Electroweak interactions - Exercices

In order to fix the notations, here we follow the notations of [1] describing a Dirac spinor as

$$\psi\left(x,t\right)=\sum_{s}\int\frac{d^{3}k}{(2\pi)^{3}\sqrt{2E_{k}}}\left(a_{k,s}u\left(k,s\right)e^{-ikx}+b_{k,s}^{\dagger}v\left(k,s\right)e^{ikx}\right)\,. \tag{1}$$

The sum runs over spin values s and $a_{k,s}^{\dagger}$ $\left(b_{k,s}^{\dagger}\right)$ creates a particle (antiparticle) of momentum $\vec{k}$ and spin s. The spinors u and v obey the Dirac equations

$$\begin{aligned}(\not{k}-m)u\left(k,s\right) &= 0\\ (\not{k}+m)v\left(k,s\right) &= 0\end{aligned} \tag{2}$$

with the sums:

$$\sum_{s}u\left(k,s\right)\bar{u}\left(k,s\right)=\not{k}+m,\quad\sum_{s}v\left(k,s\right)\bar{v}\left(k,s\right)=\not{k}-m \tag{3}$$

For a vector, we use:

$$V^{\mu}\left(x,t\right)=\sum_{\lambda}\int\frac{d^{3}k}{(2\pi)^{3}\sqrt{2E_{k}}}\left(a_{k,\lambda}\epsilon_{k,\lambda}^{\mu}e^{-ikx}+b_{k,s}^{\dagger}\epsilon_{k,\lambda}^{\mu*}e^{ikx}\right) \tag{4}$$

where the sum runs over the polarizations λ and the $\epsilon_{k,\lambda}^{\mu}$ are the polarization vectors that satisfy:

$$\epsilon_{k,\lambda}^{\mu}\epsilon_{k,\lambda'\,\mu}=-\delta_{\lambda\lambda'}\quad\text{and}\quad k_{\mu}\epsilon_{k,\lambda}^{\mu}=0\,. \tag{5}$$

Also, in order to calculate decay widths, you can make use of the following expression for the decay width [2]:

$$d\Gamma(X\to ab)=\frac{1}{32\pi^{2}}|\mathcal{M}|^{2}\frac{|\vec{p}_{a}|^{2}}{m_{X}^{2}}d\Omega \tag{6}$$

which is valid in the restframe of the particle X. $|\mathcal{M}|^{2}$ refers to the transition matrix squared summed (averaged) over final (initial) state polarization and spins and $\vec{p}_{a}$ and $d\Omega=d\phi_{a}d\cos\theta_{a}$ are the momentum and the solid angle of particle a respectively.

1 Decay of a gauge boson and number of ν_L families

1. Compute the decay width for the process:

$$W^{-}\to e^{-}\bar{\nu}_{e} \tag{7}$$

neglecting electron and the neutrino masses. For that purpose, use the lagrangian for gauge boson-lepton interactions that were obtained in the lectures. You can also use of the following tools:

(a) the sum over the polarization vectors of the on-shell W boson of 4-momentum k is given by $\sum_{\lambda}\epsilon_{k,\lambda}^{\mu}\epsilon_{k,\lambda}^{\nu\,*}=-g^{\mu\nu}+\frac{k^{\mu}k^{\nu}}{m_{W}^{2}}$

(b) tr(any odd nb. of γ's)=0

(c) $\text{tr}\left(\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^\nu\right) = 4\,\left(g^{\mu\nu}g^{\alpha\beta} - g^{\mu\alpha}g^{\nu\beta} + g^{\alpha\nu}g^{\mu\beta}\right)$

(d) $\text{tr}\left(\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^\nu\gamma^5\right) = -4i\,\epsilon^{\alpha\beta\mu\nu}$

(e) $\gamma_\mu^\dagger = \gamma_0\gamma_\mu\gamma_0$ in Dirac and Weyl representations.

2. Compute the decay width for the process:

$$Z \to \bar{\nu}\nu\,. \tag{8}$$

You can use the similitudes between the Lagrangians driving (7) and (8) for a rapid evaluation.
This is of interest because the total decay width of the Z boson can be obtained analyzing the total cross-section for e^+e^- annihilation at the Z pole obtained at LEPI and SLAC. Compare the obtained decay width with the invisible decay width of Z obtained experimentally (see e.g. PDG [2]) and deduce the number of families of active neutrinos in Nature.

3. Check 1a using the properties of the polarization vectors (5) and assuming $\sum_\lambda \epsilon^\mu_{k,\lambda}\epsilon^{\nu\,*}_{k,\lambda} = Ag^{\mu\nu} + Bk^\mu k^\nu$. Also check 1c without making use of a particular choice of representations of the γ matrices and check 1e in the Weyl representation.

2 Decay of the top quark and Goldstone bosons

1. Compute the decay width for the process:

$$t \to W^+ b \tag{9}$$

using the lagrangian for gauge boson-quarks interactions that were obtained in the lectures and assuming that the CKM matrix element $|V_{tb}| \simeq 1$ and neglecting the mass of the bottom quark.

2. Compute the decay width for the process:

$$t \to \phi^+ b\,, \tag{10}$$

where ϕ^+ is the charged Goldstone boson appearing in the decomposition of the Standard Model (SM) scalar doublet (use the top-SM scalar yukawa interactions). Compare to the result obtained in the previous exercise and discuss.

References

[1] Michael E. Peskin and Daniel V. Schroeder. An Introduction to quantum field theory. 1995.

[2] Beringer et al. (Particle Data Group). Review of particle physics. *Phys. Rev. D*, 86:010001, Jul 2012.