamma(\text{comp})$	$M_{k=3}^C$	$M_{k=4}^I$	$\Gamma(\infty)$	$\sigma_{k=3}^C(68\% \text{ CL})$	$\sigma_{k=4}^I(68\% \text{ CL})$	$\sigma_{\text{LECs}}(68\% \text{ CL})$
NVIa	394.6	0.1	n.a.	394.7	0.8(0.7)	n.a.	3.9
NVIb	395.0	0.1	n.a.	395.1	1.4(0.8)	n.a.	3.9
NVIIa	393.6	0.1	n.a.	393.7	0.8(0.7)	n.a.	3.9
NVIIb	394.0	0.1	n.a.	394.1	1.5(0.8)	n.a.	3.9
EMN450	389.8	0.1	-0.2	389.7	0.8(0.7)	0.4(0.4)	3.8
EMN500	393.4	0.1	0.2	393.7	0.8(0.7)	0.3(0.2)	3.9
EMN550	395.2	0.1	0.2	395.5	0.8(0.7)	0.4(0.2)	3.9

$$\Gamma = (393.8 \pm 4.4) \text{ s}^{-1} \quad (68\% \text{CL})$$



Impact on the MuSun Experiment



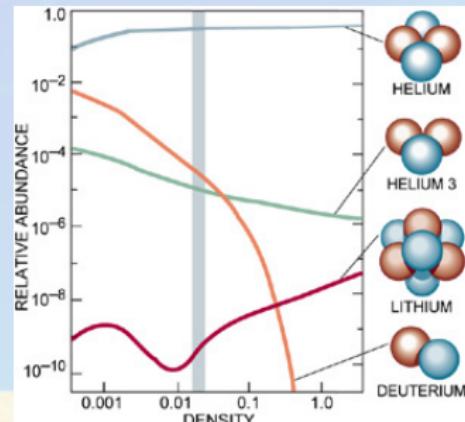
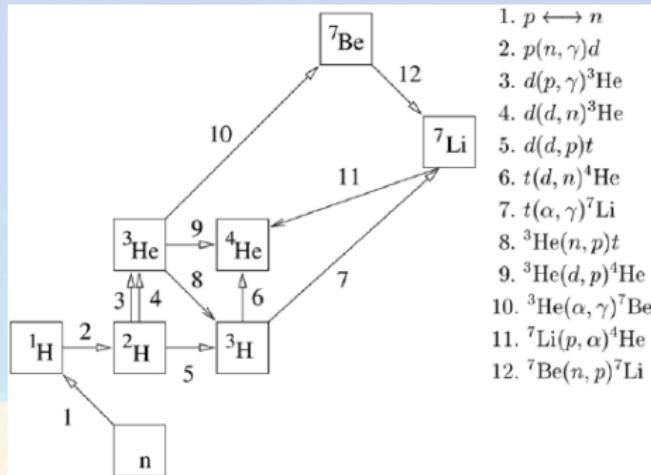
- c_D -uncertainty \rightarrow minimal impact on Γ
- present r_A -uncertainty $\rightarrow \sim 1\%$ error on Γ

$\Rightarrow r_A$ -uncertainty $\sim 10\% \rightarrow$ error on Γ of $0.6\% \ll$ MuSun quoted error (1.5%)

SELECTED RESULTS

- $A = 4$ reactions of interest for BBN

The primordial deuterium abundance



$$10^5(D/H)_{\text{exp}} = 2.527 \pm 0.030$$

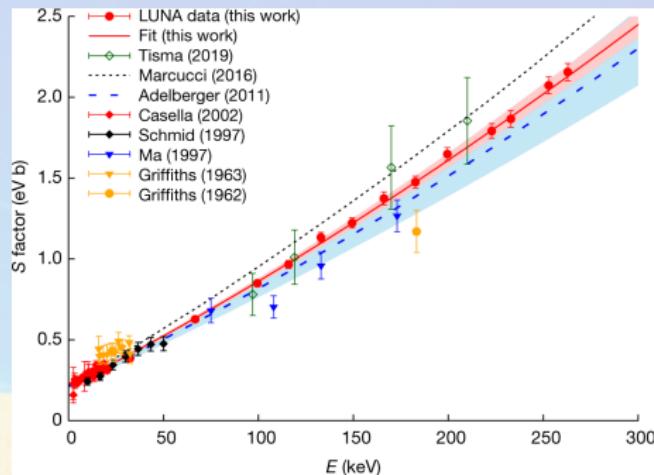
R.J. Cooke *et al.*, *Astrophys. J.* **885**, 102 (2018)

Crucial inputs for BBN

- $p(d, \gamma){}^3\text{He}$
- $d(d, p){}^3\text{H} \& d(d, n){}^3\text{He}$

LUNA experiment for $p(d, \gamma){}^3\text{He}$

The $^2\text{H}(p, \gamma)^3\text{He}$ reaction - The LUNA experiment



$$10^5(\text{D}/\text{H})_{\text{BBN}} = 2.52 \pm 0.03 \pm 0.06$$

vs.

$$10^5(\text{D}/\text{H})_{\text{exp}} = 2.527 \pm 0.030$$

----- Phenomenological approach (AV18/UIX)

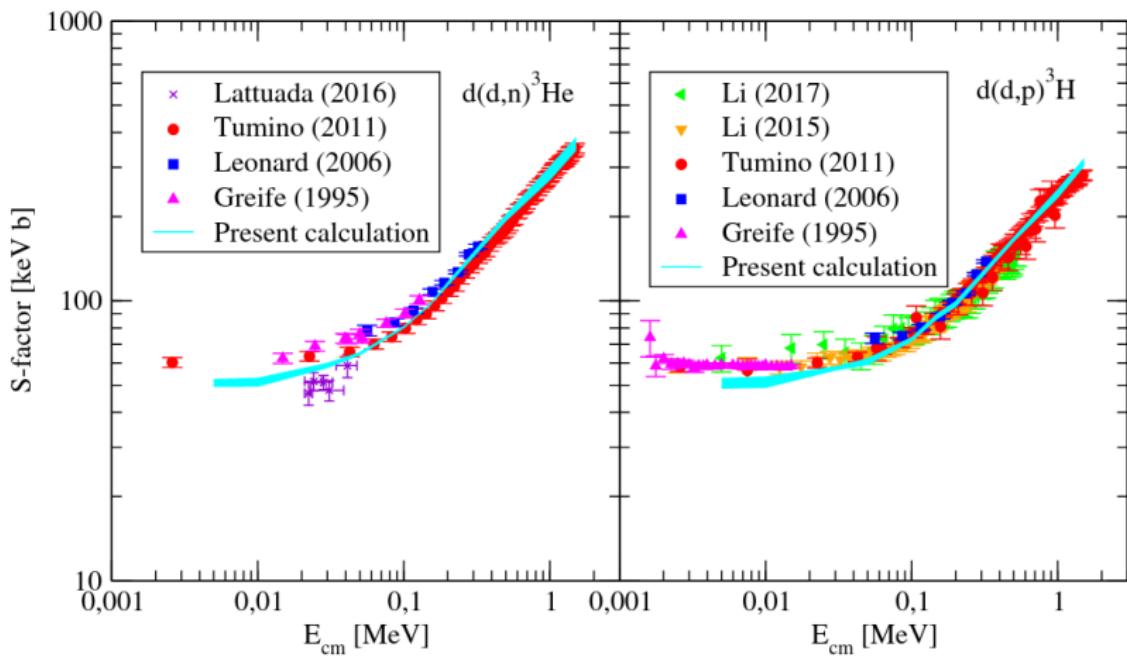
→ what is the theoretical uncertainty? $\Rightarrow \chi\text{EFT}$ (work in progress)

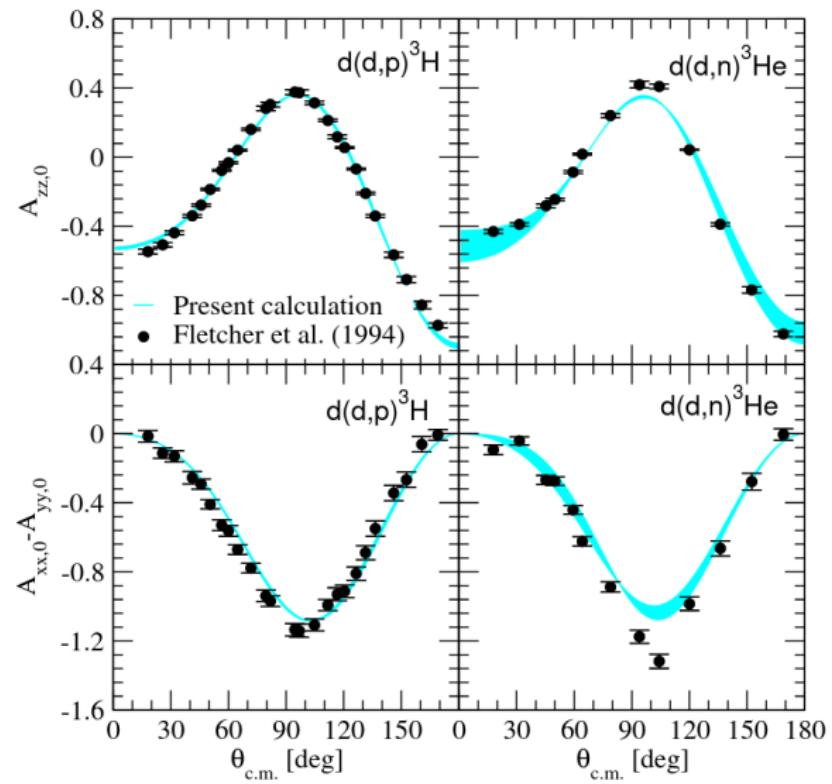
BBN error now dominated by $d(d, p)^3\text{H}$ & $d(d, n)^3\text{He}$

V. Mossa *et al.*, Nature **587**, 210 (2020)

The $d(d, p)^3\text{H}$ and $d(d, n)^3\text{He}$ processes

M. Viviani *et al.*, Phys. Rev. Lett. **130**, 122501 (2023)

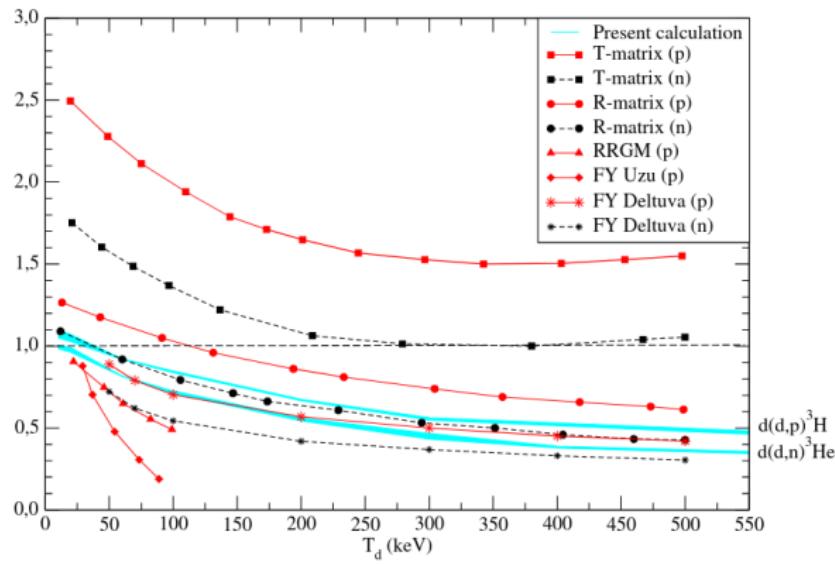




Nice agreement theory vs. experiment

The “quintic” suppression factor

$\vec{d}(\vec{d}, n)^3\text{He}$ & $\vec{d}(\vec{d}, p)^3\text{H}$ suppressed in S -wave

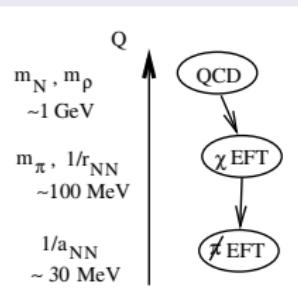


⇒ “neutron lean” reactors

Outlook

- **HH method**: systematic study of $A \geq 4$ bound- and scattering states
- Further *ab-initio* predictions in χ EFT for
 - Reactions involved in the BBN network or stellar evolution
 - e^+e^- production in $p+^7\text{Li}$ (ATOMKI) (**but also in $p+^2\text{H}$**)
 - Muon capture on $A = 3, 4, 6$ nuclei (work in progress)
 - β -decay of $6 \leq A \leq 8$ systems
- Low energies → “new” framework: \neq EFT

Pionless EFT (\neq EFT): going lower in energy ...



Advantages

- **drastic simplification in the operatorial structure** for both potentials and currents
- **faster convergence** in the HH expansion
- more direct match with lattice QCD calculations (performed at large m_π)
- large $a_{NN} \Rightarrow$ short-range NN dynamics does not decouple in the NNN sector $\Rightarrow V_{NNN}$ at LO

Local $V_{NN} + V_{NNN}$ in χ EFT

- $V_{NN} \rightarrow$ contact terms up to Q^4 (N3LO)

$$C(r) = C_0(r) P_0^\tau + C_1(r) P_1^\tau$$

$$C_\alpha(r) = \frac{e^{-(r/R_\alpha)^2}}{\pi^{3/2} R_\alpha^3}$$

Model	a	b	c	d	o
R_0 (fm)	1.7	1.9	2.1	2.3	1.54592984
R_1 (fm)	1.5	2.0	2.5	3.0	1.83039397

Model	Order	T_{lab} (MeV)	N_{np}	$\chi^2(np)/\text{datum}$	N_{pp}	$\chi^2(pp)/\text{datum}$	N	χ^2/datum
a	LO	0–1	91	5.54	157		248	
	NLO	0–15	381	1.83	394	1.53	776	1.67
	N3LO	0–25	643	1.60	451	1.24	1096	1.45
b	LO	0–1	91	37.6	157		248	
	NLO	0–15	382	1.39	395	1.09	778	1.24
	N3LO	0–25	646	1.42	452	1.06	1099	1.27
c	LO	0–1	91	24.8	157		248	
	NLO	0–15	378	2.34	392	1.97	771	2.15
	N3LO	0–25	645	1.83	453	1.33	1099	1.62
d	LO	0–1	91	41.2	157		248	
	NLO	0–15	377	10.2	392	6.88	770	8.51
	N3LO	0–25	638	2.03	446	8.09	1085	4.52
o	LO	0–1	91	2.16	157		248	
	NLO	0–15	382	1.27	394	1.08	777	1.17
	N3LO	0–25	650	1.25	452	1.10	1103	1.19

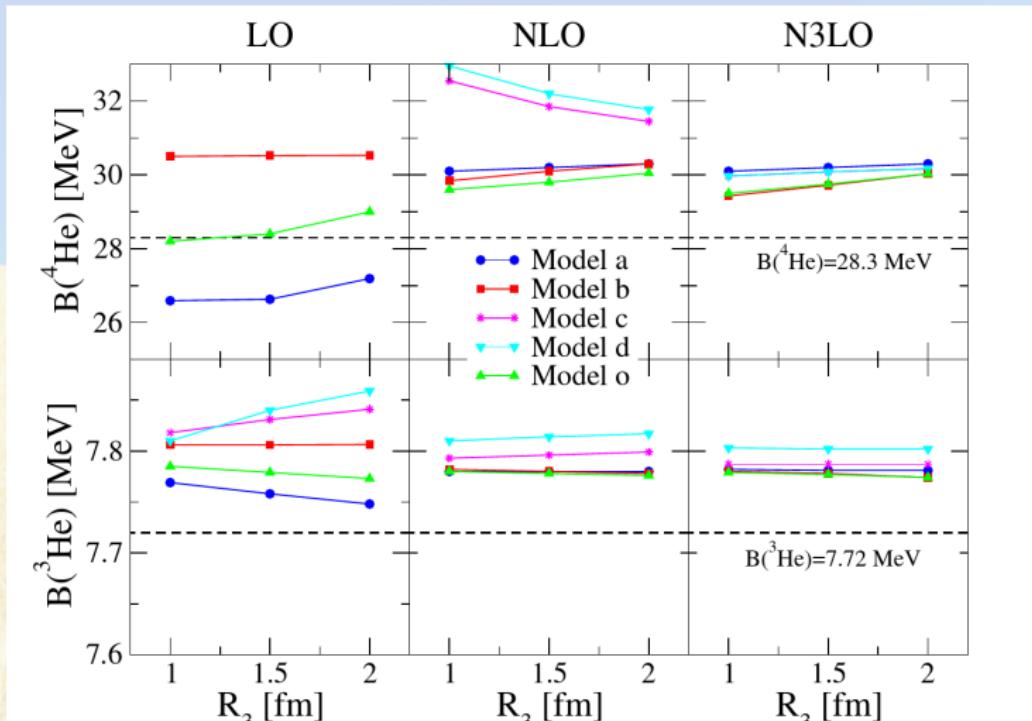
- V_{NNN} up to LO $\rightarrow c_E$ fitted to $B(^3\text{H})$

R. Schiavilla *et al.*, Phys. Rev. C **103**, 054003 (2021)

$\not\! EFT$: from few- to many-body systems (I)

R. Schiavilla *et al.*, Phys. Rev. C **103**, 054003 (2021)

- V_{NN} LO-N3LO fitted to NN systems
- V_{NNN} only at LO fitted to $B(^3\text{H}) \Rightarrow B(^3\text{He}), B(^4\text{He}), \dots$ = predictions

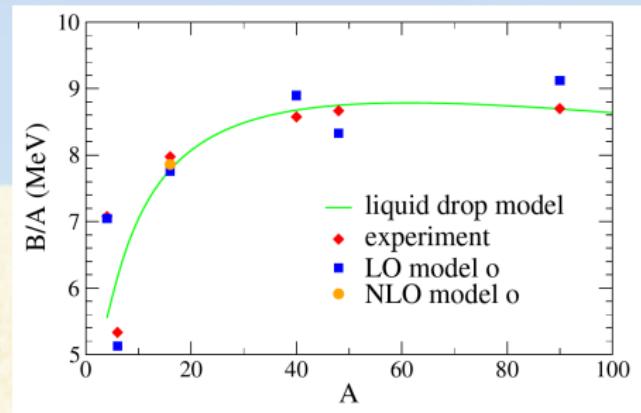


$\not\! EFT$: from few- to many-body systems (II)

R. Schiavilla *et al.*, Phys. Rev. C 103, 054003 (2021)

$V_{NN} + V_{NNN}$ applied to

- $^4\text{He}, ^6\text{Li}, ^6\text{He} \rightarrow \text{HH}$ + AFDMC (benchmark)
- $^{16}\text{O}, ^{40}\text{Ca}, ^{48}\text{Ca}, ^{90}\text{Zr} \rightarrow \text{AFDMC}$



Outlook:

- ① Go beyond $V_{NNN}(\text{LO}) \rightarrow V_{NNN}(\text{N3LO})$ (A_y -puzzle)
- ② Develop the **consistent electroweak transition operators**

In collaboration with

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- D. Logoteta (Univ. Pisa)
- L. Girlanda (Univ. del Salento)
- A. Gnech (ECT*)
- R. Schiavilla (JLab-ODU)
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Thank you, Pierre, for all your inspiring work!

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Thank you, Pierre, for all your inspiring work!

Thank you All for your attention!