Three-body projectile elastic scattering with microscopic wave functions

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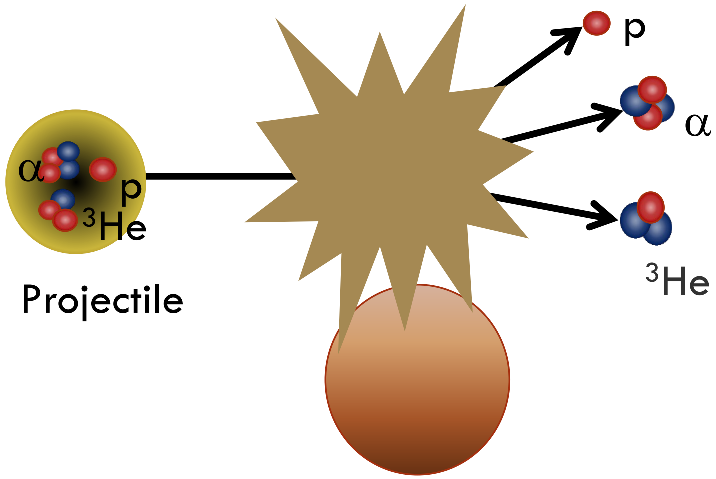
Outline

- Introduction
- Microscopic CDCC method
- Applications on the elastic scattering of the exotic nuclei
  - ^{8}B
  - ^{8}Li
- Conclusions

Describing exotic nuclei

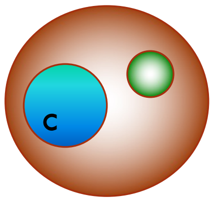


- Exotic nuclei are studied experimentally and theoretically through reactions (mostly breakup). Elastic scattering is the starting point.

- They can be seen made of few bodies and their wave function extends to large distances: An accurate description must include these characteristics into reaction models.

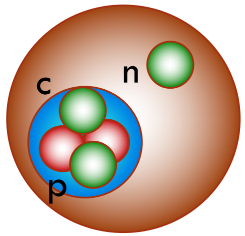
Description of the projectile: Cluster models

Non-microscopic



- Nucleus-nucleus interactions: In many cases are not well known.
- Antisymmetrization effects are simulated by a suitable choice of nucleus-nucleus potentials.
- Core excitations neglected or approximately considered.

Microscopic



- Nucleon-nucleon interactions.
- Exact account of the Pauli principle.
- Number of bodies increase: More difficult calculations.
- Core excitation are exactly taken.

The continuum discretized coupled-channel method (CDCC)

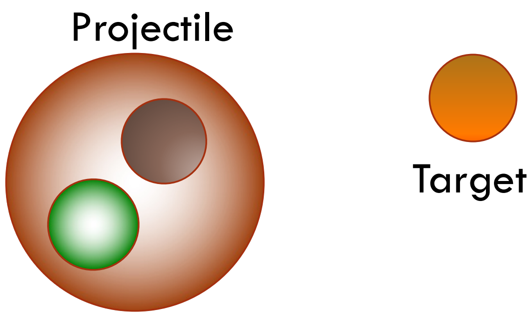
- Reaction model developed with the aim to include effects of the continuum: single channel calculations are in general unable to explain experimental cross sections of weakly bound systems.

- It is suitable for reactions around the coulomb barrier.

- This method provides an approximate solution of the three(four)–body scattering problem.

The continuum discretized coupled-channel method (CDCC)

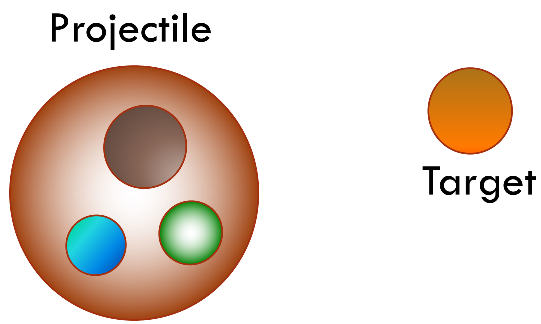
Two-body projectile



Ex:

- $(d=n+p)+^{58}$Ni → Rawitscher, PRC 9, 2210 (1974).
- $(^{19}C=^{18}C+n)+p$ → *R. Crespo et al.*, PRC 83 054613 (2011).
- $(^{11}Be=^{10}Be+n)+^{64}$Zn → *T. Druet et al.*, EPJA 48, 147 (2012).

Three-body projectile

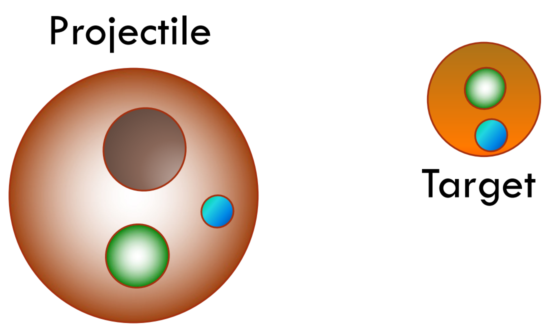


Ex:

- $(^{6}He=\alpha+n+n)+^{12}$C → *Matsumoto et al.*, NPA 738, 471 (2004).
- $(^{6}He=\alpha+n+n)+^{208}$Pb → *Rodríguez-Gallardo et al., PRC 80, 051601* (2009).
- $(^{9}Be=\alpha+\alpha+n)+^{208}$Pb→ *Descouvemont et al*, PRC 91, 024606 (2015).

The continuum discretized coupled-channel method (CDCC)

Two(three)-body projectile + Two-body target

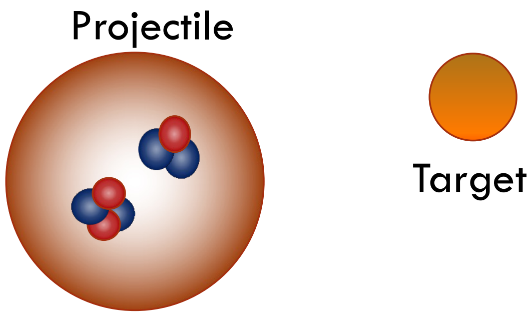


Ex:

- (^{11}Be=^{10}Be+n)+(d=n+p)→ *Descouvemont, PRC 97 064607* (2018).
- (^{11}Li=^{9}Li+n+n)+(d=n+p)→ *Descouvemont, PRC 101, 064611* (2020).

The microscopic continuum discretized coupled-channel method (MCDCC)

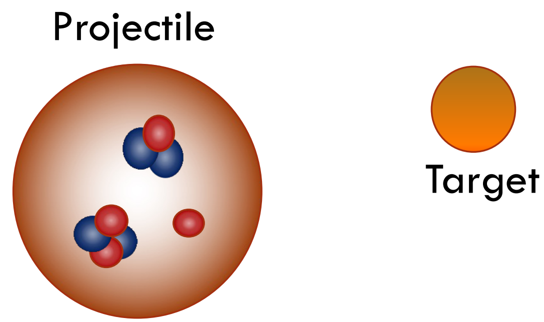
Microscopic two-body projectile



Ex:

- (^{7}Li=α+t)+^{208}Pb → *Descouvemont and M. Hussein, PRL 111, 082701* (2013).
- (^{7}Li=α+t)+^{12}C, ^{28}Si → *Pinilla & Descouvemont, PRC 89, 054615* (2014).
- (^{19}F=α+^{15}Ni)+ ^{208}Pb → *Grineviciute & Descouvemont,* PRC 90, 034616 (2014).

Microscopic three-body projectile



Ex:

- (^{9}Be=α+α+n)+^{27}Al,^{208}Pb → *Descouvemont & Itagaki, PRC 97, 014612* (2018).
- (^{8}B=α+^{3}He+p)+Target → *Descouvemont & Pinilla, FBS 60,11* (2020).
- (^{8}Li=α+t+n)+Target → *Descouvemont & Pinilla, FBS 60, 11* (2020).

The microscopic continuum discretized coupled channel method (MCDCC)

Characteristics

- High predictive power:

  Wave functions of the projectile→ nucleon-nucleon interactions

- The model is based on:

  Nucleon-Target interactions (large known set available)

  (nucleus-nucleus interactions unknown in many cases)

- Non free parameter once the *n*-*n* and nucleon-target interactions are chosen.

The continuum discretized coupled-channel method (CDCC)

We wish to solve

$$\mathrm{H_{PT}}\Phi(\xi_P,\xi_T,\boldsymbol{R}) = E_T\Phi(\xi_P,\xi_T,\boldsymbol{R}),$$

with

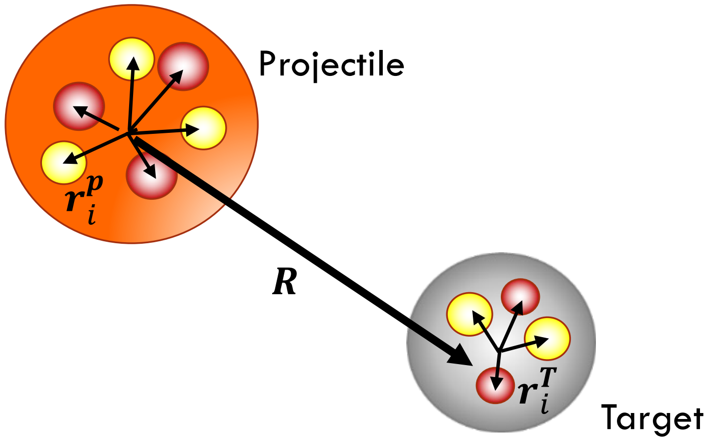
$$H_{PT} = T_R + H_P(\xi_P) + H_T(\xi_T) + V(\xi_P,\xi_T,\boldsymbol{R}).$$

Then we make the expansion

$$\Phi(\xi_P,\xi_T,\boldsymbol{R}) = \sum \underbrace{\Psi_c(\xi_P,\xi_T)}_{\text{Internal w. f.}}\underbrace{\chi_c(\boldsymbol{R})}_{\text{Relative w. f.}},$$

to get

$$\underbrace{\left[-\frac{h^2}{2\mu_{PT}}\nabla^2 + V_{cc}(\boldsymbol{R}) - (E-\epsilon_c)\right]\chi_c(\boldsymbol{R}) = \sum_{c'\neq c} V_{c'c}(\boldsymbol{R})\chi_{c'}(\boldsymbol{R})}_{\text{CDCC system of equations}}$$



$\xi_P = \{\boldsymbol{r}_i^p\} \rightarrow$ Int. coordinates of the projectile

$\xi_T = \{\boldsymbol{r}_i^T\} \rightarrow$ Int. coordinates of the target

$$H_P(\xi_P) = E_P\Psi_P(\xi_P);\ H_T(\xi_T) = E_T\Psi_T(\xi_T);$$

$E_P; E_T < 0 \rightarrow$ Bound states;

$E_P; E_T > 0 \rightarrow$ Pseudostates.

The continuum discretized coupled-channel method (CDCC)

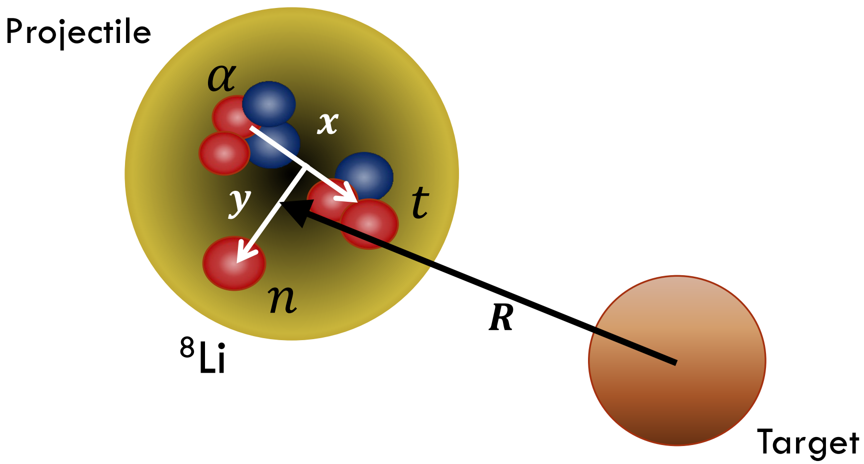
The common steps to all CDCC calculations

1. Calculation the internal wave functions of the projectile and/or the target

2. Finding of the Coupling potentials $V_{cc'}(\boldsymbol{R})$

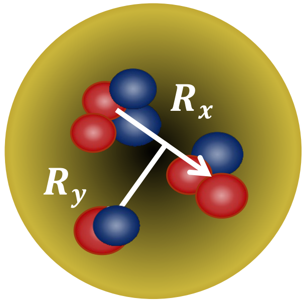
3. Solving the system of differential equations → Scattering matrix → Cross sections.

The microscopic continuum discretized coupled channel (MCDCC): Three+one



The microscopic continuum discretized coupled channel (MCDCC): Three+one

*1. Three-cluster microscopic wave function of the projectile**



Projectile

$\boldsymbol{R_x}, \boldsymbol{R_y}$ → Generator coordinates: Parameters related with the center of the clusters.

We need to solve

$$\mathrm{H}\Psi_{\mathrm{P}}^{jm\pi} = E\Psi_{\mathrm{P}}^{jm\pi},$$

with

$$H = \sum_{i=1}^{A_p} T_i + \sum_{i<j}^{A_P} v_{ij} + \sum_{i<j<k}^{A_P} v_{ijk},$$

v_{ij}, → 2B effective interactions (Cou.+Nuc), v_{ij}^{N}, → Gaussians

$$\Psi_{\mathrm{P}}^{jm\pi} = \sum_{ii'} f^{j\pi}(\boldsymbol{R}_{xi}, \boldsymbol{R}_{yi'})\Phi^{j\pi}(\mathbf{R}_{xi}, \mathbf{R}_{yi'})$$

$\Phi^{j\pi}$ → Slater determinant projected in parity and angular momentum

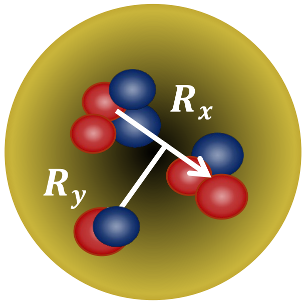
*S. korennov and P. Descouvemont, Nucl. Phys. A 740, 249 (2004), P. Descouvemont, Phys. Rev. C 99, 064308 (2019)

The microscopic continuum discretized coupled channel (MCDCC): Three+one

1. Three-cluster microscopic wave function of the projectile

To reduce calculations $\boldsymbol{R_x}, \boldsymbol{R_y} \rightarrow \boldsymbol{R}$: Hyperspherical coordinates

$$\Psi_{\mathrm{P}}^{jm\pi} = \sum_{\gamma K} \sum_{i} \underbrace{f_{\gamma K i}^{jm\pi}(R_i)}_{\text{Coeff.}} \underbrace{\Phi_{\gamma K}^{jm\pi}(R_i)}_{\text{Translational invariant Projected Slater determinants}},$$



Projectile

$\boldsymbol{R_x}, \boldsymbol{R_y} \rightarrow$ Generator coordinates: Parameters related with the center of the clusters.
$A_i \rightarrow$ Mass of cluster *i*.

with

$$\boldsymbol{X} = \sqrt{\frac{A_2 A_3}{A_2 + A_3}} \boldsymbol{R_x}; \quad \boldsymbol{Y} = \sqrt{\frac{A_1(A_2 + A_3)}{A_1 + A_2 + A_3}} \boldsymbol{R_y}; \quad R = \sqrt{X^2 + Y^2};$$

$$\gamma = \{l_x, l_y, L, S\}; \quad K = l_x + l_y + 2n$$

The microscopic continuum discretized coupled channel (MCDCC): Three+one

1. *Three-cluster microscopic wave function of the projectile*

Thus, the problem reduces to an eigenvalue problem

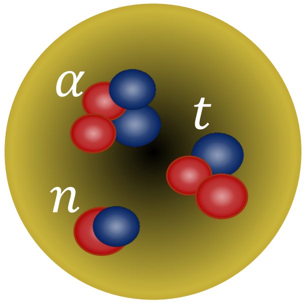
$$\sum_{\gamma K}\sum_{i} f_{\gamma K i}^{jm\pi}\left(\underbrace{\left\langle \Phi_{\gamma K}^{jm\pi}\middle| H \middle| \Phi_{\gamma K}^{jm\pi}\right\rangle} - E^{j\pi}\underbrace{\left\langle \Phi_{\gamma K}^{jm\pi}\middle| \Phi_{\gamma K}^{jm\pi}\right\rangle}\right) = 0$$

Seven dimensional integrals:

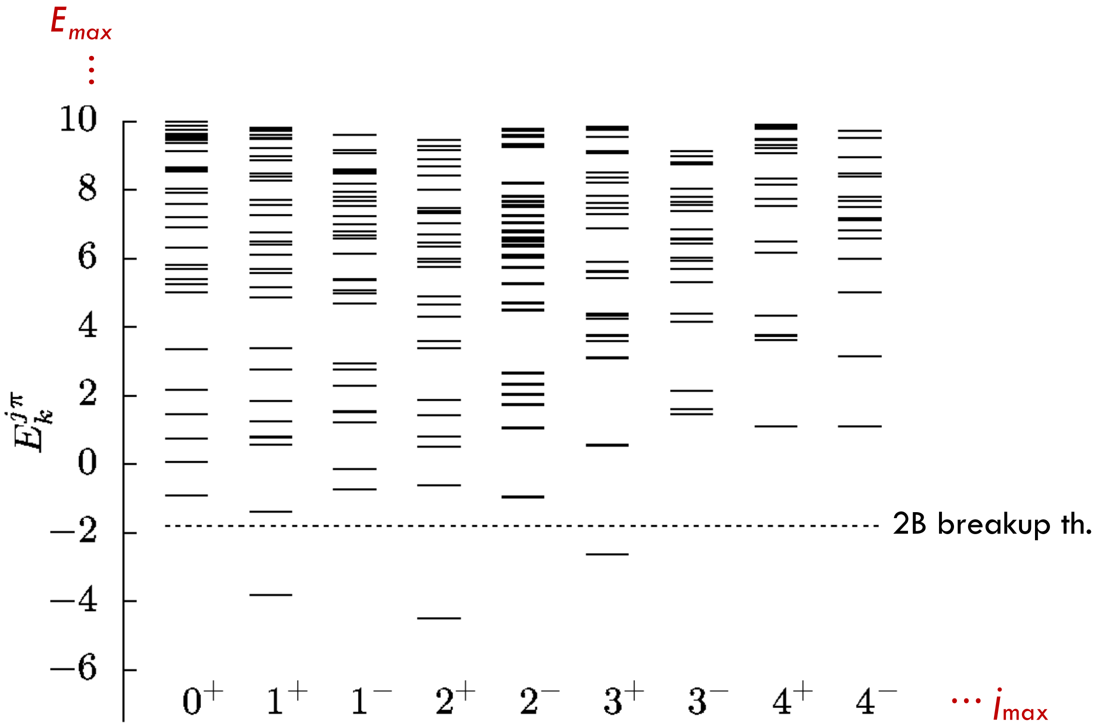
Long numerical calculations

The microscopic continuum discretized coupled channel (MCDCC): Three+one

1. *Three-cluster microscopic wave function of the projectile*



Projectile



^{8}Li=α+t+n spectrum computed in the GCM method

The microscopic continuum discretized coupled channel (MCDCC)

2. Coupling potentials

They are computed from the single folding of the microscopic proton ρ^p and neutron ρ^n densities of the projectile.

$$V_{cc'}(\boldsymbol{R}) = \sum_{i=1}^{A_P} \left\langle \Psi_{\mathrm{P}}^{j'm'\pi'} \middle| V_{iT}(\boldsymbol{r}_i - \boldsymbol{R}) \middle| \Psi_{\mathrm{P}}^{jm\pi} \right\rangle$$

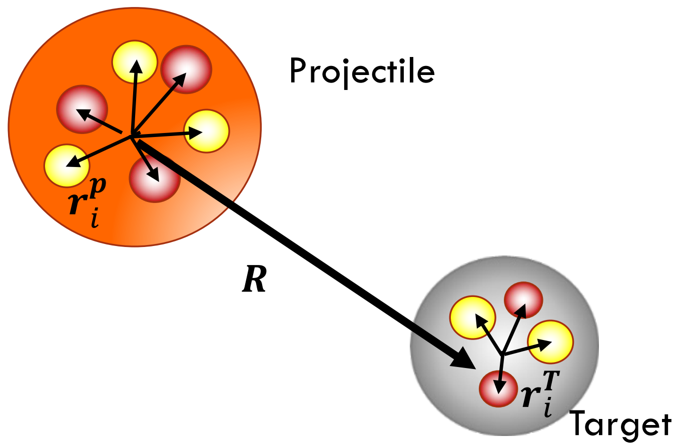
$$V_{cc'}(\boldsymbol{R}) = \int d\boldsymbol{u} [\underbrace{V_{nT}(\boldsymbol{u})}_{\text{Neutron-target potential}} \rho^n(\boldsymbol{R} - \boldsymbol{u}) + \underbrace{V_{pT}(\boldsymbol{u})}_{\text{Proton-target potential}} \rho^p(\boldsymbol{R} - \boldsymbol{u})]$$

The microscopic continuum discretized coupled channel (MCDCC)

3. Solving the coupled differential equations

After projections on angular momentum and the internal state of the projectile we end up with

$$\left[-\frac{h^2}{2\mu_{PT}}\left(\frac{d^2}{dR^2}-\frac{L(L+1)}{R^2}\right)+V_{cc}(R)-(E-\epsilon_c)\right]\chi_c(R)$$
$$=\sum_{c'\neq c} V_{c'c}(R)\chi_{c'}(R)$$



The microscopic continuum discretized coupled channel (MCDCC)

The R-matrix method

Internal region

$$\chi_c(R) = \sum_{i=1}^{N} c_i\, f_i(\rho)$$

Unknown

Nuclear + Coulomb + Centrifugal potentials

a

External region

$$\chi_c(R) = (R \rightarrow \infty)$$

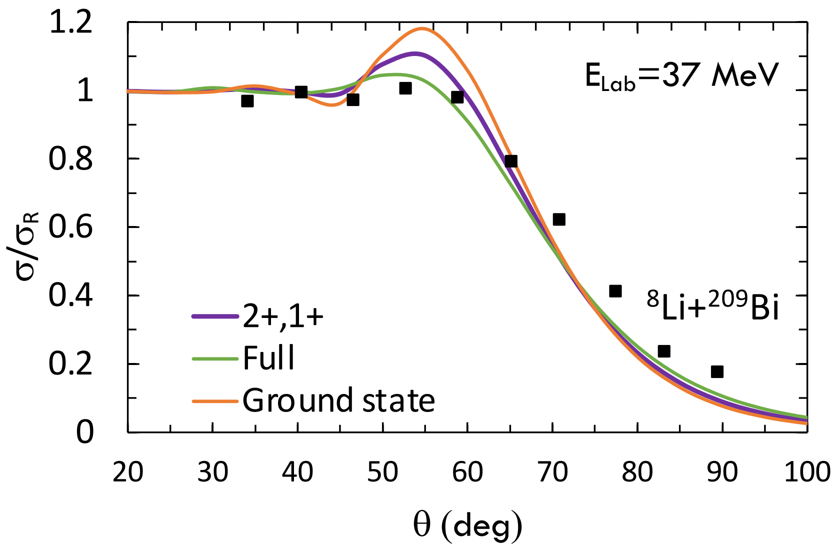
known

Coulomb + Centrifugal potentials

R

From the matching: R-matrix ⟶ Collision matrix ⟶ Cross sections.

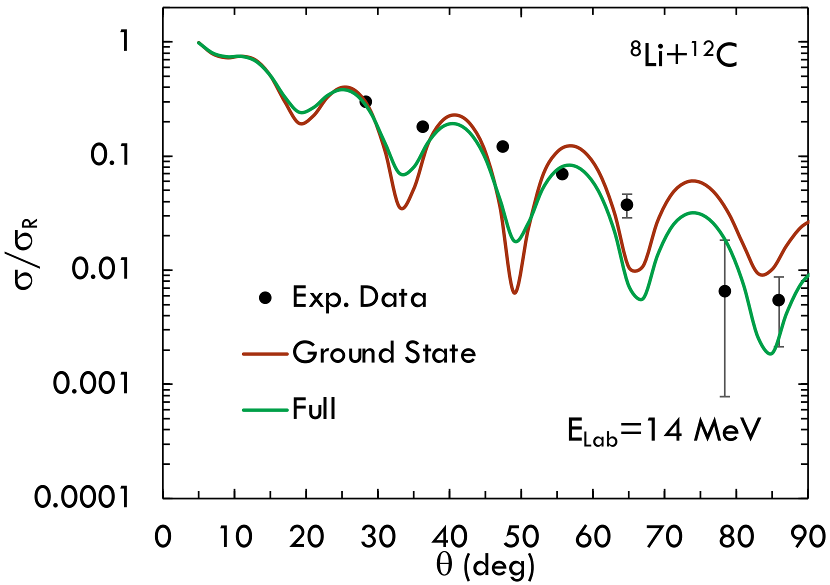
MCDCC elastic scattering of (^{8}Li=α+t+n)+target



Convergence with the maximum angular momentum of the pseudostates (E_{max}=5 MeV). Experimental data taken from *K. J. Kook et al., Phys. Rev. C 97, 021601 (2018).*

- ✓ Fair agreement with experimental data: No fitting parameters
- ✓ Influence of the continuum channels

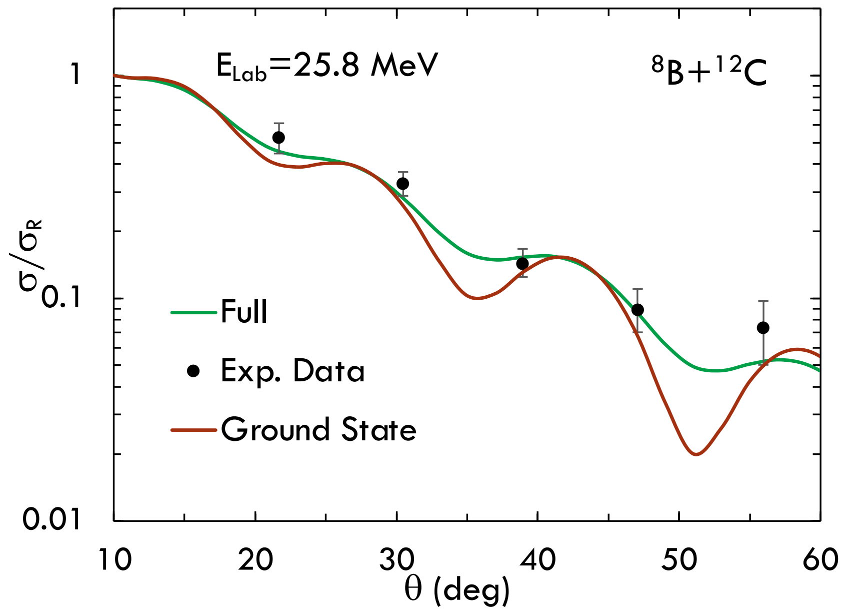
MCDCC elastic scattering of (^{8}Li=α+t+*n*)+target

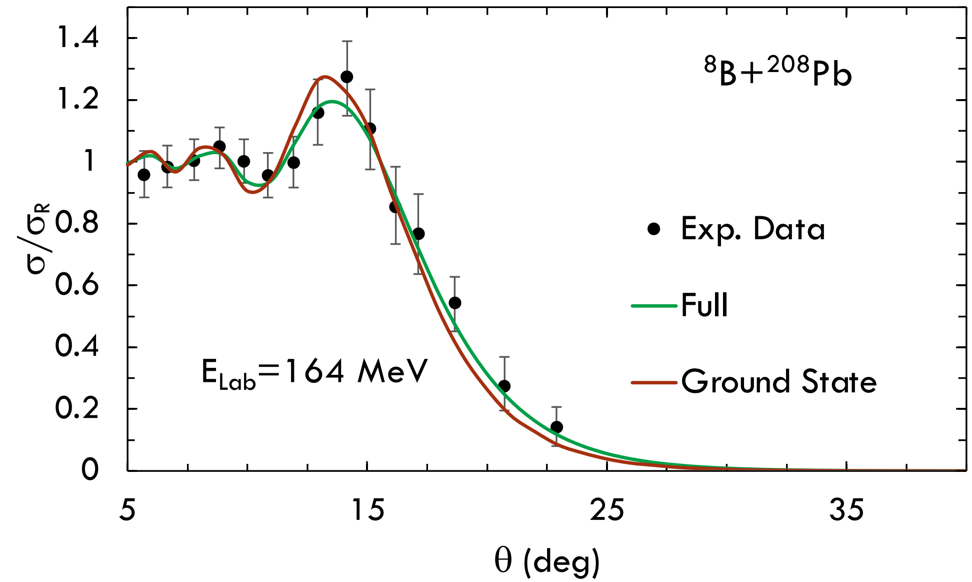


Single channel Vs full calculation (E_{max}=10 MeV; $j^{\pi}_{max} = 4^{\pm}$). Experimental data taken from *A. Barioni et al., Phys. Rev. C 80, 034617 (2009).*

- ✓ Fair agreement with experimental data: No fitting parameters
- ✓ Influence of the continuum channels

MCDCC elastic scattering of ($^8B=\alpha+^3He+p$)+target





Single channel Vs full calculation. Experimental data taken from *A. Barioni et al., Phys. Rev. C 84, 014603 (2011)* *(left)*, *Y. Y. Yang et al., Phys. Rev. C 87, 044613 (2013) (right)*.

Conclusions

- We introduce a precise microscopic wave function of the projectile (microscopic three-cluster model) to study the elastic scattering of ^{8}B and ^{8}Li.
- We observe an influence of the continuum states of the projectile on the elastic scattering.
- We get a fair agreement with the experimental data without any adjustable parameter.
- We can predict cross sections to be further measured.

Thank you!