Three-body reactions of astrophysical interest

Stefan Typel

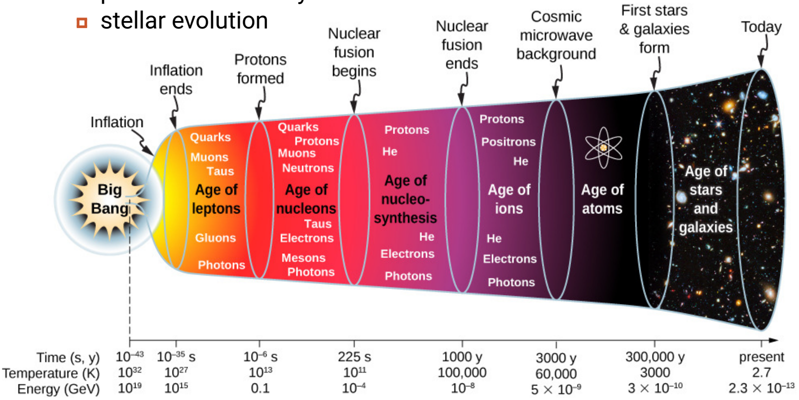
**Exploring low-energy nuclear properties:
latest advances on reaction mechanisms
with light nuclei**

Workshop dedicated to Pierre Descouvemont

Université Libre de Bruxelles
Campus de la Plaine
Bruxelles, Belgium
June 1 - 2, 2023

History of the Cosmos

- origin of chemical elements
  - primordial nucleosynthesis
  - stellar evolution

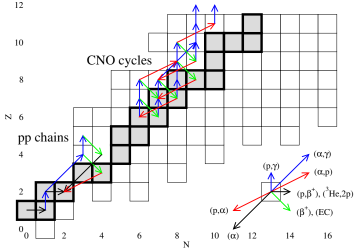


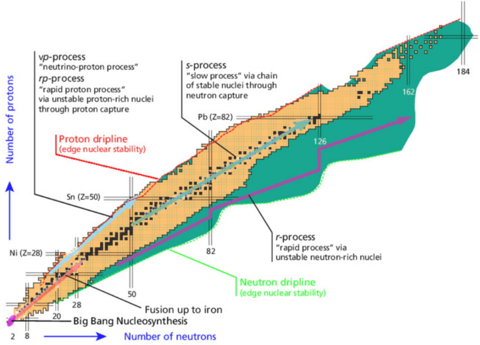
source: phys.libretexts.org

Stellar Nucleosynthesis

TECHNISCHE UNIVERSITÄT DARMSTADT

- different types of reactions (pure nuclear, electromagnetic, weak)
- different processes





Nuclear Reactions of Astrophysical Interest

- reactions of the form $a + b \to c + d + \dots$
  with two nuclei in initial state and two or more nuclei in final state
  - suppression of cross section at low energies due to Coulomb repulsion
  - reaction rate $R(T) \propto n_a n_b$ with densities of nuclei n_a and n_b
  - three-body aspect: strong clustering in nuclei a or b
    (e.g. in transfer reactions)
- reactions $a + b + c \to d + e + \dots$ with three nuclei in initial state
  - reaction rate $R(T) \propto n_a n_b n_c$
  - stronger Coulomb suppression and temperature dependence
    than two-body reactions
  - high densities and temperatures needed to be efficient
  - relevant cases for nucleosynthesis?

Three-Body Reactions of Astrophysical Interest

TECHNISCHE UNIVERSITÄT DARMSTADT

- charged-particle reactions:
  - most important example: triple-α process

    ^{4}He + ^{4}He + ^{4}He $\rightarrow$ ^{12}C + 7.275 MeV

    strong enhancement of cross section due to
    - formation of ^{8}Be ground state as resonance in $\alpha + \alpha$ system
    - 'Hoyle' resonance in ^{12}C just above 3α threshold
- reactions with neutron in entrance channel:
  - relevant for astrophysics only when unstable neutrons are available (e.g. primordial nucleosynthesis)
  - of possible interest: n + ^{3}He + ^{4}He $\rightarrow$ ^{4}He + ^{4}He

    via $3/2^{-}$ resonance in ^{5}He
    - magnitude and energy dependence of theoretical cross section?
    - experimental study with Trojan-horse method

Transition Rate of Reaction
n + ^{3}He + ^{4}He $\to$ ^{4}He + ^{4}He

TECHNISCHE UNIVERSITÄT DARMSTADT

- general form in relative coordinates

$$dw_{134\to 44} = \frac{2\pi}{\hbar} \frac{1}{(2J_1+1)(2J_3+1)(2J_4+1)} \sum_{M_1 M_3 M_{4i}} \sum_{M_{4f} M'_{4f}} \int \frac{d^3 p_{44}}{(2\pi\hbar)^3} \left|T_{fi}\right|^2 \delta(E_{14}+E_{35}-E_{44}+Q_{134\to 44})$$

with

  - averaging over initial and summation over final spin projections
  - energy conservation with Q value $Q_{134\to 44} = (m_1 + m_3 - m_4)c^2$
  - energies $E_{ij} = p_{ij}^2/(2\mu_{ij})$, momenta p_{ij}, reduced masses $\mu_{ij} = m_i m_j/(m_i + m_j)$
  - T-matrix elements T_{fi}

Transition Rate of Reaction
n + ^{3}He + ^{4}He $\to$ ^{4}He + ^{4}He

TECHNISCHE UNIVERSITÄT DARMSTADT

- integration over E_{44} in final state $\Rightarrow$

$$\frac{dw_{134\to 44}}{d\Omega_{44}}(\vec{p}_{14}, \vec{p}_{35}) = \frac{2\pi}{\hbar} \frac{\mu_{44} p_{44}}{(2\pi\hbar)^3} \frac{1}{(2J_1+1)(2J_3+1)(2J_4+1)} \sum_{M_1 M_3 M_{4i}} \sum_{M_{4f} M'_{4f}} |T_{fi}|^2$$

with unit $[dw_{134\to 44}/d\Omega_{44}] = L^6 T^{-1}$

- 'pseudo' cross section of reaction ^{3}He + ^{5}He $\to$ ^{4}He + ^{4}He

$$\sigma_{35\to 44}(E_{35}) = \frac{\mu_{35}}{p_{35}} \int \frac{d^3 p_{14}}{(2\pi\hbar)^3} \frac{dw_{134\to 44}}{d\Omega_{44}}(\vec{p}_{14}, \vec{p}_{35})$$

with fixed direction of $\vec{p}_{35}$ and unit $[\sigma_{35\to 44}] = L^2$

Astrophysical Reaction Rate
n + ^{3}He + ^{4}He $\to$ ^{4}He + ^{4}He

TECHNISCHE UNIVERSITÄT DARMSTADT

- total transition rate

$$w_{134\to44}(\vec{p}_{14},\vec{p}_{35}) = \int d\Omega_{44}\,\frac{dw_{134\to44}}{d\Omega_{44}}(\vec{p}_{14},\vec{p}_{35}) \qquad [w_{134\to44}] = L^6T^{-1}$$

- astrophysical reaction rate

$$R_{134\to44}(T) = \frac{n_1n_3n_4}{1+\delta_{13}+\delta_{14}+\delta_{34}+2\delta_{13}\delta_{14}}\langle w_{134\to44}\rangle \qquad [R_{134\to44}] = L^{-3}T^{-1}$$

with Maxwellian-averaged transition rate

$$\langle w_{134\to44}\rangle = \int \frac{d^3p_{35}}{(2\pi\mu_{35}kT)^{3/2}}\int\frac{d^3p_{14}}{(2\pi\mu_{14}kT)^{3/2}} \exp\left(-\frac{p_{35}^2}{2\mu_{35}kT}-\frac{p_{14}^2}{2\mu_{14}kT}\right) w_{134\to44}(\vec{p}_{14},\vec{p}_{35})$$

T-Matrix Element
n + ^{3}He + ^{4}He $\to$ ^{4}He + ^{4}He

TECHNISCHE UNIVERSITÄT DARMSTADT

- post-form distorted-wave Born approximation

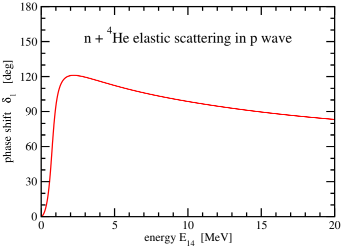
$$T_{fi} = \langle \Phi_4 \Phi_{4'} \chi^{(-)}_{44'}(\vec{p}_{44}) | W | \Phi_3 \Phi^{(+)}_5(\vec{p}_{14}) \chi^{(+)}_{35}(\vec{p}_{35}) \rangle$$

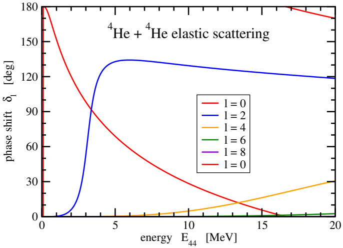
with
  - intrinsic cluster wave functions Φ_i (Gaussians, adjusted to charge radii)
  - ^{5}He resonance wave function $\Phi^{(+)}_5(\vec{p}_{14}) = \Phi_{4'} \psi^{(+)}_{14'}(\vec{p}_{14})$
  - distorted waves $\chi^{(+)}_{35}(\vec{p}_{35})$, $\chi^{(-)}_{44'}(\vec{p}_{44})$
  - potentials (Gaussians with parameters depth and radius)
    - V_{14} and V_{44} adjusted to resonance properties for $l_{14} = 1$ and $l_{44} = 0, 2$
    - V_{35} from scaling (same volume integral as V_{44})
    - (optical) potentials $U_{ij} = V_{ij}$
    - transition potential $W = V_{44} - U_{44} \approx V_{14}$
  - partial-wave expansions with $l_{44} = 0, 2, 4, 6, 8$

Phase Shifts and Resonance Properties

TECHNISCHE UNIVERSITÄT DARMSTADT

- phase shifts





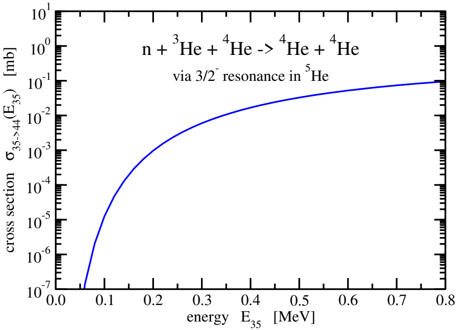
- resonance properties

system	resonance J^{π}	energy E_{th}	width Γ_{th}	energy E_{exp}	width Γ_{exp}
n + ^{4}He	$3/2^-$	0.735 MeV	0.648 MeV	0.735 MeV	0.648 MeV
^{4}He + ^{4}He	0^+	91.84 keV	4.74 eV	91.84 eV	5.57 eV
	2^+	3.122 MeV	1.048 MeV	3.122 MeV	1.513 MeV

Reaction Cross Section

- n + ^{3}He + ^{4}He $\rightarrow$ ^{4}He + ^{4}He via $\frac{3}{2}^-$ resonance in ^{5}He
  - strong suppression at low energies
  - experimental measurement ?
  - direct experiment ?
    - no ^{5}He target
    - ^{5}He beam ?
  - indirect experiment: Trojan-Horse Method (THM)
    - ^{9}Be nucleus as Trojan horse with strong ^{5}He + ^{4}He cluster structure

$\Rightarrow$ study ^{9}Be(^{3}He,$\alpha\alpha$)^{4}He transfer reaction at quasifree scattering conditions



Indirect Methods for Nuclear Astrophysics – General Characteristics

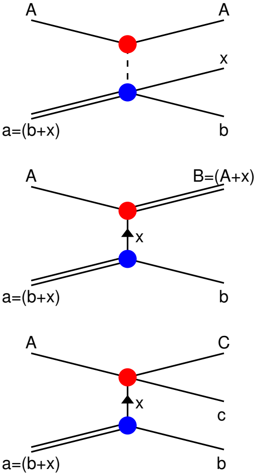
TECHNISCHE UNIVERSITÄT DARMSTADT

- two-body reaction at low energies is replaced by **three-body reaction** at 'high' energies
- relation of cross sections is found with help of direct reaction theory
- theoretical approximations essential
- treatment as transfer reactions: transfer of virtual particle
  - rearrangement reaction $\Rightarrow$ nucleus *x*
  - radiative capture reaction $\Rightarrow$ photon γ
- study of peripheral reactions
  - asymptotics of wave functions relevant
  - selection of suitable kinematic conditions important

Indirect Methods for Nuclear Astrophysics – Examples

TECHNISCHE UNIVERSITÄT DARMSTADT

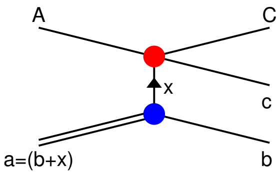
- radiative capture reactions $b(x,\gamma)a$
  - **Coulomb dissociation method:**
    transfer of virtual photon
    $\Rightarrow$ absolute S factor as function of energy
  - **ANC method:**
    transfer of virtual nucleus to bound state
    $\Rightarrow$ absolute S factor at zero energy
- rearrangement reactions $A(x,c)C$
  - **Trojan-horse method:**
    transfer of virtual nucleus to scattering state
    $\Rightarrow$ energy dependence of S factor



Trojan-Horse Method – Introduction

TECHNISCHE UNIVERSITÄT DARMSTADT

- method introduced by Gerhard Baur
  (Physics Letters B 178 (1986) 135)
- basic idea
  - study breakup reaction $A + a \to C + c + b$ to extract cross section of astrophysical charged-particle reaction $A + x \to C + c$ with Trojan horse $a = b + x$ and spectator b
  - establish relation of cross sections with help of direct reaction theory



- specific features
  - small relative energies in A + x system accessible
  - surface dominated reaction
    $\Rightarrow$ reduction of suppression by Coulomb barrier
  - 'high' relative energy in A + a system
    $\Rightarrow$ no electron screening

Trojan-Horse Method – Theory I

- transfer reaction to continuum state
  - general cross section (without spins)

$$d\sigma = \frac{2\pi}{\hbar} \frac{\mu_{Aa}}{p_{Aa}} \int \frac{d^3 p_{Bb}}{(2\pi\hbar)^3} \frac{d^3 p_{Cc}}{(2\pi\hbar)^3} \left| T_{fi} \right|^2 \delta(E_{Aa} - E_{Bb} - E_{Cc} + Q)$$

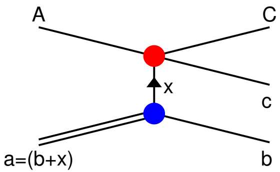
with $B = C + c$ and $Q = (m_a + m_A - m_b - m_c - m_C)c^2$

  - T-matrix element in post formulation

$$T_{fi} = \langle \Psi^{(-)}_{Cc} \phi_b \exp\left(i\vec{p}_{Bb} \cdot \vec{r}_{Bb}/\hbar\right) \left| V_{Bb} \right| \Psi^{(+)}_{Aa} \rangle$$

with full scattering wave function $\Psi^{(+)}_{Aa}$

  - scattering wave function $\Psi^{(-)}_{Cc}$ contains information on reaction $A + x \to C + c$



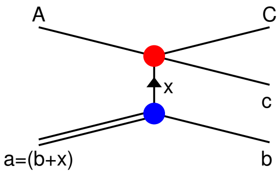
Trojan-Horse Method – Theory II

- transformation of T-matrix element (Gell-Mann–Goldberger relation)
- distorted-wave Born approximation (DWBA)
- approximation of potential

$$\Rightarrow \quad T_{fi} = \langle \Psi^{(-)}_{Cc} \phi_b \chi^{(-)}_{Bb} | V_{xb} | \phi_A \phi_a \chi^{(+)}_{Aa} \rangle$$

- introduce momentum distribution W_a of Trojan-horse nucleus a

$$V_{xb}\phi_a = \int \frac{d^3q}{(2\pi)^3} W_a(\vec{q}) \, \exp(i\vec{q} \cdot \vec{r}_{xb}) \, \phi_x \, \phi_b$$



- surface approximation
  $\Rightarrow$ use asymptotic form of $\Psi^{(-)}_{Cc}$

Trojan-Horse Method – Theory III

TECHNISCHE UNIVERSITÄT DARMSTADT

- simplification in plane-wave approximation
  - analytic integration over $\vec{r}_{Bb}$ and $\vec{q}$ $\Rightarrow$ factorization of T-matrix element

$$T_{fi} = W_a(\vec{Q}_{Bb}) \, \langle \Psi_{Cc}^{(-)} | \exp(i\vec{Q}_{Aa} \cdot \vec{r_{Ax}}/\hbar) \phi_A \phi_x \rangle$$

with $\vec{Q}_{Bb} = \vec{p}_{Bb} - \frac{m_b}{m_x+m_b}\vec{p}_{Aa}$ and $\vec{Q}_{Aa} = \vec{p}_{Aa} - \frac{m_A}{m_A+m_x}\vec{p}_{Bb}$

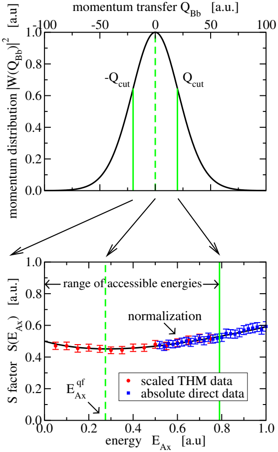
  - factorization of cross section ($\sim$ impulse approximation)

$$\frac{d^3\sigma}{dE_{Cc}d\Omega_{Cc}d\Omega_{Bb}} = K \left| W_a(\vec{Q}_{Bb}) \right|^2 \frac{d\sigma^{HOES}}{d\Omega_{Cc}}(A + x \to C + c)$$

    - kinematic factor K
    - momentum distribution $|W_a(\vec{Q}_{Bb})|^2$ of Trojan-horse nucleus a
    - half-off-energy-shell cross section $\frac{d\sigma^{HOES}}{d\Omega_{Cc}}(A + x \to C + c)$ of reaction $A + x \to C + c$ $(Q_{Aa}^2/(2\mu_{Ax}) + m_A + m_x \neq p_{Cc}^2/(2\mu_{Cc}) + m_C + m_c)$

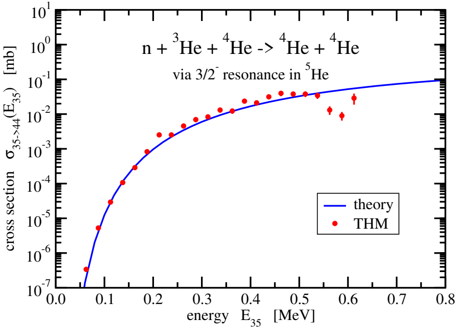
Trojan-Horse Method – Application

- dominance of quasifree scattering
  $\Rightarrow$ small momentum transfer to spectator b
  $\Rightarrow \vec{Q}_{Bb} \approx 0 \Rightarrow$ specific kinematic conditions
- well-clustered Trojan-horse nucleus a
  $\Rightarrow$ peak of $|W_a(\vec{Q}_{Bb})|^2$ at $\vec{Q}_{Bb} \approx 0$
  for s-wave ground states (^{2}H, ^{6}Li, ...)
- cutoff in Q_{Bb} determines range of accessible energies E_{Ax}
- transformation of $\frac{d\sigma^{HOES}}{d\Omega_{Cc}}$ to on-shell cross section $\frac{d\sigma}{d\Omega_{Cc}}$ with penetrability factor
- normalization of cross section to direct data at high energies



Trojan-Horse Method – Application to ^{5}He + ^{3}He $\to$ ^{4}He + ^{4}He Reaction

- Experimental study of Trojan-horse reaction ^{9}Be(^{3}He,$\alpha\alpha$)^{4}He
  - 4 MeV ^{3}He beam @ RBI, Zagreb; Trojan-horse nucleus ^{9}Be = ^{5}He + ^{4}He
  - momentum transfer $Q_{Bb} \leq$ 40 MeV/c $\Rightarrow$ selection of kinematics
  - test of quasi-free reaction mechanism $\Rightarrow$ Treiman-Yang criterion
  - extraction of cross section for ^{5}He + ^{3}He $\to$ ^{4}He + ^{4}He
  - normalization to theory data
- details:
  C. Spitaleri et al.
  Eur. Phys. J. A 57 (2021) 20



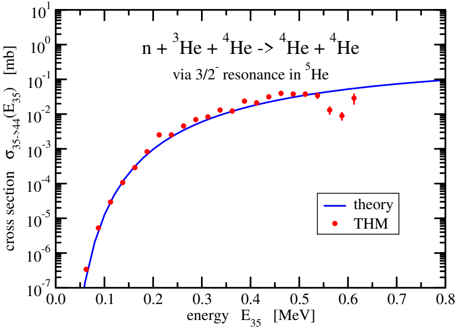
Trojan-Horse Method – Application to ^{5}He + ^{3}He $\rightarrow$ ^{4}He + ^{4}He Reaction

TECHNISCHE UNIVERSITÄT DARMSTADT

- Experimental study of Trojan-horse reaction ^{9}Be(^{3}He,$\alpha\alpha$)^{4}He
  - 4 MeV ^{3}He beam @ RBI, Zagreb; Trojan-horse nucleus ^{9}Be = ^{5}He + ^{4}He
  - momentum transfer $Q_{Bb} \leq 40$ MeV/c $\Rightarrow$ selection of kinematics
  - test of quasi-free reaction mechanism $\Rightarrow$ Treiman-Yang criterion
  - extraction of cross section for ^{5}He + ^{3}He $\rightarrow$ ^{4}He + ^{4}He
  - normalization to theory data

$\Rightarrow$ energy dependence of theory and experiment consistent

$\Rightarrow$ validity test of THM with unstable ^{5}He nucleus



Conclusions

TECHNISCHE UNIVERSITÄT DARMSTADT

- three-body aspects important in several reactions of astrophysical interest
  - three particles in initial/final states
  - strong clustering in participating nuclei
- direct reactions with three particles in initial state, e.g.,
  - triple-α process $\Rightarrow$ nucleosynthesis of ^{12}C
  - n + ^{3}He + ^{4}He $\to$ ^{4}He + ^{4}He reaction via $\frac{3}{2}^{-}$ resonance in ^{5}He
- indirect methods with three particles in final state, e.g.,
  - Trojan-horse method
    - transfer reaction to continuum at quasifree scattering conditions
    - only simplified theoretical approach so far
      $\Rightarrow$ improved treatment necessary to check approximations
- future: application of Faddeev approach?

Thank You for Your Attention!

Presentation supported by the
Deutsche Forschungsgemeinschaft
(DFG, German Research Foundation)
Project-ID 279384907 — SFB 1245.