

# Hard diffractive scattering from soft color screening effects

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# Basics of Quantum ChromoDynamics – reminder

## Group theory:

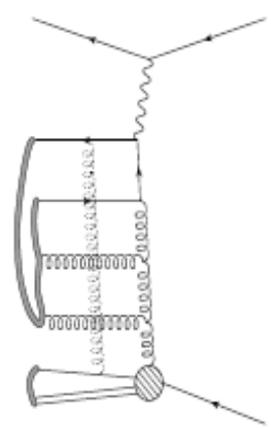
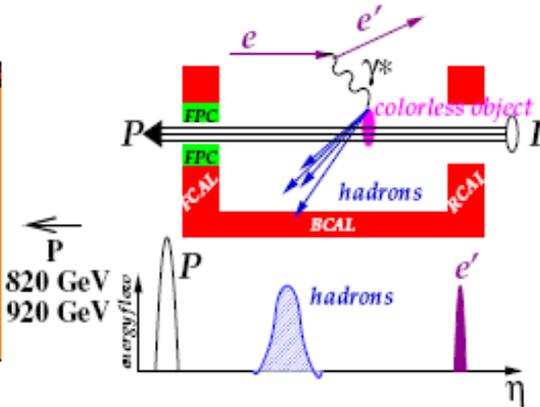
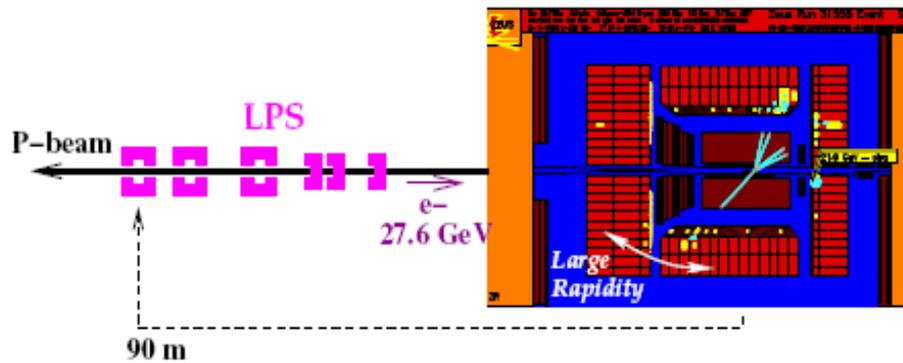
- Colour charge is the source of the field
- Gauge symmetry  $\rightarrow$  group  $SU(3)_C$   
representations **3, 8, 6, 10, 15** . . .
- Fundamental representation:  
triplet **3**  $\rightarrow$  *red, green, blue* quarks  
antitriplet  $\bar{\mathbf{3}}$   $\rightarrow$  anti-quarks
- Local gauge symmetry  $\Rightarrow$  interactions  
8 generators of  $SU(3)_C$   
 $\hookrightarrow$  octet (**8**) representation  
8 gluons carrying colour–anticolour

## Dynamics:

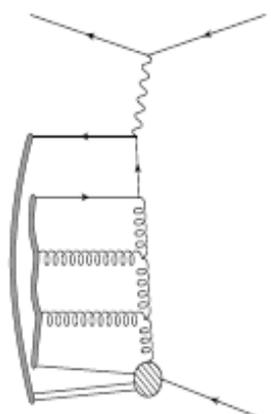
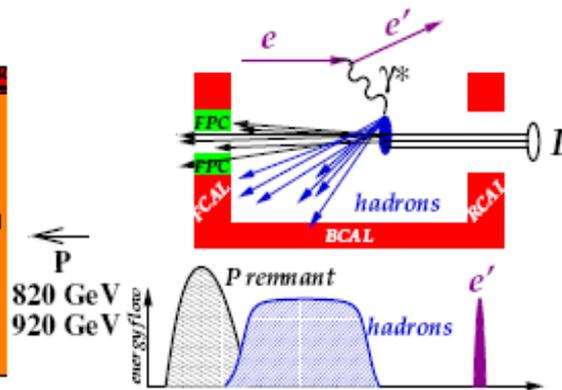
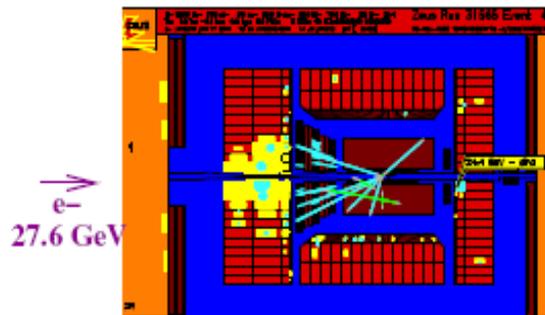
- Coupling  $\alpha_s(Q^2) = \frac{12\pi}{(33-2n_f) \ln(Q^2/\Lambda^2)}$   
depends on momentum transfer  $Q^2$
- QCD  $\Lambda \sim 200$  MeV
- Hard QCD,  $Q^2 \gg \Lambda^2$ : small coupling  
 $\Rightarrow$  perturbation theory  
**Asymptotic Freedom**
- Soft QCD,  $Q^2 \sim \Lambda^2$ : large coupling  
 $\Rightarrow$  lattice theory or **models**  
**Confinement**
- QCD potential  $V(r) = -\frac{\alpha_s}{r} + \kappa r$

# Diffractive rapidity-gap events in $ep$ at HERA

## 1. Diffractive scattering

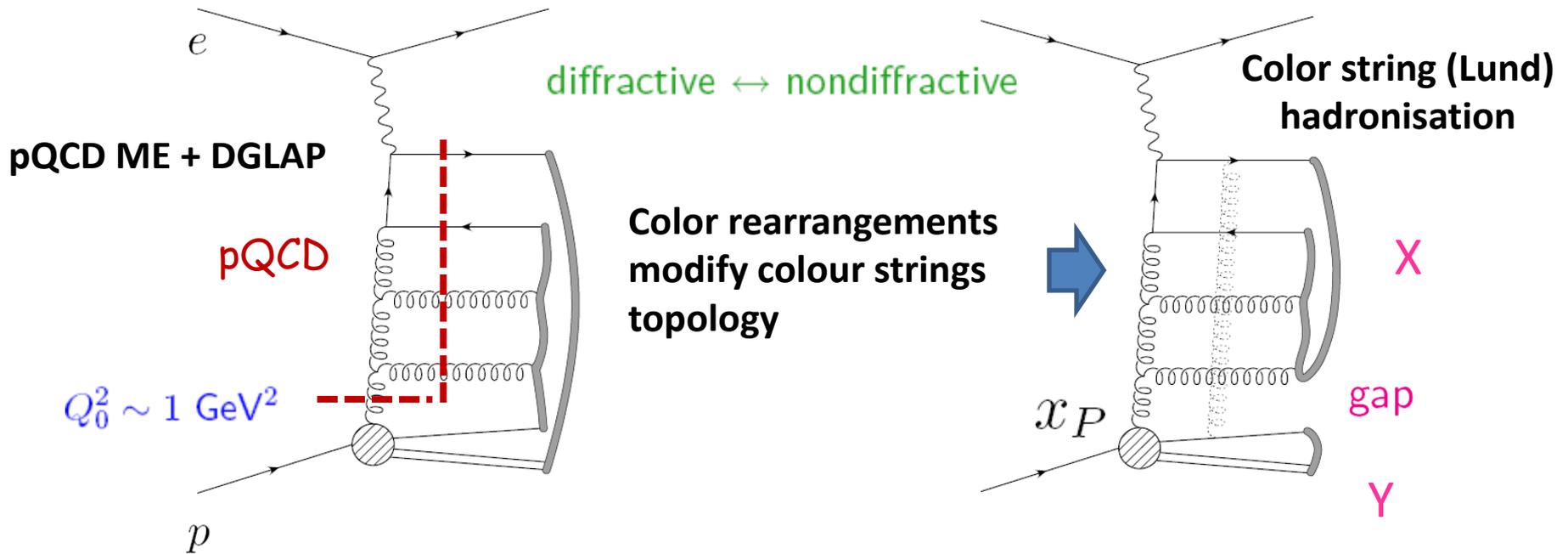


## 2. Non-diffractive scattering



# Diffractive Deep Inelastic Scattering: motivation

- ✓ The success of **Soft Color Interaction (SCI) model** (Edin, Ingelman, Rathsman'97)



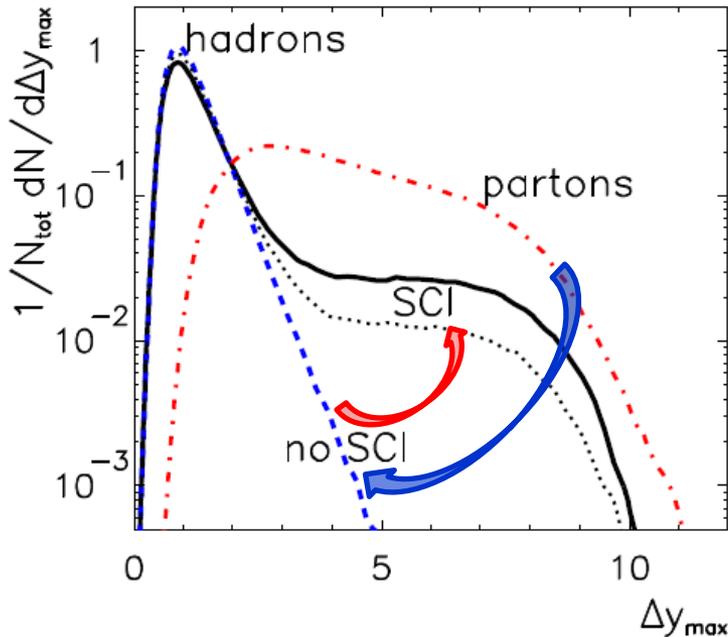
- **Soft interactions among the final state partons and proton remnants** ( $\Rightarrow$  proton color field) at **small momentum transfers**  $< 1 \text{ GeV}$
- **Hard pQCD part (small distances) is not affected** by soft interactions (large distances)
- **Single parameter** - probability for soft colour-anticolour (gluon) exchange
- **Single model describing all final states: both diffractive and nondiffractive**

# Soft Colour Interaction model (SCI)

Add-on to Lund Monte Carlo's LEPTO ( $ep$ ) and PYTHIA ( $p\bar{p}$ )

ME + DGLAP PS  $> Q_0^2$  → SCI model → String hadronisation  $\sim \Lambda$   
colour ordered parton state → rearranged colour order → modified final state

Size  $\Delta y_{max}$  of largest gap in DIS events



SCI  $\Rightarrow$  plateau in  $\Delta y_{max}$   
characteristic for diffraction

Small parameter sensitivity

—  $P = 0.5$

...  $P = 0.1$

Gap-size is infrared sensitive observable !

Large gaps at parton level  
normally string across  $\rightarrow$  hadrons fill up  
SCI  $\rightarrow$  new string topologies, some with gaps

Gap events not 'special', but  
fluctuation in colour/hadronisation

# Jets, $W$ , $Z$ , $b\bar{b}$ , $J/\psi$ in diffractive gap events at the Tevatron

$$R_{\text{hard}} = \frac{1}{\sigma_{\text{hard}}^{\text{tot}}} \int_{x_{F\text{min}}}^1 dx_F \frac{d\sigma_{\text{hard}}}{dx_F}$$

$R_{\text{hard}}[\%]$		Exp. observed	SCI
dijets	CDF	$0.75 \pm 0.10$	0.7
W	CDF	$1.15 \pm 0.55$	1.2
W	DØ	$1.08^{+0.21}_{-0.19}$	1.2
$b\bar{b}$	CDF	$0.62 \pm 0.25$	0.7
Z	DØ	$1.44^{+0.62}_{-0.54}$	1.0
$J/\psi$	CDF	$1.45 \pm 0.25$	1.4

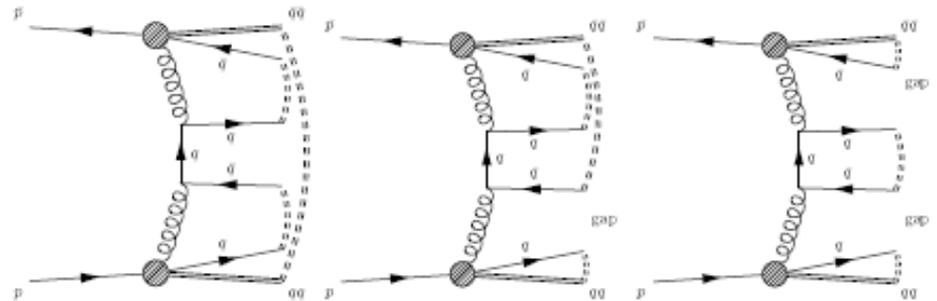
↑

predictions

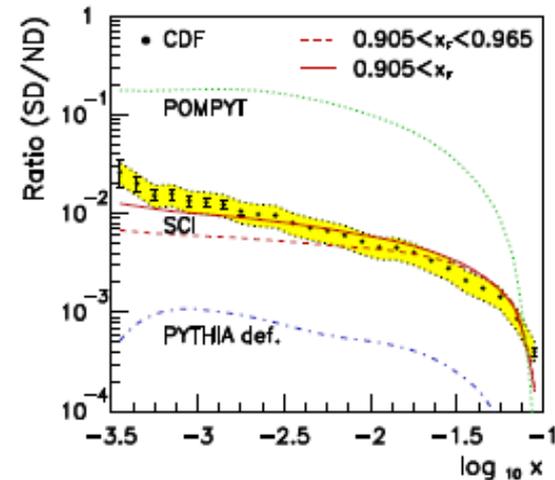
SCI  $\rightarrow$  gap &  $c\bar{c}$  colour octet  $\rightarrow$  singlet  $\rightarrow J/\psi$

SCI model OK, also for two-gap (DPE) events

Pomeron model too high, PYTHIA too low



$R_{\text{dijets}}$  vs  $x$  of parton in  $\bar{p}$

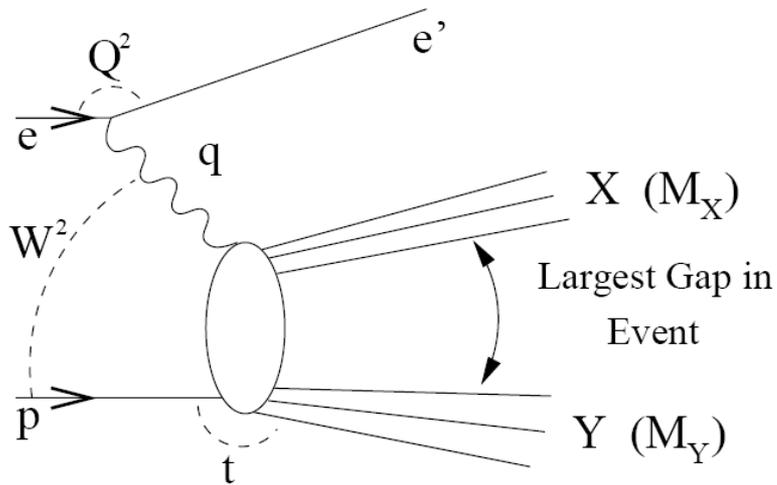


SCI model phenomenologically successful — Why ?

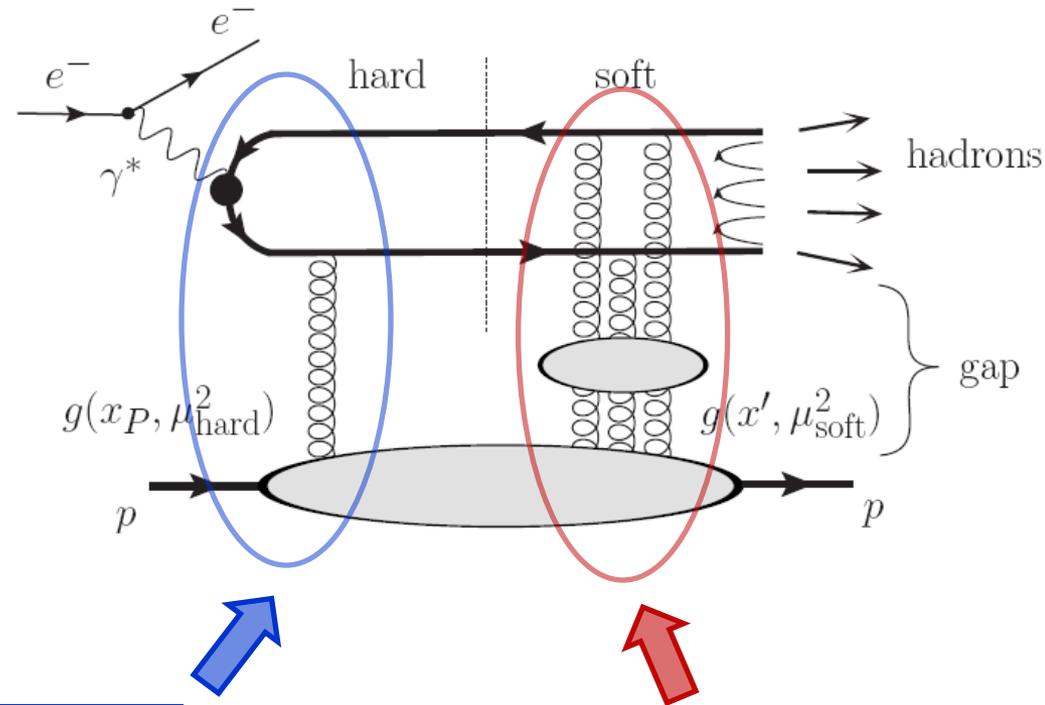
Captures most essential QCD dynamics  $\Rightarrow$  theory emerging . . .

# QCD rescattering theory

## Diffractive DIS at HERA



## QCD rescattering model



Hard part  
conventional  
(small distance)

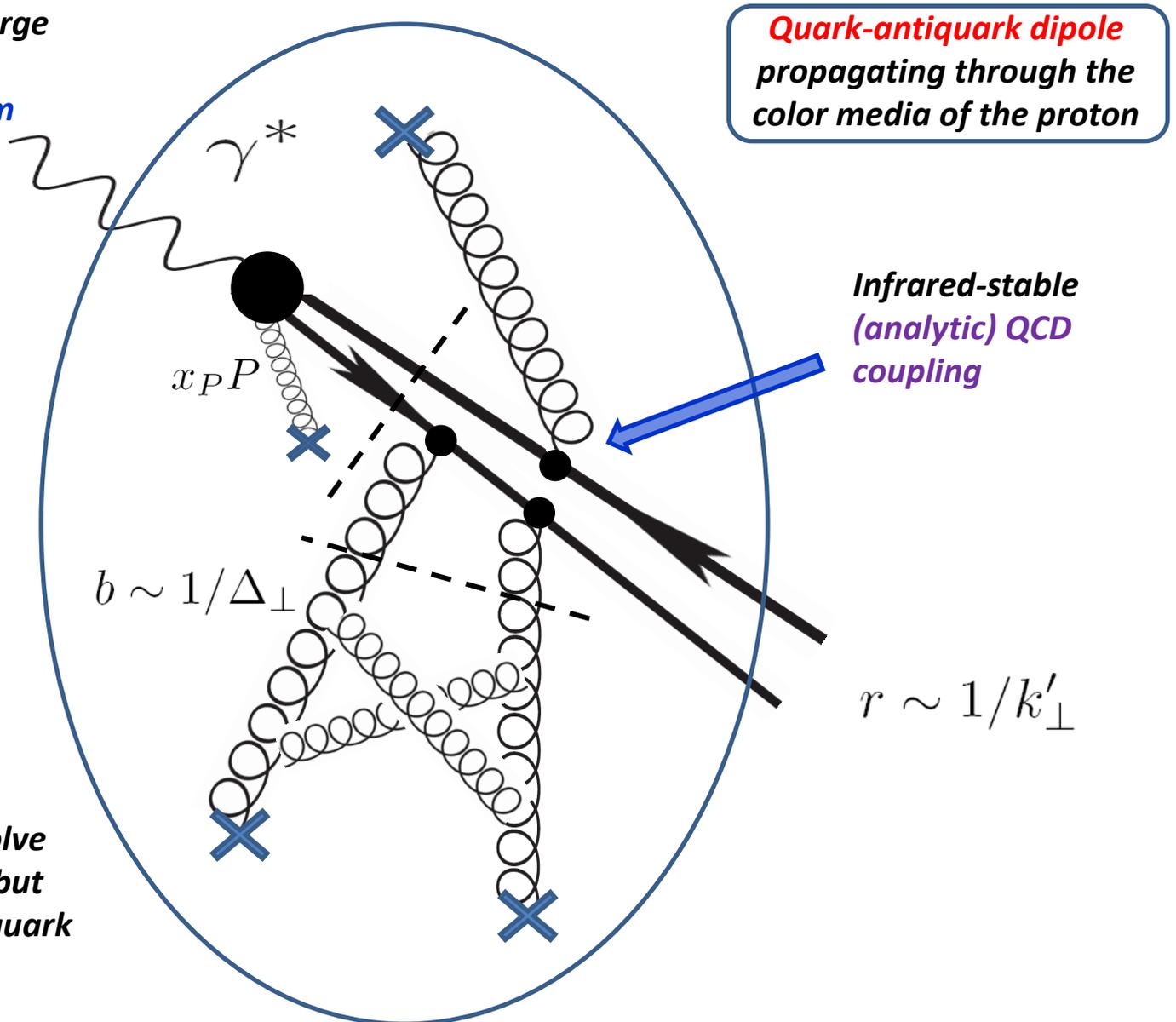
Soft part:  
color-screening (octet)  
multigluon exchange  
(large distance)

# Hard diffractive scattering in the dipole picture

Compensation of the large photon virtuality by **longitudinal momentum transfer** in the single hard interaction

**t- and s-channel factorisation**

Soft gluons cannot resolve quarks **dynamically** → but they always couple to quark current!



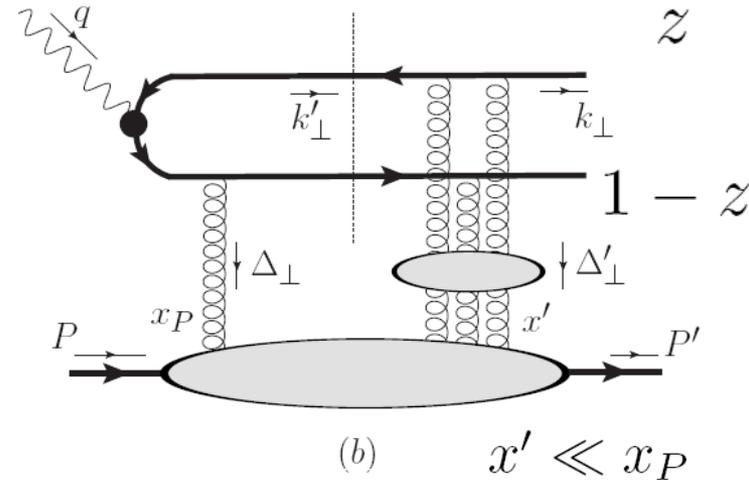
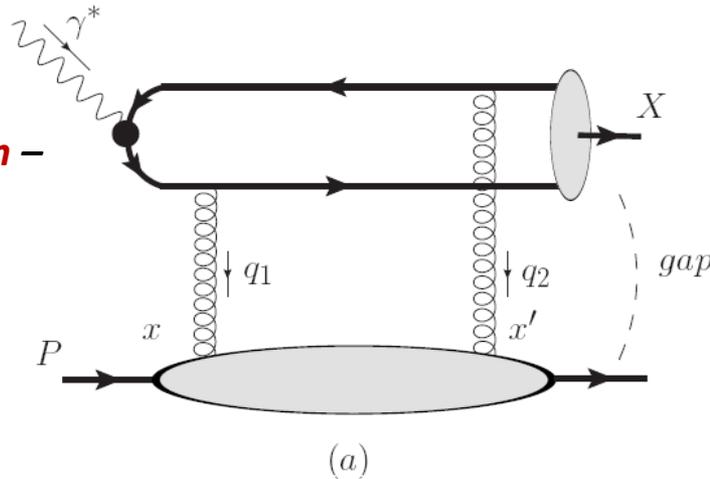
# Diffractive DIS at HERA: kinematics

**Basic variables:**  $x \equiv \frac{Q^2}{2Pq} = \frac{Q^2}{Q^2 + W^2}, \quad \beta = \frac{Q^2}{Q^2 + M_X^2}, \quad x_P = \frac{x}{\beta}, \quad t = (P' - P)^2$

$$Q^2 = -q^2$$

**Leading contribution – by quark dipole**

$$\beta = x/x_P \rightarrow 1$$



**Invariant mass of X system and c.m.s energy**

$$M_X^2 = \frac{1 - \beta}{\beta} Q^2, \quad W^2 \simeq \frac{Q^2}{x_P \beta},$$

$$\varepsilon^2 = z(1 - z)Q^2 + m_q^2, \quad k_\perp^2 = z(1 - z)M_X^2 - m_q^2$$

**The hard QCD factorization scale = quark virtuality!**

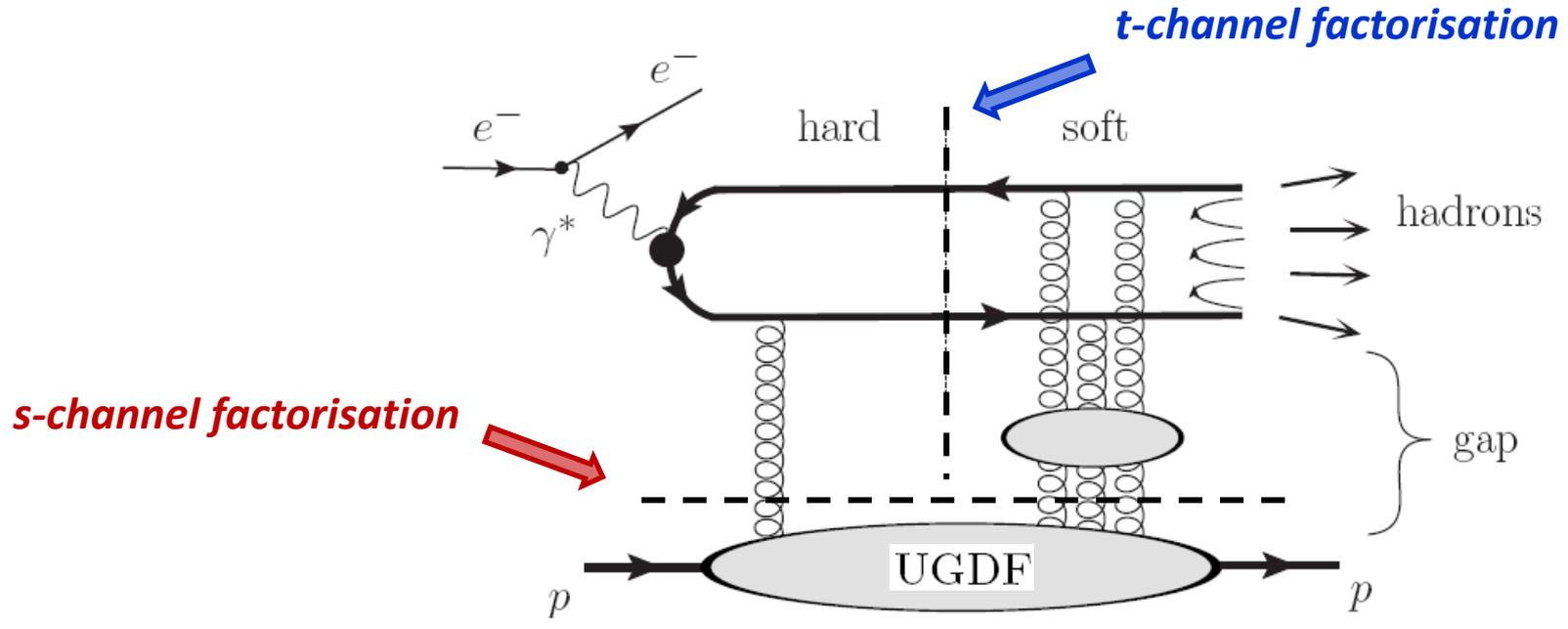
**Working domain of interest:**

$$x_P \ll 1, \quad M_X \ll W$$

$$|t| \ll Q^2, M_X^2$$

$$\mu_F^2 = k_\perp^2 + \varepsilon^2 = z(1 - z) \frac{Q^2}{\beta}$$

# Hard-soft factorization scheme



✓ The total **amplitude**

loop integration + cutting rules

$$M(\delta) \sim \int \frac{d^2 \Delta_{\perp}}{(2\pi)^2} \cdot M^{hard}(\Delta_{\perp}) \cdot M^{soft}(\delta - \Delta_{\perp}) \mathcal{F}_g^{off}$$

$$\delta \equiv \sqrt{-t} = |\Delta_{\perp} + \Delta'_{\perp}|$$

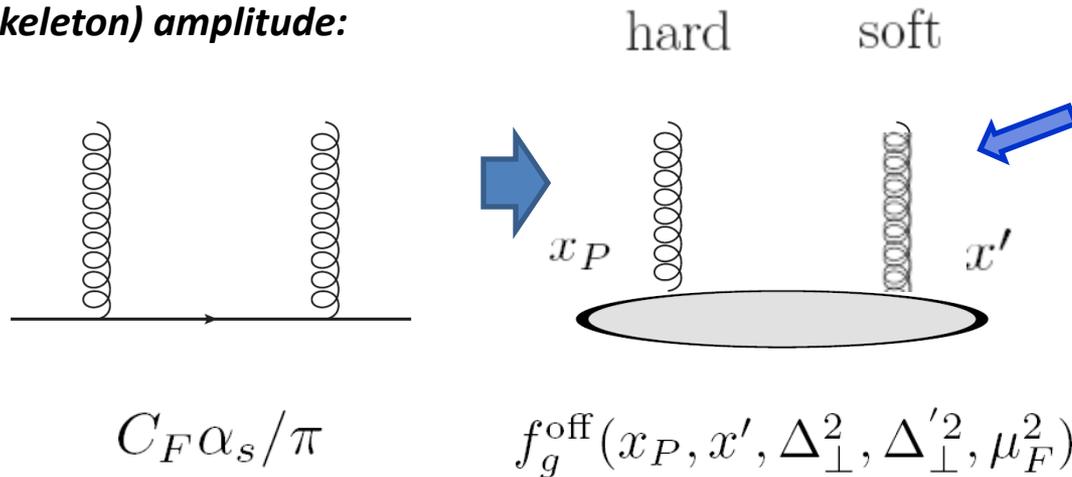
**factorisation**  
of the **b-dependence**

to the **impact parameter representation** →

$$M(\delta, \mathbf{k}_{\perp}) \sim \int d^2 r d^2 b e^{-i\mathbf{k}_{\perp} \mathbf{r}} e^{-i\mathbf{k} \mathbf{b}} \hat{M}^{hard}(\mathbf{b}, \mathbf{r}) \hat{M}^{soft}(\mathbf{b}, \mathbf{r}) \mathcal{V}(\mathbf{b}, \mathbf{r})$$

# Generalized (skewed) unintegrated gluon density

Partonic (skeleton) amplitude:



**“fat” soft gluon – effectively carries color octet charge in the limit of small  $r \sim 1/k'_{\perp}$**

**Notion of “hardness” is different w.r.t. the standard one:**

- \* **“hard” gluon** in our case – the gluon which takes the largest **longitudinal** momentum, compensating the quark virtuality
- \* **“hard” scale** is related **with longitudinal momentum transfer** given by  $x_P$  (similarity with Durham model for CEP of Higgs)

**Off-diagonal UGDF currently unknown → different models are applied!**

# UGDF model and impact parameter representation

We model the *skewedness effect* using *positivity constraints* (Pire, Soffer, Teryaev'99) as

$$\mathcal{F}_g^{\text{off}} \simeq \sqrt{\mathcal{F}_g(x_P, \Delta_\perp^2, \mu_F^2) \mathcal{F}_g(x', \Delta_\perp'^2, \mu_{\text{soft}}^2)},$$

*Infrared behavior:*

$$\frac{f_g(x, \Delta_\perp^2)}{\Delta_\perp^2} \equiv \mathcal{F}(x, \Delta_\perp^2) \rightarrow \text{const}, \quad \Delta_\perp^2 \rightarrow 0$$

*Impact parameter representation:*

$$\mathcal{V}(\mathbf{b}, \mathbf{r}) = \frac{1}{\alpha_s(\mu_{\text{soft}}^2)} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} \sqrt{x_P} \mathcal{F}_g^{\text{off}} \times \{e^{-i\mathbf{r}\Delta_\perp} - e^{i\mathbf{r}\Delta_\perp}\} e^{i\mathbf{b}\Delta_\perp}.$$

*Gaussian Ansatz:*

$$\sqrt{x_P} \mathcal{F}_g^{\text{off}} \simeq \sqrt{x_P g(x_P, \mu_F^2) \boxed{x' g(x', \mu_{\text{soft}}^2)} f_G(\Delta_\perp^2)},$$

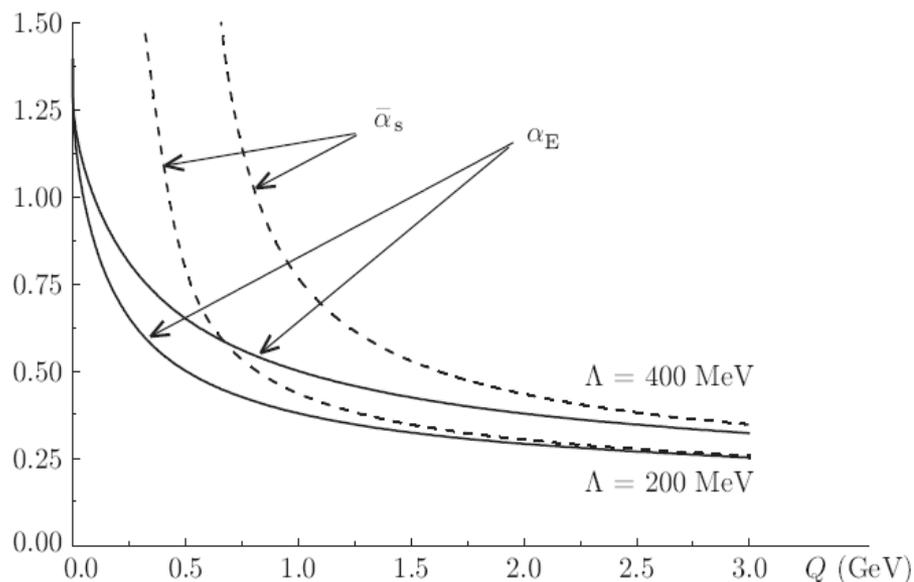
$$f_G(\Delta_\perp^2) = 1/(2\pi\rho_0^2) \exp(-\Delta_\perp^2/2\rho_0^2),$$

*Soft hadronic scale – transverse proton radius*  $r_p \sim 1/\rho_0$ .

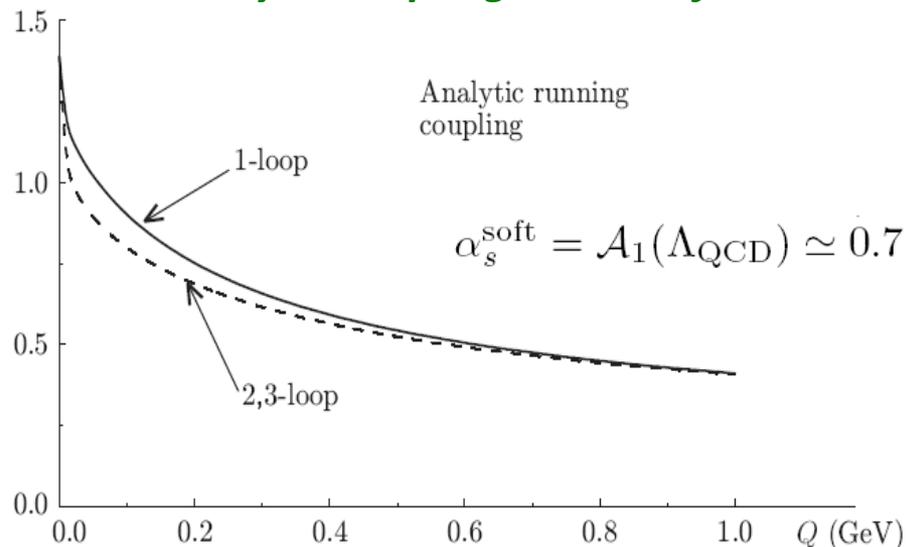
*Diffractive slope (ZEUS)*

$$\sim \exp(B_D t) \quad B_D = 1/\rho_0^2 \simeq 6.9 \pm 0.2 \text{ GeV}^2 \quad \rho_0 \simeq 380 \text{ MeV}$$

# QCD coupling at low scales



## Analytic coupling at the soft scale



## One-loop analytic coupling

$$\alpha_E(Q^2) = \frac{1}{\pi} \int_0^\infty d\sigma \frac{\rho(\sigma)}{\sigma + Q^2},$$

$$\rho_k(\sigma) = \text{Im } \bar{\alpha}_s^k(-\sigma - i\epsilon)$$

$$\alpha_E^{(1)}(Q^2) = \frac{1}{\beta_0} \left[ \frac{1}{\ln(Q^2/\Lambda^2)} + \frac{\Lambda^2}{\Lambda^2 - Q^2} \right].$$

**It can be important to parameterize PDFs using ghost-free analytic QCD approach...**

# The hard scattering amplitude

✓ **Hard part**

Factorization condition in the impact parameter space

$$\Delta_{\perp} \sim \Delta'_{\perp} \ll k'_{\perp} \sim k_{\perp} \longrightarrow r \ll b$$

Diagram illustrating the hard scattering process. A wavy line with momentum  $q$  enters from the top left and hits a black semi-circular target. A horizontal arrow labeled  $k'_{\perp}$  points to the right from the center of the target. Below the target, a vertical chain of circles represents a propagator with momentum  $\Delta_{\perp}$  pointing downwards. At the bottom, a horizontal arrow labeled  $P$  enters from the left and hits a grey semi-circular target. A vertical arrow labeled  $x_P$  points upwards from the center of the grey target to the propagator. A dashed line on the right groups the diagram and the integral equation.

$$M_{L,T}^{hard}(\Delta_{\perp}, k'_{\perp}) = \int d^2 r d^2 \mathbf{b} \hat{M}_{L,T}^{hard}(\mathbf{b}, \mathbf{r}) e^{-i\mathbf{r}\mathbf{k}'_{\perp}} e^{-i\mathbf{b}\Delta_{\perp}}$$

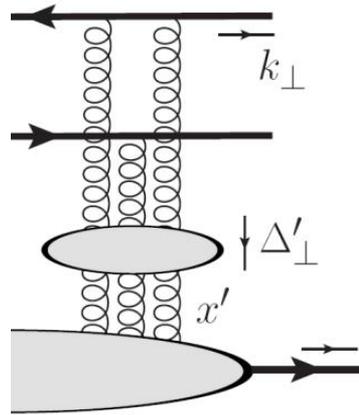
$r \sim 1/k'_{\perp}$   
 $b \sim 1/\Delta_{\perp}$

$$\hat{M}_L^{hard} = i\mathcal{C} \alpha_s(\mu_F^2) \sqrt{\beta} W^3 z^{3/2} (1-z)^{3/2} K_0(\varepsilon r)$$

$$\hat{M}_{T,\pm}^{hard} = i\mathcal{C} \alpha_s(\mu_F^2) \sqrt{\frac{2\beta}{1-\beta}} \frac{1}{\sqrt{x_P}} W^2 z^{1/2} (1-z)^{3/2} \varepsilon K_1(\varepsilon r) \frac{r_x \pm ir_y}{r}$$

# Soft gluon scattering and “exponentiation”

- ✓ Soft gluon exchanges generate only **the phase shifts** – to be **resummed to all orders!**



the large  $N_c$  limit – planar diagrams only!

$$M^{soft}(\Delta'_{\perp}, k_{\perp}) =$$

$$e^{-ir\mathbf{k}'_{\perp}} M_1^{soft} = \mathcal{A} e^{-ir\mathbf{k}_{\perp}} \frac{1}{\Delta'^2_{\perp}} \left[ e^{-ir\Delta'_{\perp}} - 1 \right],$$

$$e^{-ir\mathbf{k}'_{\perp}} M_2^{soft} = \frac{\mathcal{A}^2}{2!} e^{-ir\mathbf{k}_{\perp}} \times$$

$$\int \frac{d^2\Delta'_{2\perp}}{(2\pi)^2} \frac{1}{\Delta'^2_{1\perp} \Delta'^2_{2\perp}} \left[ e^{-ir\Delta'_{\perp}} - e^{-ir\Delta'_{2\perp}} - e^{-ir\Delta'_{1\perp}} + 1 \right]$$

**etc ...**

**Fourier transform** →

$$e^{-ir\mathbf{k}'_{\perp}} \hat{M}_1^{soft} = e^{-ir\mathbf{k}_{\perp}} \mathcal{A} \cdot \mathcal{W}(\mathbf{b}, \mathbf{r}),$$

$$e^{-ir\mathbf{k}'_{\perp}} \hat{M}_2^{soft} = e^{-ir\mathbf{k}_{\perp}} \frac{\mathcal{A}^2 \cdot \mathcal{W}(\mathbf{b}, \mathbf{r})^2}{2!}, \quad \dots$$

**Soft gluon rescattering amplitude** →

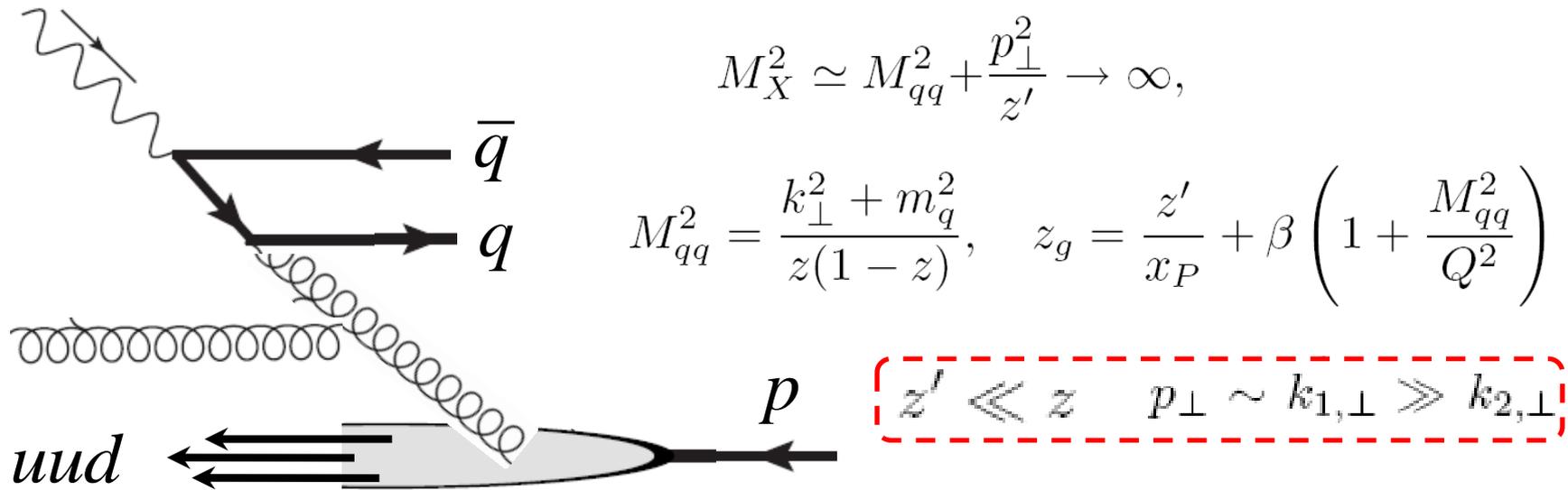
$$e^{-ir\mathbf{k}'_{\perp}} \hat{M}^{soft}(\mathbf{b}, \mathbf{r}) = -e^{-ir\mathbf{k}_{\perp}} (1 - e^{\mathcal{A} \cdot \mathcal{W}(\mathbf{b}, \mathbf{r})})$$

$$\mathcal{A} = ig_s^2 C_F / 2 \quad \mathcal{W}(\mathbf{b}, \mathbf{r}) = \frac{1}{2\pi} \ln \frac{|\mathbf{b} - \mathbf{r}|}{|\mathbf{b}|}$$

**Inspired by Brodsky et al, PRD65, 114025 (2002)**

## Gluonic contribution @ large $M_X$

Gluon radiated from “hard” gluon is far away in  $p$ -space from  $q\bar{q}$   
 → leading contribution to large  $M_X$



→ Altarelli-Parisi splitting  $\otimes$   $q\bar{q}$ -dipole  $\otimes$  multiple gluon exchange

$$x_P F_{q\bar{q}g}^{D(4)} \simeq \frac{1}{N_c^2} \int \frac{dt_g dz_g}{t_g + m_g^2} P_{gg}(z_g) \frac{\alpha_s(t_g)}{2\pi} x_P F_{q\bar{q}}^{D(4)}$$

# Diffractive structure function: results

Data are given in terms of *the reduced cross section*

$$\sqrt{s} = 318 \text{ GeV}$$

$$x_P \sigma_r^{D(3)} = x_P F_{q\bar{q},T}^{D(3)} + \frac{2-2y}{2-2y+y^2} x_P F_{q\bar{q},L}^{D(3)} + x_P F_{q\bar{q}g}^{D(3)} \quad y = Q^2/(sx_B) \leq 1$$

**Quark dipole contribution:**

$$x_P F_L^{D(4)} = \mathcal{S} Q^4 M_X^2 \int_{z_{min}}^{\frac{1}{2}} dz (1-2z) z^2 (1-z)^2 |J_L|^2$$

$$x_P F_T^{D(4)} = 2\mathcal{S} Q^4 \int_{z_{min}}^{\frac{1}{2}} dz (1-2z) \{(1-z)^2 + z^2\} |J_T|^2$$

$$\mathcal{S} = \sum_q e_q^2 / (2\pi^2 N_c^3)$$

**Amplitudes:**

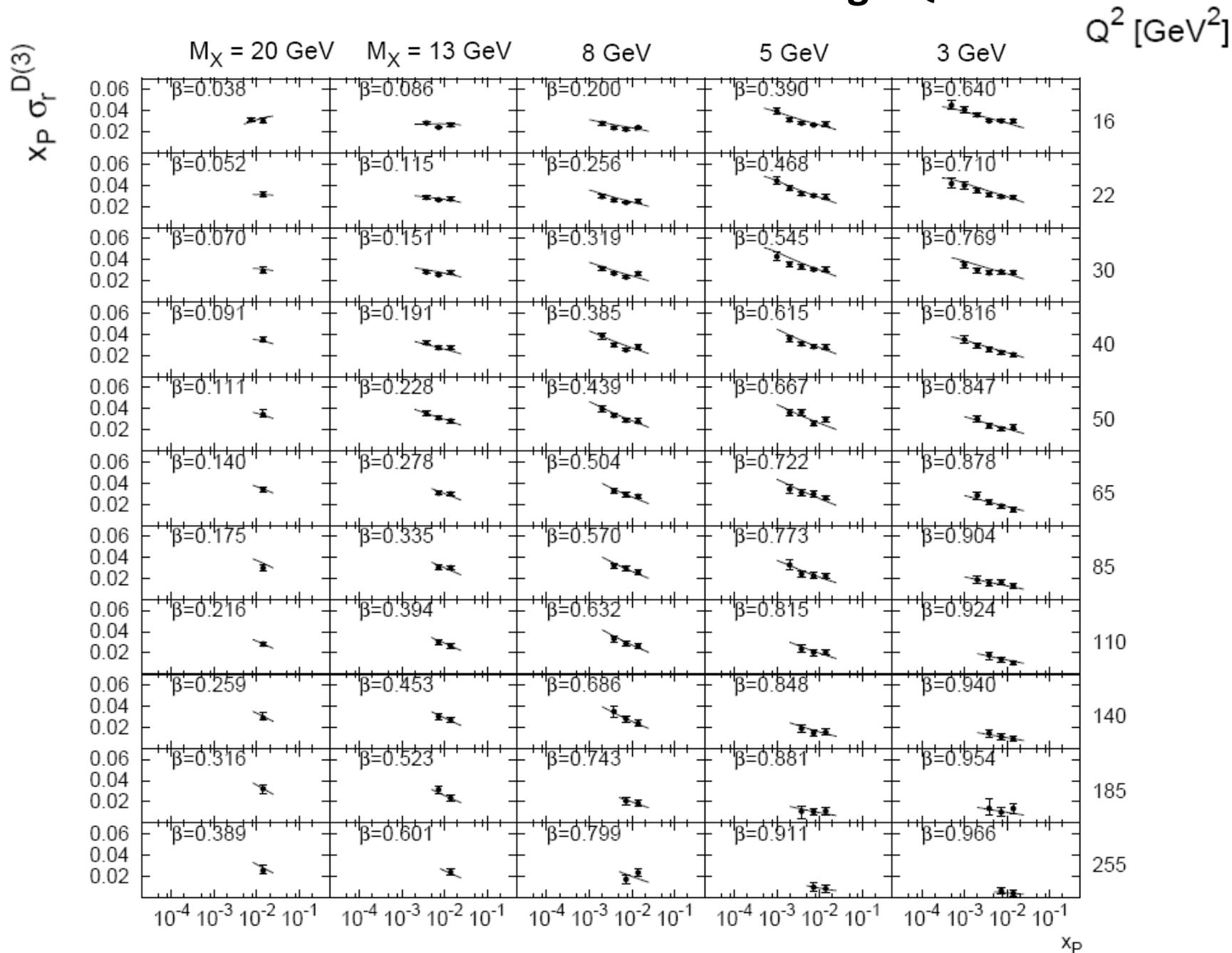
$$J_L = i\alpha_s(\mu_F^2) \int d^2\mathbf{r} d^2\mathbf{b} e^{-i\delta\mathbf{b}} e^{-i\mathbf{r}\mathbf{k}_\perp} K_0(\varepsilon r) \\ \times \mathcal{V}(\mathbf{b}, \mathbf{r}) [1 - e^{\mathcal{A}\mathcal{W}}],$$

$$J_T = i\alpha_s(\mu_F^2) \int d^2\mathbf{r} d^2\mathbf{b} e^{-i\delta\mathbf{b}} e^{-i\mathbf{r}\mathbf{k}_\perp} \varepsilon K_1(\varepsilon r) \\ \times \frac{r_x \pm ir_y}{r} \mathcal{V}(\mathbf{b}, \mathbf{r}) [1 - e^{\mathcal{A}\mathcal{W}}].$$

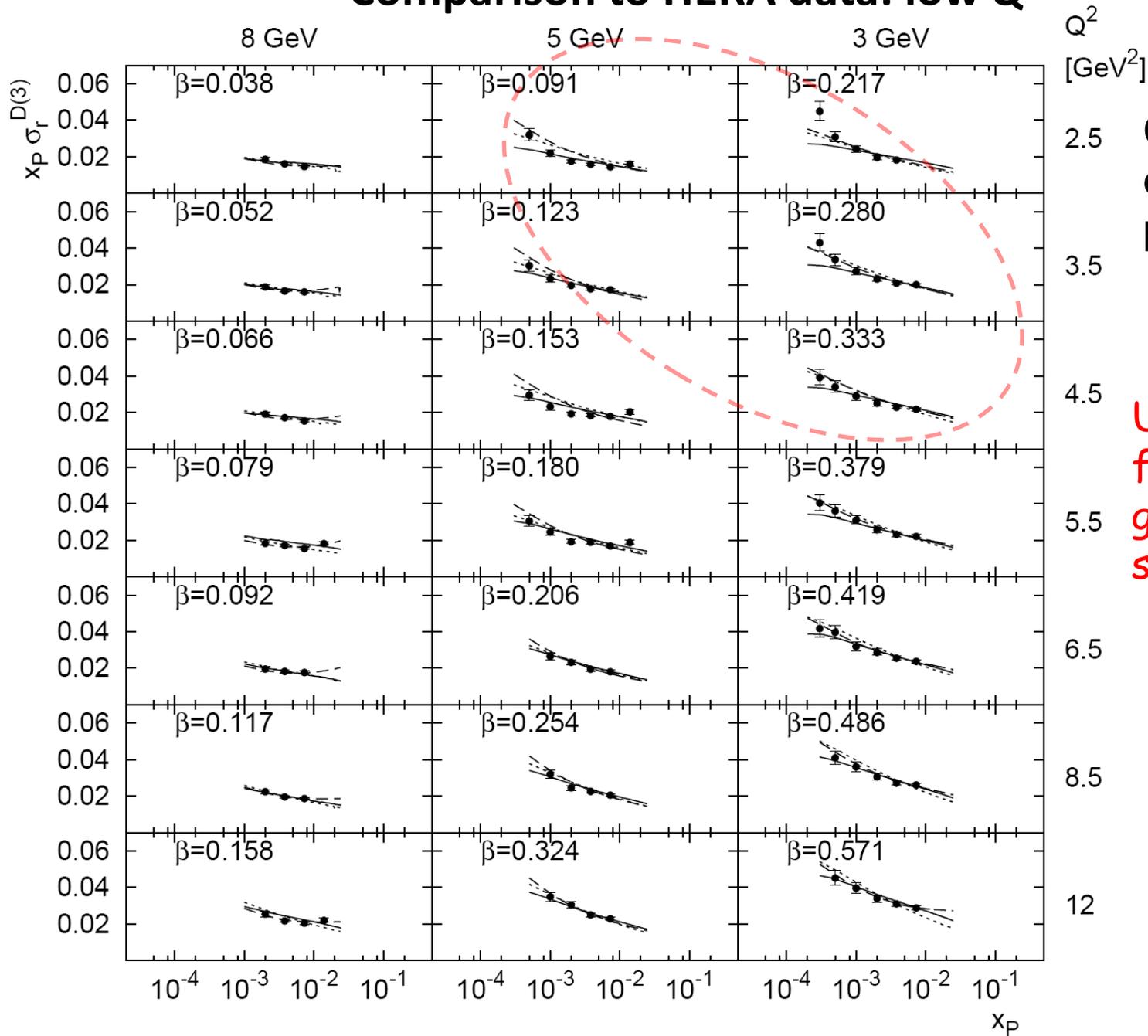
**Gluonic dipole contribution:**

$$x_P F_{q\bar{q}g}^{D(4)} \simeq \frac{1}{N_c^2} \int \frac{dt_g dz_g}{t_g + m_g^2} P_{gg}(z_g) \frac{\alpha_s(t_g)}{2\pi} x_P F_{q\bar{q}}^{D(4)}$$

# Diffractive structure function: large $Q^2$



# Comparison to HERA data: low $Q^2$



Curves for different  $xg(x, \mu)$  parametrizations

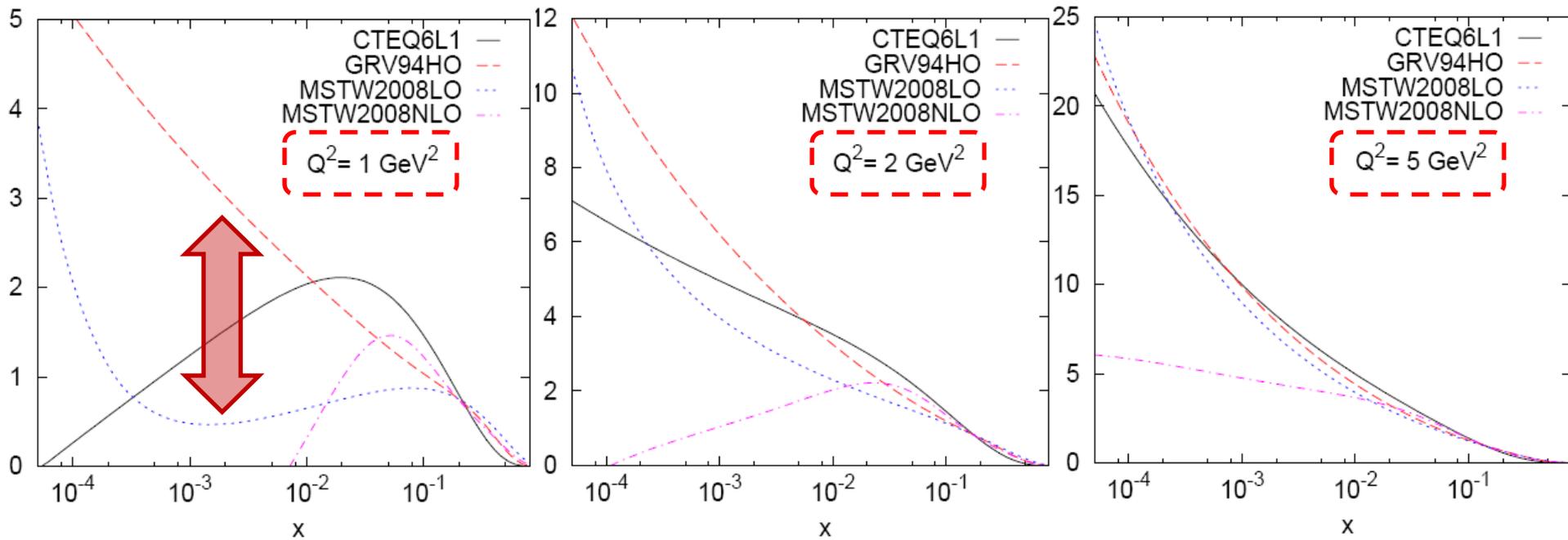


Uncertainty from unknown gluon density at small  $x$  & scale



Possibility to extract  $g(x, \mu_F)$  at  $x \sim 10^{-4}$ ,  $\mu_F < \sim 1$  GeV

# Glueon density parametrizations at low- $x$ and low $Q^2$

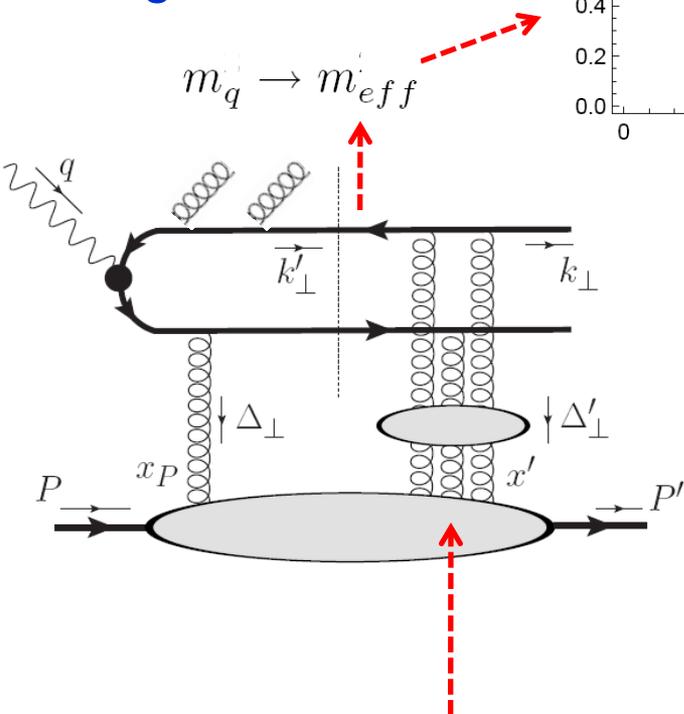
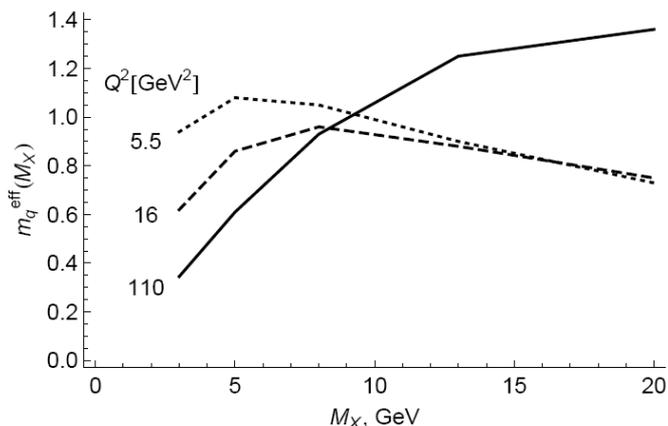
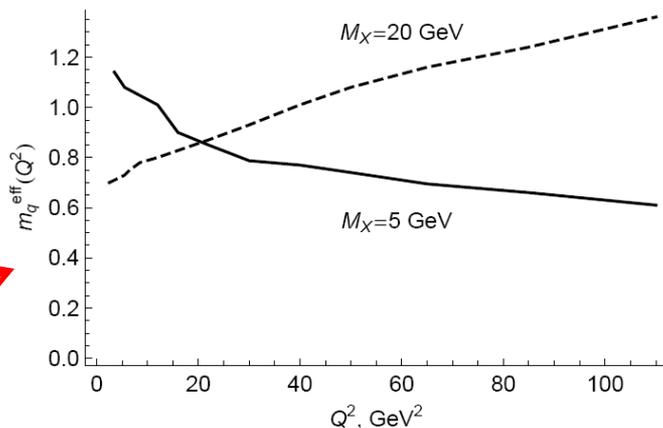


Large differences at  $x < \sim 10^{-2}$  and  $Q^2 < \sim 2 \text{ GeV}^2$  !!

→ Unknown gluon density in this region !!!

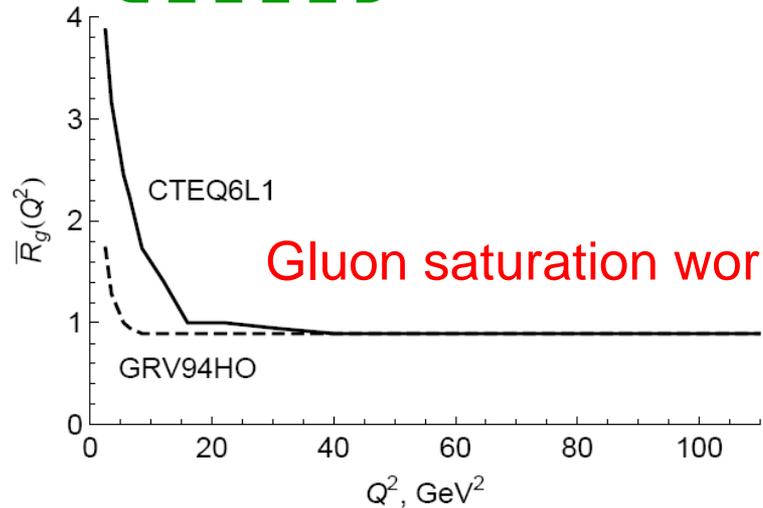
# Model parameters

Effective mass  $m_{eff}$  of quark in dipole from dressing-up with gluon radiation



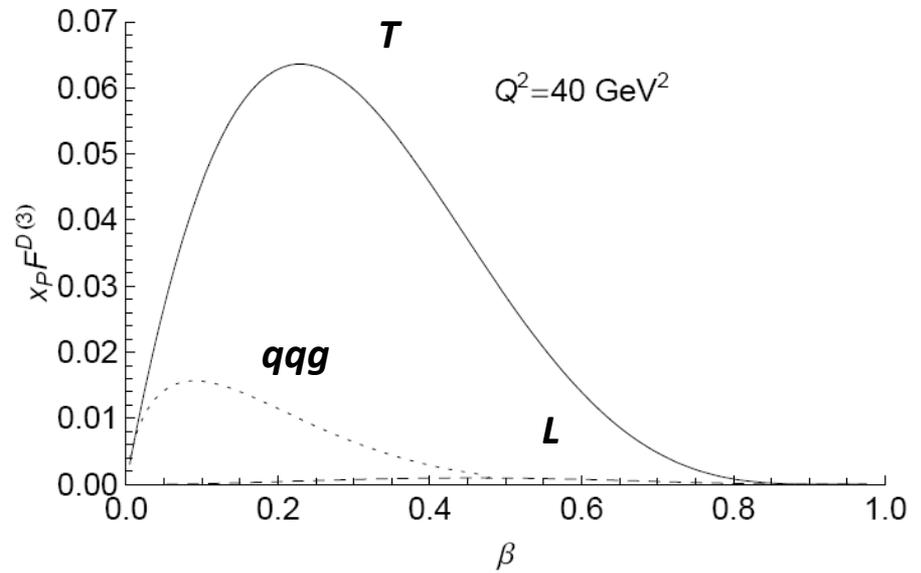
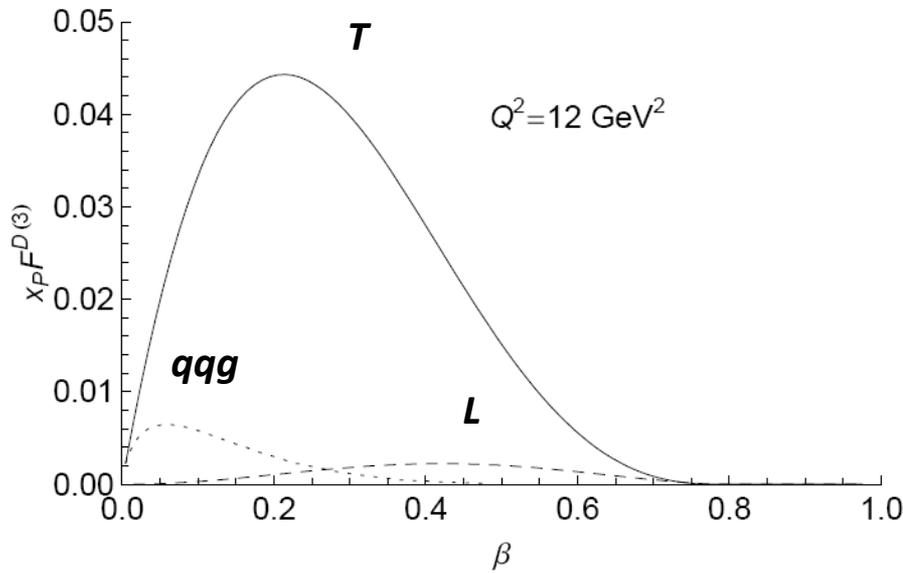
Soft gluon density function  $R_g$  = constant  $\approx 1$ , except at small  $Q^2$

$$\sqrt{x_P} \mathcal{F}_g^{\text{off}} \simeq \left[ \bar{R}_g(x', \mu_{\text{soft}}^2) \right] \sqrt{x_P g(x_P, \mu_F^2) f_G(\Delta_\perp^2)}$$



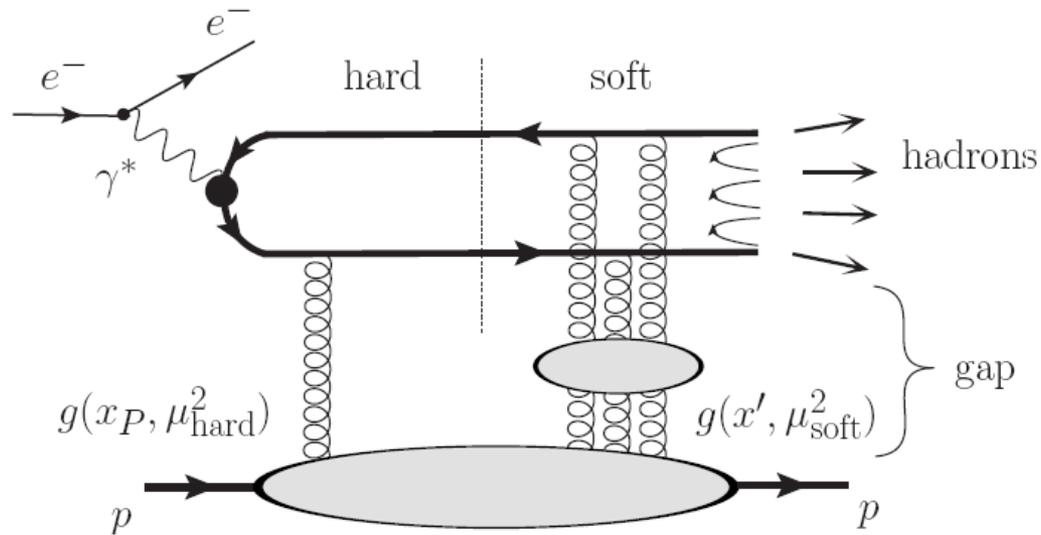
Gluon saturation works!

# Photon polarization contributions and mass spectrum



Gluonic contribution increases at high  $M_x$  and  $Q^2$  !!!

# Proposal for “New” SCI Monte-Carlo technique



## ***Probability for soft color exchanges:***

- 1. ...is actually dependent on dynamics of multiple gluon interactions and kinematics of emerging partons, and can be constrained from the first principles;***
- 2. ...should be written in terms of infrared finite dipole-target or dipole-dipole rescattering;***
- 3. ...can be written in terms of normalized sensitivity of the hard subprocess part to small fluctuations of transverse momenta of dipole constituents due to soft exchanges.***

Work in progress...

# Summary

- ✓ We constructed **the QCD based model** for the soft gluon rescattering.
- ✓ The model works basically well and leads to a **good description of the HERA data** on the diffractive structure function **in almost all bins** in photon virtuality and invariant mass of the final hadronic system.
- ✓ At lower  $x_P$  and  $Q^2$  **the uncertainties in conventional parton densities** become significant, and some improvement of the data description is still needed.